CARIBBEAN EXAMINATIONS COUNCIL

CAPE®
Pure Mathematics

SYLLABUS
SPECIMEN PAPER
MARK SCHEME
SUBJECT REPORTS

Macmillan Education
4 Crinan Street, London, N1 9XW
A division of Macmillan Publishers Limited
Companies and representatives throughout the world

www.macmillan-caribbean.com

ISBN 978-0-230-48247-0

First published 2014
This revised version published December 2022

Designed by Macmillan Publishers Limited
Cover design by Macmillan Publishers Limited and Red Giraffe

CAPE® Pure Mathematics Free Resources

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Pure Mathematics

Mathematics is one of the oldest and most universal means of creating, communicating, connecting and applying structural and quantitative ideas. Students doing this syllabus will have already been exposed to Mathematics in some form mainly through courses that emphasise skills in using mathematics as a tool, rather than giving insight into the underlying concepts.

This syllabus will not only provide students with more advanced mathematical ideas, skills and techniques, but encourage them to understand the concepts involved, why and how they "work" and how they are interconnected. It is also to be hoped that, in this way, students will lose the fear associated with having to learn a multiplicity of seemingly unconnected facts, procedures and formulae. In addition, the course should show them that mathematical concepts lend themselves to generalisations, and that there is enormous scope for applications to solving real problems. The course is therefore intended to provide quality in selected areas rather than in a large number of topics.

The syllabus is arranged into two (2) Units, each Unit consists of three Modules.

Unit 1: Algebra, Geometry and Calculus

Module 1 – Basic Algebra and Functions
Module 2 – Trigonometry, Geometry and Vectors
Module 3 – Calculus I

Unit 2: Complex Numbers, Analysis and Matrices

Module 1 – Complex Numbers and Calculus II
Module 2 – Sequences, Series and Approximations
Module 3 – Counting, Matrices and Differential Equations

CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Advanced Proficiency Examination®

SYLLABUS

PURE MATHEMATICS

CXC A6/U2/*21*

Effective for examinations from May–June *2023*

CAPE®

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NOTE TO TEACHERS AND LEARNERS

This document CXC A6/U2/*21* replaces CXC A6/U2/12 issued in 2012.

Please note that the syllabus has been revised and amendments are indicated by italics.

First issued 1999
Revised 2004
Revised 2007
Amended 2012
Revised 2021

Please check the website www.cxc.org for updates on **CXC®**'s syllabuses.

Please access relevant curated resources to support teaching and learning of the syllabus at https://learninghub.cxc.org/

For access to short courses, training opportunities and teacher orientation webinars and workshops go to our Learning Institute at https://cxclearninginstitute.org/

PLEASE NOTE

This icon is used throughout the syllabus to represent key features which teachers and learners may find useful.

Introduction

The Caribbean Advanced Proficiency Examination® (**CAPE®**) is designed to provide certification of the academic, vocational and technical achievement of students in the Caribbean who, having completed a minimum of five years of secondary education, wish to further their studies. The examinations address the skills and knowledge acquired by students under a flexible and articulated system where subjects are organised in 1-Unit or 2-Unit courses with each Unit containing three Modules. Subjects examined under **CAPE®** may be studied concurrently or singly.

The Caribbean Examinations Council offers three types of certification at the **CAPE®** level. The first is the award of a certificate showing each **CAPE®** Unit completed. The second is the **CAPE®** Diploma, awarded to candidates who have satisfactorily completed at least six Units, including Caribbean Studies. The third is the **CXC®** Associate Degree, awarded for the satisfactory completion of a prescribed cluster of *ten* **CAPE®** Units including Caribbean Studies, Communication Studies *and Integrated Mathematics. Integrated Mathematics is not a requirement for the* **CXC®** *Associate Degree in Mathematics.* The complete list of Associate Degrees may be found in the **CXC®** Associate Degree Handbook.

For the **CAPE®** Diploma and the **CXC®** Associate Degree, candidates must complete the cluster of required Units within a maximum period of five years*. To be eligible for a* **CXC®** *Associate Degree, the educational institution presenting the candidates for the award, must select the Associate Degree of choice at the time of registration at the sitting (year) the candidates are expected to qualify for the award.* Candidates will not be awarded an Associate Degree for which they were not registered.

Pure Mathematics Syllabus

◆ RATIONALE

The Caribbean society is an integral part of an ever-changing world. The impact of globalisation on most societies encourages this diverse Caribbean region to revisit the education and career opportunities of our current and future citizens. A common denominator is for Caribbean societies to create among its citizens a plethora of quality leadership with the acumen required to make meaningful projections and innovations for further development. Further, learning appropriate problem-solving techniques, inherent to the study of mathematics, is vital for such leaders. Mathematics promotes intellectual development, is utilitarian and applicable to all disciplines. Additionally, its aesthetics and epistemological approaches provide solutions fit for any purpose. Therefore, Mathematics is the essential tool to empower people with the knowledge, competencies and attitudes that are required for academia as well as quality leadership for sustainability in this dynamic world.

Students doing this syllabus will have already been exposed to Mathematics in some form mainly through courses that emphasise skills in using mathematics as a tool, rather than giving insight into the underlying concepts. *This course of study is designed to utilise learner-centered approaches to teaching, learning and assessment to expand students' mathematical knowledge by exposing them to more advanced mathematical ideas, skills, and techniques, and to enhance their understanding of why and how the concepts are interconnected and can be utilised. This approach will minimise the fear associated* with having to learn a multiplicity of seemingly unconnected facts, procedures and formulae. In addition, *this* course *will* show them that mathematical concepts lend themselves to generalisations, and that there is enormous scope for applications to solving *real-world* problems.

This course of study incorporates the features of the Science, Technology, Engineering, and Mathematics (STEM) principles. On completion of this syllabus, students will be able to make a smooth transition to further studies in Mathematics and other related subject areas or move on to career choices where a deeper knowledge of the general concepts of Mathematics is required. It will enable students to develop and enhance twenty-first century skills including critical and creative thinking, problem solving, logical reasoning, modelling, collaboration, decision making, research, information and communications technological competencies which are integral to everyday life and for life-long learning. Students will be exposed to the underlying concepts of Mathematics to foster a deeper understanding and greater appreciation of the subject. This course provides insight into the exciting world of advanced mathematics, thereby equipping students with the tools necessary to approach any mathematical situation with confidence.

This Syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government. *This person will* demonstrate multiple literacies, independent and critical thinking*;* and *question the beliefs and practices of the past and present bringing this to bear on the* innovative application of science and technology to problem solving. Such a person should also demonstrate a positive work *ethic* and values and displays creative imagination *and entrepreneurship. In keeping with the UNESCO Pillars of Learning, on completion of this course of study, students will learn to know, learn to do, learn to be, learn to live together, and learn to transform themselves and society.*

AIMS

The syllabus aims to:

1. provide understanding of mathematical concepts and structures, their development and the relationships between them;

2. enable the development of skills in the use of mathematical tools;

3. *make Mathematics fun, interesting, and recognizable;*

4. develop an appreciation of the idea of mathematical proof, the internal logical coherence of Mathematics, and its consequent universal applicability;

5. develop the ability to make connections between distinct concepts in Mathematics, and between mathematical ideas and those pertaining to other disciplines;

6. *develop skills such as, critical and creative thinking, problem solving, logical reasoning, modelling, collaboration, decision making, research, information communication and technological competences which are integral to everyday life and for life-long learning;*

7. *develop positive intrinsic mathematical values, such as, accuracy and rigour;*

8. develop the skills of recognising essential aspects of concrete, real-world problems, formulating these problems into relevant and solvable mathematical problems and mathematical modelling;

9. *integrate Information and Communications Technology (ICT) tools and skills in the teaching and learning processes; and,*

10. *prepare* students *for* advanced courses in Mathematics *and related areas*.

SKILLS AND ABILITIES TO BE ASSESSED

The assessment will test candidates' skills and abilities in relation to three cognitive levels.

1. **Conceptual knowledge** - the ability to **recall** and understand appropriate facts, concepts and principles in a variety of contexts.

2. **Algorithmic knowledge** - the ability to **manipulate** mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.

3. **Reasoning** - the ability to **select** appropriate strategy or select, **use** and **evaluate** mathematical models and **interpret** the results of a mathematical solution in terms of a given real-world problem and engage in problem-solving.

PREREQUISITES OF THE SYLLABUS

Any person with a **good** grasp of the contents of the syllabus of *the Caribbean Secondary Education Certificate **(CSEC®)** course in Mathematics*, or equivalent, should be able to undertake the course. *However, persons with a **good** grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate **(CSEC®)** course in Additional Mathematics would be better prepared to pursue this course of study.* Successful participation in the course will also depend on the possession of good verbal and written communication skills.

STRUCTURE OF THE SYLLABUS

The syllabus is arranged into two (2) Units, Unit 1 which will lay the foundation, and Unit 2 which expands on, and applies, the concepts formulated in Unit 1.

It is therefore recommended that Unit 2 be taken after satisfactory completion of Unit 1 or a similar course. Completion of each Unit will be separately certified.

Each Unit consists of three Modules.

Unit 1: Algebra, Geometry and Calculus, contains three Modules each requiring approximately 50 hours. The total teaching time is therefore approximately 150 hours.

Module 1 - Basic Algebra and Functions
Module 2 - Trigonometry, *Coordinate* Geometry and Vectors
Module 3 - Calculus I

Unit 2: Complex Numbers, Analysis and Matrices, contains three Modules, each requiring approximately 50 hours. The total teaching time is therefore approximately 150 hours.

Module 1 - Complex Numbers and Calculus II
Module 2 - Sequences, Series and Approximations
Module 3 - Counting, Matrices and Differential Equations

APPROACHES TO TEACHING THE SYLLABUS

The Specific Objectives indicate the scope of the content and activities that should be covered. Teachers are encouraged to utilise a learner-centered approach to teaching and learning. They are also encouraged to model the process for completing, solving, and calculating mathematical problems. It is recommended that activities to develop these skills be incorporated in every lesson using collaborative, integrative and practical teaching strategies. Note as well that additional notes and the formulae sheet are included in the syllabus.

RECOMMENDED 2-UNIT OPTIONS

1. Pure Mathematics Unit 1 **AND** Pure Mathematics Unit 2.

2. Applied Mathematics Unit 1 **AND** Applied Mathematics Unit 2.

3. Pure Mathematics Unit 1 **AND** Applied Mathematics Unit 2.

◆ MATHEMATICAL MODELLING

Mathematical Modelling should be used in both Units 1 and 2 to solve real-world problems.

A. The topic Mathematical Modelling involves the following steps:

1. identify a real-world situation to which modelling is applicable;
2. carry out the modelling process for a chosen situation to which modelling is applicable; and,
3. discuss and evaluate the findings of a mathematical model in a written report.

B. The Modelling process requires:

1. a clear statement posed in a real-world situation, and identification of its essential features;
2. translation or abstraction of the problem, giving a representation of the essential features of the real-world;
3. solution of the mathematical problem (analytic, numerical, approximate);
4. testing the appropriateness and the accuracy of the solution against behaviour in the real-world; and,
5. refinement of the model as necessary.

C. Consider the two situations given below.

1. A weather forecaster needs to be able to calculate the possible effects of atmospheric pressure changes on temperature.
2. An economic adviser to the Central Bank Governor needs to be able to calculate the likely effect on the employment rate of altering the Central Bank's interest rate.

In each case, people are expected to predict something that is likely to happen in the future. Furthermore, in each instance, these persons may save lives, time, and money or change their actions in some way as a result of their predictions.

One method of predicting is to set up a <u>mathematical model</u> of the situation. A mathematical model is not usually a model in the sense of a scale model motor car. A mathematical model is a way of describing an underlying situation mathematically, perhaps with equations, with rules or with diagrams.

Some examples of mathematical models are:

1. <u>Equations</u>

(a) Business

A recording studio invests \$25 000 to produce a master CD of a singing group. It costs \$50.00 to make each copy from the master and covers the operating expenses. We can model this situation by the equation or mathematical model,

$$C = 50.00\,x + 25\,000$$

where C is the cost of producing x CDs. With this model, one can predict the cost of producing 60 CDs or 6 000 CDs.

Is the equation $x + 2 = 5$ a mathematical model? Justify your answer.

(b) Banking

Suppose you invest \$100.00 with a commercial bank which pays interest at 12% per annum. You may leave the interest in the account to accumulate. The equation $A = 100(1.12)^n$ can be used to model the amount of money in your account after n years.

2. <u>Table of Values</u>

Traffic Management

The table below shows the safe stopping distances for cars recommended by the Highway Code.

Speed m/h	**Thinking Distance m**	**Braking Distance m**	**Overall Stopping Distance m**
20	6	6	12
30	9	14	23
40	12	24	36
50	15	38	53
60	18	55	73
70	21	75	96

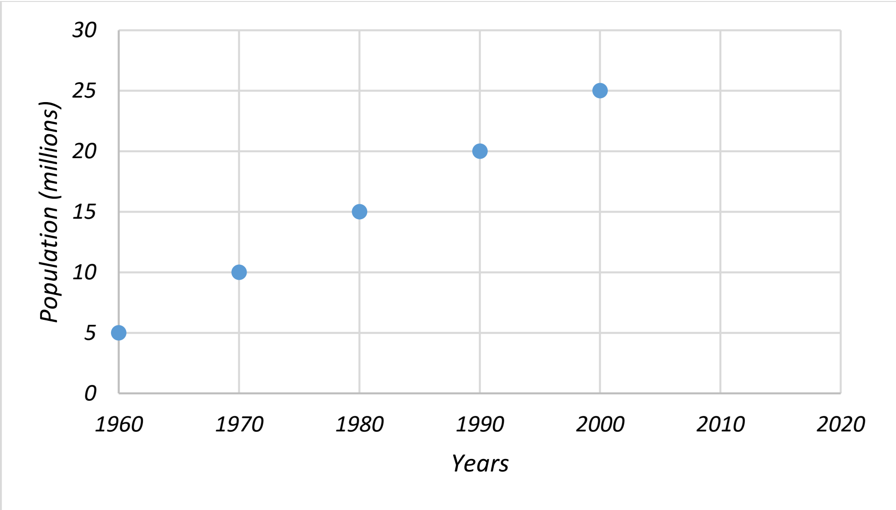
We can predict our stopping distance when travelling at 50 m/h from this model.

3. Rules of Thumb

You might have used some mathematical models of your own without realising it; perhaps you think of them as "rules of thumb". For example, in the baking of hams, most cooks use the rule of thumb, "bake ham fat side up in roasting pan in a moderate oven (160°C) ensuring 25 to 40 minutes per ½ kg". The cook is able to predict how long it takes to bake *the* ham without burning it.

4. Graphs

Not all models are symbolic in nature; they may be graphical. For example, the graph below shows the population at different years for a certain country.



RESOURCE

Hartzler, J. S. and Swetz, F. Mathematical Modelling in the Secondary School Curriculum, A Resource Guide of Classroom Exercises, Vancouver, United States of America: National Council of Teachers of Mathematics, Incorporated, Reston, 1991.

◆ UNIT 1: ALGEBRA, GEOMETRY AND CALCULUS
MODULE 1: BASIC ALGEBRA AND FUNCTIONS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. develop the ability to construct simple proofs of mathematical assertions;

2. understand the concept of a function;

3. be confident in the manipulation of algebraic expressions and the solutions of equations and inequalities;

4. understand the properties and significance of the exponential and logarithm functions; *and,*

5. develop the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES	**CONTENT**
Students should be able to:	
1. Reasoning and Logic	
1.1 identify simple and compound propositions;	*Simple propositions, compound propositions, and connectives (disjunction, negation, conditional, bi-conditional).*
1.2 establish the truth value of compound statements using truth tables;	Truth tables *for compound statements*.
1.3 state the converse, contrapositive and inverse of statements*; and,*	Converse, contrapositive *and inverse* of: (a) *conditional statements; and,* (b) *bi-conditional* statements.
1.4 determine whether two statements are logically equivalent.	Logical equivalence, including *tautology and contradiction.* Identities involving propositions.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
2. The Real Number System – ℝ	
2.1 use the concepts of *arithmetic operations;*	Applications of the concepts of identity, closure, inverse, commutativity, associativity, distributivity addition, multiplication and other simple binary operations. Axioms of the system - including commutative, associative and distributive laws; non-existence of the multiplicative inverse of zero.
2.2 perform binary operations;	*B*inary operation*s.*
2.3 perform *basic* operations involving surds;	*Surds.* *Basic operations (addition, subtraction, multiplication and rationalization) involving surds.*
2.4 use the summation notation ($\sum$); and,	*The summation of number series.* *Interpretation of the summation notation.* *Natural number series.* *Expansion of a series.* $$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$
2.5 *prove mathematical statements.*	*Proof by mathematical induction (addition and divisibility).* *Other m*ethods of proof *include* direct, counter-examples.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
3. Algebraic Operations	
3.1 use the Factor Theorem;	*Using* Factor Theorem to: *(a) find factors; and,* *(b) evaluate unknown coefficients*.
3.2 *use* the Remainder Theorem;	*Using* Remainder *Theorem to find the remainder.*
3.3 extract all factors of $a^n - b^n$ for positive integers n ≤ 6; and,	*Factorisation.*
3.4 use the *order* of polynomial expressions.	*Order of Polynomial expressions.*
4. Functions	
4.1 define mathematical *notations and terms related to functions;*	*Terms: function, ordered pairs, domain, range, one-to-one function (injective function), into function, onto function (surjective function), many-to-one, one-to-one and onto function (bijective function), composition and inverse of functions.*
4.2 *determine whether or not a given simple function is into, onto or one-to-one;* 4.3 *determine if an inverse exists for a function;*	*Functions:* (a) *i*njective; (b) surjective; (c) bijective; and, (d) inverse.
4.4 *determine the inverse of the function f; and,*	*Composite inverse.* If g is the inverse function of f, then $f[g(x)] = x$, for all *x,* in the domain of g.
4.5 *draw graphs to show the relationship between a function and its inverse.*	*Transformation of a graph and its inverse.* *Graphs to show relationship between the function of $y = f(x)$ given in graphical form and the inverse of $f(x)$, that is,* $y = f^{-1}(x)$.

SPECIFIC OBJECTIVES		CONTENT
Students should be able to:		
5.	**Exponential and Logarithmic Functions**	
5.1	*simplify expressions by using laws of logarithms;*	*Laws of logarithms applied to problems:* (a) $\log_c (PQ) = \log_c P + \log_c Q$; (b) $\log_c \left(\frac{P}{Q}\right) = \log_c P - \log_c Q$; *and,* (c) $\log_c P^a = a \log_c P$.
5.2	define an exponential function;	*Properties of the exponential functions.* $y = a^x \; for\, a \in R, x \in R$
5.3	sketch the graph of *exponential functions*;	*Exponential functions and their graphs.* Graphs of the functions. $y = a^x \; for\, x > 0$
5.4	define logarithmic functions;	Properties of the logarithmic functions.
5.5	*sketch the graph of logarithmic functions;*	*Natural logarithmic functions and their graphs* $y = log_a x$
5.6	define *the* exponential function;	*Properties of the* exponential functions. $y = e^x$ and its inverse $y = \ln x$, where $\ln x = \log_e x$
5.7	use the fact that $y = \ln x \Leftrightarrow x = e^y$ *to convert functions;*	*Convert from logarithmic to exponential functions.* *Convert from exponential to logarithmic functions.*
5.8	use logarithms to solve equations of the form $a^x = b$; and,	*Logarithmic solutions.*
5.9	solve problems involving changing of the base of a logarithm.	*Change of base.* $\log_c P = \frac{\log_a P}{\log_a c}$

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
6. The Modulus Function	
6.1 define the modulus function;	Definition of the modulus function. $$\lvert x\rvert = \begin{cases} x \ \ if\ x \geq 0 \\ -x\ if\ x < 0 \end{cases}$$
6.2 use the properties *of the Modulus function;*	*Properties:* (a) $\lvert x\rvert$ is the positive square root of x^2; (b) $\lvert x\rvert < \lvert y\rvert$ if and only if *(iff)* $x^2 < y^2$; and, (c) $\lvert x\rvert < \lvert y\rvert \Leftrightarrow$ iff $-y < x < y$.
6.3 *derive the property of the modulus function;*	*Properties of the modulus function.* $$\lvert x+y\rvert \leq \lvert x\rvert + \lvert y\rvert$$
6.4 *draw graphs to show the relationship between a function and its modulus; and,*	*Transformation of the graphs of:* $$y = f(x) \ to\ y = \lvert f(x)\rvert$$
6.5 solve equations and inequalities involving *simple rational and* modulus functions.	Solving equations and inequalities, using algebraic or graphical methods.
7. Cubic Functions and Equations	
7.1 use the relationship among roots of an equation.	Sum of roots, the product of the roots, the sum of the product of the roots pair-wise and the coefficients of $ax^3 + bx^2 + cx + d = 0$. *Use notation of* $\sum\alpha$, $\sum\alpha\beta$ and $\alpha\beta\gamma$ *for* sum of roots, the sum of the product of the roots pair-wise and the product of the roots respectively.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Teachers are encouraged to incorporate the use of PowerPoint Presentations or YouTube Videos to engage students in a brief review of the number systems.*

2. *Students should be encouraged to practice different methods of proof, for example, to prove that the product of two consecutive integers is an even integer and to disprove by counter-example the statement* $n^2 + n - 1$ *is prime* $\forall\, n \in Z$.

3. *Students should be encouraged to work in groups to solve practice questions, using the information below. Students should* explore mapping properties of quadratic functions which:

   (a) will, or will not, be injective, depending on which subset of the real line is chosen as the domain;

   (b) will be surjective if its range is taken as the co-domain (completion of the square is useful here); and,

   (c) if both injective and surjective, will have an inverse function which can be constructed by solving a quadratic equation.

   Example: Use the function $f : A \to B$ given by $f(x) = 3x^2 + 6x + 5$, where the domain A is alternatively the whole of the real line, or the set $\{x \in \mathbb{R} \mid x \geq -1\}$, and the co-domain B is $\mathbb{R}$ or the set $\{y \in \mathbb{R} \mid y \geq 2\}$.

4. *Students should conduct an internet search on* the theory of the quadratic equation and the nature of its roots. *A class discussion should be initiated, and students asked to creatively present their findings. Teacher can also use the responses as an introduction to the lesson topic.*

5. *Guide students to apply their understanding of reasoning and logic by completing the following activity. Let p be the statement "it is cold" and let q be the statement "it is raining",
Write each of the following statements in symbolic form using p and q.*

   (a) *It is cold and it is raining.*
   (b) *It is cold but it is not raining.*
   (c) *It is false that it is cold, or it is raining.*
   (d) *It is neither cold nor raining.*
   (e) *It is not true that it is not cold or not raining.*

6. *Students should be encouraged to apply their knowledge of logarithmic function to respond to worded problems. For example, students may be given the following statement: A Jeep Wrangler cost $30,788 in 2005 in a certain country. 2 years later the book value on the vehicle was $18,000. Students should be encouraged to work together to:*

   (a) *determine the values of a and k in the exponential model of the book value* $V = ae^{kt}$*; and,*

   (b) *find the book value of the vehicle after 1 year and after 3 years.*

RESOURCES

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UNIT 1
MODULE 2: TRIGONOMETRY, *COORDINATE* GEOMETRY AND VECTORS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. develop the ability to manipulate and describe the behaviour of trigonometric functions;

2. develop the ability to establish trigonometric identities;

3. acquire the skills to solve trigonometric equations;

4. develop the ability to represent and deal with objects in two and three dimensions through the use of coordinate geometry and vectors;

5. acquire the skills to conceptualise and to manipulate objects in two and three dimensions; and,

6. develop the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
1. Trigonometric Functions, Identities and Equations (all angles will be assumed to be in radians unless otherwise stated)	
1.1 *use reciprocal functions of* $\cos x$, $\sin x$ *and* $\tan x$;	*Reciprocal* functions:
1.2 *sketch graphs of reciprocal functions of* $\cos x$, $\sin x$ *and* $\tan x$;	(a) $sec\ x$; (b) $cosec\ x$; *and,* (c) $cot\ x$.
1.3 *derive* compound-angle formulae;	Compound-angle formulae for sin $(A \pm B)$, cos $(A \pm B)$, tan $(A \pm B)$.
1.4 *use compound-angle formulae;*	*Double-angle formulae.*
1.5 derive identities for the *multiple-angle formulae;*	Multiple-angle formulae. sin kA, cos kA, tan kA, for $k \in \mathbb{Q}$.

SPECIFIC OBJECTIVES		CONTENT
Students should be able to:		

Trigonometric Functions, Identities and Equations (all angles will be assumed to be in radians unless otherwise stated) (cont'd)

1.6	*use the factor formulae;*	$\sin A \pm \sin B$, $\cos A \pm \cos B$.
1.7	*prove trigonometric identities;*	Trigonometric identities $\cos^2\theta + \sin^2\theta \equiv 1$, $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$, $1 + \tan^2\theta \equiv \sec^2\theta$.
1.8	*determine* the general solution of *trigonometric* equations;	General solutions of simple trigonometric equations: $\sin(k\theta \pm \alpha) = s$; $\cos(k\theta \pm \alpha) = s$; $\tan(k\theta \pm \alpha) = s$; *and,* where $0 < \alpha < \frac{\pi}{2}$; *k is positive and* $0 \leq \theta \leq 2\pi$.
1.9	*express* $a\cos\theta + b\sin\theta$ *in the forms* $r\sin(\theta \pm \alpha)$ *and* $r\cos(\theta \pm \alpha)$;	Expression of $a\cos\theta + b\sin\theta$ in the forms $r\sin(\theta \pm \alpha)$ and $r\cos(\theta \pm \alpha)$, where r is positive, $0 < \alpha < \frac{\pi}{2}$.
1.10	*determine the general solution for* $a\cos\theta + b\sin\theta = c$, *for* $a, b, c \in \mathbb{R}$;	
1.11	*solve trigonometric equations for a given range; and,*	*Using general solutions of trigonometric equations for a given range.*
1.12	*obtain maximum or minimum values of functions of the form* $a\cos\theta + b\sin\theta$ *for* $0 \leq \theta \leq 2\pi$.	*Using* maximum and minimum values of functions of $\sin\theta$ and $\cos\theta$.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
2. *Coordinate* Geometry	
2.1 *derive equations of circles;*	*Equation of a circle.* *Centre and radius of a circle.* *Graph of a circle.*
2.2 *derive* equations of tangents and normals to *a* circle;	Properties of the circle, tangents and normals.
2.3 *determine* the points of intersection of a curve with a straight line;	Intersections between lines and curves.
2.4 *determine* the points of intersection of two curves;	
2.5 obtain the Cartesian equation of a curve given its parametric representation; *and,*	Cartesian equations of curves. Parametric representations of curves (including the parabola and ellipse).
2.6 *determine the loci of points satisfying given properties.*	*Line equidistant from two given points, circle, distance of a general point from two given points.*
3. Vectors	
3.1 express a vector in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ *or* $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$;	Vectors in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ *where **i**, **j** and **k** are unit vectors in the directions of* x-, y- *and* z-*axis respectively.*
3.2 *perform basic operations on vectors;*	*Equality of vectors.* *Addition of vectors.* *Subtraction of vectors.* *Multiplication of a vector by a scalar quantity.*

SPECIFIC OBJECTIVES		CONTENT
Students should be able to:		
Vectors (cont'd)		
3.3	*calculate* the magnitude and direction of a vector;	*Magnitude* and direction of a vector.
3.4	derive unit vectors, position vectors and displacement vectors;	Position vectors, unit vectors, displacement vectors.
3.5	use unit vectors, position vectors and displacement vectors;	*Application of the position vectors, unit vectors, displacement vectors.*
3.6	*calculate* the angle between two given vectors;	Scalar (Dot) Product.
3.7	*determine the equation of a line with given conditions;*	Equation of a line *in the following forms:* *(a)* *vector;* *(b)* *parametric; and,* *(c)* *cartesian.* *Conditions for equation of a line:* *(a)* *a point on the line and a vector parallel to the line; or,* *(b)* *two points on the line.*
3.8	determine *the relationship between* two lines; and,	*Parallel lines, intersecting lines, skewed lines.*
3.9	*determine* the equation of *a* plane, in the form $ax + by + cz = d$, or $\mathbf{r.n} = d$.	Equation of a plane: *(a)* *given a point in the plane and the normal to the plane; and,* *(b)* *given 3 points in the plane and the normal to the plane (cross product will not be required).*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Teachers are encouraged to engage students in a guided class discussion after demonstrating the process to* master proofs of Trigonometric Identities using identities such as the formulae for: sin $(A \pm B)$, cos $(A \pm B)$, tan $(A \pm B)$, sin $2A$, $\cos 2A$, tan $2A$. *Reinforce by showing that* the identity $\frac{1 - \cos 4\theta}{\sin 4\theta} \equiv \tan 2\theta$ can be established by deducing that $\cos 4\theta \equiv 1 - 2\sin^2 2\theta$ and $\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$.

2. *Students should be engaged in activities requiring them to* derive trigonometric functions sin x and cos x for angles x of any value (including negative values), using the coordinates of points on the unit circle.

3. *Teachers are encouraged to incorporate the use of PowerPoint Presentations or Open Educational Resources such as YouTube videos to i*ntroduce students to the three-dimensional axis and *help them* understand how to plot vectors in three dimensions. *Students should then be engaged in practice activities.*

4. *Teachers are encouraged to incorporate the use of explainer videos to reinforce the concept of Coordinate Geometry. Students should then be encouraged to work in groups to solve worded problems and provide explanations on how they arrived at the answers. For example:*

   The coordinates of the point L, M, and N are (8,-2), (-8, 10) and (14,6) respectively.

   (a) *Use your knowledge of gradients to show that the three points form a right-angle triangle, and state which point has the right angle.*

   (b) *A circle passes through the points L, M, and N. Determine the coordinates of O, the centre of the circle.*

   (c) *A point P lies on the circumference of the circle:*

      (i) determine the coordinates of P if OP = LN; and,

      (ii) describe the shape of OLNP.

UNIT 1
MODULE 2: *TRIGONOMETRY, COORDINATE* GEOMETRY AND VECTORS (cont'd)

RESOURCES

Bahall, D.	*Pure Mathematics Unit 1 for CAPE Examinations. Macmillan Publishers Limited, 2013.*
Bostock, L. and Chandler, S.	*Core Mathematics for Advanced-Levels (4th Ed.).* United Kingdom: *Oxford University Press, 2015.*
Campbell, E.	Pure Mathematics for CAPE, Vol. 1. Jamaica: LMH Publishing Limited, 2007.
Toolsie, Raymond	*Pure Mathematics: A Complete Course for CAPE Unit 1. Trinidad, West Indies: Caribbean Educational Publishers Ltd, 2003.*

Website:

https://www.mathworks.com/products/matlab.html?s_tid=hp_products_matlab

UNIT 1
MODULE 3: CALCULUS I

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *understand the concept of continuity of a function from its graph;*

2. *develop* the ability to *identify and determine* the limits (when they exist) of functions in simple cases;

3. know the relationships between the derivative of a function at a point and the behaviour of the function and its tangent at that point;

4. know the relationship between integration and differentiation;

5. *know the relationship between integration and areas under the curve, and volumes of revolution; and,*

6. *develop* the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
1. Limits	
1.1 use graphs to determine *the limit of a function*;	Concept of limit of a function.
1.2 describe the behaviour of a function $f(x)$;	*Graphs of functions:* *(a) with and without limits; and,* *(b) as x gets arbitrarily close to some given fixed number, using a descriptive approach.*
1.3 use the limit notation;	Limit Theorems: $\lim_{x \to a} f(x) = L,\ f(x) \to L$ as $x \to a$;
1.4 use limit theorems in simple problems;	if $\lim_{x\to a} f(x) = F, \lim_{x\to a} g(x) = G$; and, then $\lim_{x\to a} kf(x) = kF, \lim_{x\to a} f(x)g(x) = FG$.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
Limits (cont'd)	
	where k is a constant $\lim_{x\to a}\{f(x)+g(x)\} = F + G$ and, provided $G \neq 0, \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{F}{G}$.
1.5 use the fact that $\lim_{x\to 0}\frac{\sin x}{x} = 1$; *and,*	*Geometric approach to limits.* *(L'Hopital's rule will not be tested).*
1.6 *determine points of continuity and discontinuity of functions.*	Continuity and Discontinuity. *Left-hand or right-hand limits.* *Using graphs.* *Using algebraic functions.*
2. Differentiation I	
2.1 define the derivative of a function at a point as a limit;	The Derivative as a limit.
2.2 use differentiation notations;	*First derivatives:* $\frac{dy}{dx}$ OR $f'x$.
2.3 differentiate simple functions from first principles;	Gradient *function*, differentiation from first principles. *Functions such as:* (a) $f(x) = k$ where $k \in \mathbb{R}$; (b) $f(x) = x^n$, where $n \in$ {-3, -2, -1, - ½, ½, 1, 2, 3}; (c) $f(x) = \sin x$; and, (d) $f(x) = \cos x$.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
Differentiation I (cont'd)	
2.4 use the *sum*, product and quotient rules *of* differentiation;	Differentiation *of polynomial functions*.
2.5 differentiate sums, products and quotients of trigonometric functions;	*Trigonometric functions.*
2.6 apply the chain rule in differentiation;	*Chain rule of differentiation of:* (a) *composite functions (substitution); and,* (b) *functions given by parametric equations.*
2.7 solve problems involving rates of change;	Rates of change.
2.8 use the sign of the *first* derivative to investigate *when* a function is increasing or decreasing;	*Increasing and decreasing functions (simple polynomial functions).*
2.9 apply the concept of stationary points;	*Stationary (critical) points (maximum, minimum and point of inflection).*
2.10 calculate second derivatives;	Second derivatives of *functions (simple polynomial functions, sine and cosine functions)*.
2.11 interpret the significance of the sign of the second derivative;	
2.12 use the sign of the second derivative to determine the nature of stationary points;	*Second derivative sign test.*
2.13 sketch graphs *of given functions;*	*Curve sketching, including horizontal and vertical asymptotes:* *(a) polynomials;* *(b) rational functions; and,* *(c) trigonometric functions.* The features of the function and its first and second derivatives.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
Differentiation I (cont'd)	
2.14 describe the behaviour of graphs for large values of the independent variable; *and,*	*Graphs of polynomials, rational functions and trigonometric functions.* *Behaviours of functions at their end points.*
2.15 *use differentiation to determine* equations of tangents and normals to curves.	*Application of differentiation to* tangents and *n*ormals to curves*.*
3. Integration I	
3.1 *show* integration as the reverse process of differentiation;	Integration as the reverse of differentiation.
3.2 *use the integration notation* $\int f(x)\mathrm{d}x$*;*	
3.3 *determine the indefinite integral;*	*The indefinite integral including the constant of integration.* *Indefinite integral as a family of functions which differ by a constant.*
3.4 *calculate the constant of integration given certain conditions;*	*Conditions of integration.* *The constant of integration.*
3.5 demonstrate *the* use of integration theorems;	Linearity of integration. (a) $\int cf(x)\,\mathrm{d}x = c\int f(x)\,\mathrm{d}x$, where c is a constant; *and,* (b) $\int\{f(x) \pm g(x)\}\,\mathrm{d}x = \int f(x)\,dx \pm \int g(x)\,dx$

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
Integration I (cont'd)	
3.6 determine *indefinite integrals using integration theorems;*	Polynomial *functions*. Simple trigonometric functions: $sin(ax+b), cos(ax+b)$
3.7 integrate using *given* substitution;	Integration *using the method of* substitution.
3.8 use the results *of definite integrals;*	Definite integrals. (a) $\int_a^b f(x)\,\mathrm{d}x = \int_a^b f(t)\,\mathrm{d}t$; (b) $\int_0^a f(x)\,\mathrm{d}x = \int_0^a f(a-x)\,\mathrm{d}x$ *for* $a >$ 0*; and,* (c) $\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$ where $F'(x) = f(x)$.
3.9 apply integration to *areas under the curve*;	Applications of integration *to* areas: *(a) bounded by the curve and one or more lines*; *and,* *(b) between two curves.*
3.10 calculate volumes of revolutions;	*Applications of integration to volumes of revolution.* *Rotating regions about the x-axis or y-axis and a line.*

SPECIFIC OBJECTIVES

Students should be able to:

Integration I (cont'd)

3.11 formulate a differential equation of the form $y' = f(x)$;

3.12 solve differential equations of the form $y' = f(x)$; and,

3.13 interpret solutions from differential equations of the form $y' = f(x)$.

CONTENTS

Simple first order differential equations of the type $y' = f(x)$, where $f(x)$ is a polynomial or a trigonometric function, with or without initial boundary conditions.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to work in small groups. Each group should be given the following worded problem. Teachers are also encouraged to utilise similar problems to reinforce concepts.*

   A certain chemical substance dissolves in water at a rate proportional to the product of the amount (in grams) undissolved and (1/2 – D), where D is the ratio of the amount (in grams) dissolved to the amount (in grams) of water. When 30 grams of the substance are agitated initially with 100 grams of water, it is discovered that 10 grams of the substance are dissolved after 2 hours.

   Students should be encouraged to discuss the problem and respond to the questions below. Once completed each group can be asked to explain how they arrive at their answer:

   (a) *develop a differential equation showing this information;*

   (b) *solve, completely, this differential equation; and,*

   (c) *calculate the approximate amount of grams of the chemical that would have been dissolved after 5 hours.*

2. *Incorporate the use of real-world problems that require students to analyse data and propose solutions.*

3. *The following statement can be projected or written for students to read, 'A hemispherical bowl of radius r centimetres is being filled with molten lead at a constant rate'. Students should then be encouraged to:*

   (a) *obtain an expression for the volume of lead in the bowl when the depth of the lead in the bowl is h centimetres;*

   (b) *show that between the time when the molten lead is halfway to the top of the bowl, and the time when the bowl is about to overflow, the rate at which the depth is rising has fallen by a quarter (1/4).; and,*

   (c) *recommend why the rate will be reducing.*

   Selected students or groups can be asked to respond to the questions and explain how they arrived at their answer.

4. *Students should be encouraged to work in groups. Each group should be given a topic to guide their research, for example one group can be asked to look at differentiation and another integration. Groups should be encouraged to gather data on definitions, formulas, and the importance of each area of study. Students should also creatively present their findings to the class.*

RESOURCES

Bahall, D.	*Pure Mathematics Unit 1 for CAPE Examinations. Macmillan Publishers Limited, 2013.*
Bostock, L., and Chandler, S.	*Core Mathematics for Advanced-Levels (4th Ed).* United Kingdom: *Oxford University Press, 2015.*
Campbell, E.	*Pure Mathematics for CAPE (Vol. 1). Jamaica: LMH Publishing Limited, 2007.*
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Toolsie, Raymond	*Pure Mathematics: A Complete Course for CAPE Unit 1. Trinidad West Indies: Caribbean Educational Publishers Ltd, 2003.*

UNIT 2: COMPLEX NUMBERS, ANALYSIS AND MATRICES
MODULE 1: COMPLEX NUMBERS AND CALCULUS II

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand *how to write* complex *roots of* quadratic equations;
2. develop the ability to represent objects geometrically through the use of complex numbers;
3. be confident in using the techniques of differentiation and integration; and,
4. develop the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES		CONTENT
Students should be able to:		
1.	**Complex Numbers**	
1.1	express complex numbers in the form a + bi where a and b are real numbers;	*Real and imaginary parts of a complex number.* *Rectangular form of a complex number.*
1.2	*perform the arithmetic processes on complex numbers in the form a + bi, where a and b are real numbers;*	*Addition of complex numbers in the form a + bi where a and b are the real and imaginary parts, respectively, of the complex number.* *Subtraction of complex numbers in the form a + bi where a and b are the real and imaginary parts, respectively, of the complex number.* *Multiplication of complex numbers in the form a + bi where a and b are the real and imaginary parts, respectively, of the complex number.* *Division of complex numbers in the form a + bi where a and b are the real and imaginary parts, respectively, of the complex number.*
1.3	*compute* the roots of the general quadratic equation *using complex numbers;*	Nature of roots of a quadratic equation, *imaginary roots,* sums and products of roots. $ax^2 + bx + c = 0$, when $b^2 - 4ac < 0$

SPECIFIC OBJECTIVES		CONTENT
Students should be able to:		
Complex Numbers (cont'd)		
1.4	use the concept that complex roots of equations with constant coefficients occur in conjugate pairs;	Conjugate pairs of complex roots.
1.5	calculate the square root of a complex number;	*Square root of a complex number.*
1.6	*calculate* the modulus of a given complex number;	The modulus of a complex number.
1.7	*calculate* the principal value of the argument θ of a non-zero complex number where $-\pi < \theta \leq \pi$;	*The argument of a complex number.*
1.8	represent complex numbers on an Argand diagram;	Representation of complex numbers (*their sums, differences, and product)* on an Argand diagram.
1.9	interpret modulus and argument of complex numbers on an Argand diagram;	
1.10	*determine* the locus of *z* on the Argand diagram;	Locus of a point. The set of all points z (locus of *z*) on the Argand diagram such that *z* satisfies given properties. *Description of the locus z satisfying the equations:* *(a)* $\|z-a\| = k$; *(b)* $\|z-a\| = \|z-b\|$; *and,* *(c)* $arg(z-a) = \alpha$ *where a and b are complex numbers, k is a constant and $-\pi < \alpha < \pi$.*
1.11	apply De Moivre's theorem for integral values of n; and,	De Moivre's theorem for integral *values of* n.
1.12	use $e^{ix} = \cos x + i \sin x$, for real x.	Polar-argument and exponential forms of complex numbers.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
2. Differentiation II	
2.1 determine the derivative of $e^{f(x)}$; *2.2 determine the derivative of ln f (x);*	Application of the chain rule to differentiation *of exponential and logarithmic functions where f (x) is a differentiable function of* x *(polynomial or trigonometric).*
2.3 apply the chain rule to *differentiation of* parametric equations;	First derivative of a function which is defined parametrically. Gradients of tangents and normal *of parametric equations*.
2.4 use the concept of implicit differentiation;	Implicit differentiation *with the assumption that one of the variables is a function of the other.*
2.5 differentiate inverse trigonometric functions;	Differentiation of inverse trigonometric functions *(*$\sin^{-1}x$*,* $\cos^{-1}x$*,* $\tan^{-1}x$*).*
2.6 differentiate any combinations of *functions;*	Differentiation of combinations of functions- polynomials, trigonometric, exponential and logarithmic.
2.7 *determine* second derivatives, $f''(x)$; and,	Second derivative *(that is,* $f''(x)$ OR $\frac{d^2y}{dx^2}$*) of the functions in 2.4, 2.5, 2.6).*
2.8 determine the first and second partial derivatives of *u = f(x, y).*	*Partial derivatives and notations.*
3. Integration II	
3.1 *decompose a rational function into a sum of partial fractions;*	*Rational functions (proper and improper) whose denominators are:*
3.2 integrate rational functions;	*(a) distinct linear factors;* *(b) repeated linear factors;* *(c) distinct quadratic factors;* *(d) repeated quadratic factors; and,*

SPECIFIC OBJECTIVES		**CONTENT**
Students should be able to:		
Integration II (cont'd)		
		(e) combinations of (a) to (d) above (repeated factors will not exceed power 2).
3.3	integrate trigonometric functions using appropriate trigonometric identities;	*Trigonometric identities.*
3.4	integrate exponential functions and logarithmic functions;	*Exponential and logarithmic functions.*
3.5	*integrate functions* of the form $\frac{f'(x)}{f(x)}$;	$\int \frac{f'(x)}{f(x)} dx = \ln\left\| f(x) \right\| + C$
3.6	use *appropriate* substitutions to integrate functions;	Integration by substitution. *(the substitution may be given)*
3.7	use integration by parts for combinations of functions;	Integration by parts.
3.8	integrate inverse trigonometric functions;	*Inverse trigonometric functions.*
3.9	*use integration to* derive reduction formulae;	*Reduction formulae.*
3.10	use reduction formulae to *obtain* integrals *of higher power*; and,	Integration *using* reduction formula.
3.11	*use the trapezium rule as a method for approximating the area under the graph of a function.*	Area under the graph of a continuous function (Trapezium Rule).

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Encourage students to calculate the principal argument by either solving:*

   (a) *the simultaneous equations*

   $$\cos\theta = \frac{Re(z)}{|z|} \text{ and } \sin\theta = \frac{Im(z)}{|z|}, \text{ with } -\pi < \theta \leq \pi;$$

   or,

   (b) *the equation*

   $$\tan\theta = \frac{Im(z)}{Re(z)} \text{ for } Re(z) \neq 0 \text{ and } -\pi < \theta \leq \pi;$$

   together with the representation of z on the Argand diagram.

2. *Allow students to find the loci of z-satisfying equations such as:*

   (a) $|z - a| = k$;

   (b) $|z - c| = |z - b|$; and,

   (c) $\arg(z - a) = \alpha$.

3. *Use* $e^{ix} = \cos x + i\sin x$ *in integration of* $\int e^{ax}\cos bx\,dx$ *and* $\int e^{ax}\sin bx\,dx$.

4. *Encourage students to work in groups. Each group should be encouraged to explain how De Moivre's Theorem can be used to evaluate* $(-3 + 3i)^{12}$*. Selected groups should be encouraged to present the results of their discussion.*

RESOURCES

Bahall, D.	*Pure Mathematics Unit 2 for CAPE Examinations. Macmillan Publishers Limited, 2013.*
Bostock, L. and Chandler, S.	*Core Mathematics for Advanced-Levels (4th Ed).* United Kingdom: *Oxford University Press, 2015.*
Bradie, B.	*Rate of Change of Exponential Functions*: A Precalculus Perspective, Mathematics Teacher Vol. 91(3), p. 224 – 237.

Campbell, E.	*Pure Mathematics for CAPE, Vol. 2. Jamaica:* LMH Publishing Limited, 2007.
Caribbean Examinations Council	*The Exponential and Logarithmic Functions – An Investigation. Barbados: Caribbean Examinations Council,* 1998*.*
Martin, A., Brown, K., Rigby, P. and Ridley, S.	Pure Mathematics. *Cheltenham, United Kingdom*: *Stanley Thornes (Publishers) Limited, 2000.*
Toolsie, Raymond	*Pure Mathematics: A Complete Course for CAPE Unit 2. Trinidad West Indies: Caribbean Educational Publishers Ltd, 2003.*

UNIT 2
MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *understand* the concept of a sequence as a function from the natural numbers to the real numbers;

2. *understand* the difference between sequences and series;

3. distinguish between convergence and/or divergence of some standard series or sequences;

4. apply successive approximations to roots of equations; and,

5. *develop* the ability to use concept to model and solve real-world problems.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
1. Sequences	
1.1 *define a sequence* $\{a_n\}$;	Definition of a sequence in *terms of* a_n *where* n *is a positive integer.*
1.2 *use the formula for the* n^{th} *term to write a specific term in the sequence*;	*Sequences defined by recurrence relations.*
1.3 *describe the behaviour of convergent and divergent sequences;*	*Convergent and divergent sequences.* Limit of a sequence. *(test of divergence/convergence is not required)*
1.4 *identify periodic and oscillating sequences; and,*	*Periodic and oscillating sequences.*
1.5 *apply mathematical induction to establish properties of sequences.*	*Using the process of mathematical induction.*

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
2. Series	
2.1 *distinguish between series and sequences;*	*Definition of a series, as the sum of the terms of a sequence.* *Differences between series and sequences.*
2.2 *write the general term of a series, using the summation notation;*	*Arithmetic and Geometric series.* *Making distinctions between arithmetic and geometric series.*
2.3 *calculate* the m^{th} partial sum S_m as the sum of the first m terms of *a given series;*	*Series.* *Use of summation notation* $\sum$. $S_m = \sum_{r=1}^{m} a_r;$
2.4 apply mathematical induction to establish properties of *sequences and* series;	*Further a*pplications of mathematical induction to *sequences and* series.
2.5 find the sum to infinity of a convergent series;	*Convergence and/or divergence of series to which the method of differences can be applied.*
2.6 apply the method of differences to appropriate series, and find their sums;	
2.7 use the Maclaurin theorem for the expansion of series; and,	The Maclaurin series.
2.8 use the Taylor theorem for the expansion of series.	The Taylor series.

SPECIFIC OBJECTIVES	**CONTENT**
Students should be able to:	
3. The Binomial Theorem	
3.1 *use properties of n!* and $\binom{n}{r}$ or ${}^{n}C_{r}$, where $n, r \in Z^{+}$;	Factorials and Binomial coefficients; their interpretation and properties.
3.2 expand $(a + b)^n$ for $n \in \mathbb{Q}$; and,	The Binomial Theorem.
3.3 apply the Binomial Theorem to real world problems.	Applications of the Binomial Theorem.
4. Roots of Equations	
4.1 *use intermediate value theorem to* test for the existence of a root of $f(x) = 0$ where f is continuous;	Intermediate Value Theorem.
4.2 use interval bisection to find an approximation for a root in a given interval;	Interval Bisection.
4.3 use linear interpolation to find an approximation for a root in a given interval;	Linear interpolation.
4.4 explain, in geometrical terms, the working of the Newton-Raphson method;	Newton - Raphson Method *(including failure cases)*.
4.5 use the Newton-Raphson method to compute successive approximations to the roots of $f(x) = 0$, where f is differentiable; and,	
4.6 use a given iteration to determine a root of an equation to a specified degree of accuracy.	*Using the method of* Iteration.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the learning activities listed below.

1. *Teachers are encouraged to use instructional videos to introduce Binomial Theorem. Appropriate examples should be utilised to guide students through the process.*

2. *Teachers are encouraged to use examples to concretize students understanding of the Value Theorem. The following scenario can then be projected and used to guide a discussion,* 'A taxi is travelling at 5 km/h at 8:00 a.m. Fifteen minutes later the speed is 100 km/h. Since the speed varies continuously, clearly at some time between 8:00 a.m. and 8:15 a.m. the taxi was travelling at 75 km/h.' Note that the taxi could have travelled at 75 km/h at more than one time between 8:00 a.m. and 8:15 a.m.

3. *Students should be encouraged to illustrate the Intermediate Value Theorem using examples of continuous functions using the* example: $f\ (x) = x^2 - x - 6$ examined on the intervals (-2.5, -1.5) and (2.5, 3.5).

4. *Guide students in determining an interval in which a real root lies. If* f *(a) and* f (*b*) *are of opposite signs, and* f *is continuous, then a < x < b, for the equation* f (*x*) = 0.

5. *Students should be encouraged to work in small groups to investigate* $x = \frac{a+b}{2}$ *and note the resulting sign to determine which side of* $\frac{a+b}{2}$ *the root lies. Students should be further encouraged to repeat the method until the same answer to the desired degree of accuracy is obtained.*

6. *Teachers are encouraged to engage students in activities on linear interpolation.*

7. *Teachers are encouraged incorporate the use of real-world problems that require students to analyse data and propose solutions.*

RESOURCES

Bahall, D.	*Pure Mathematics Unit 2 for CAPE Examinations. Macmillan Publishers Limited, 2013.*
Bostock, L. and Chandler, S.	*Core Mathematics for Advanced-Levels (4th Ed).* United Kingdom: *Oxford University Press, 2015.*
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Toolsie, Raymond	*Pure Mathematics: A Complete Course for CAPE Unit 2. Trinidad West Indies: Caribbean Educational Publishers Ltd, 2003.*

UNIT 2
MODULE 3: COUNTING, MATRICES AND *DIFFERENTIAL EQUATIONS*

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *develop* the ability to analyse and solve simple problems dealing with choices and arrangements;

2. *develop an understanding of* the algebra of matrices;

3. *develop* the ability to analyse and solve systems of linear equations;

4. *develop skills to model some real-world phenomena by means of differential equations, and solve these; and,*

5. *develop* the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
1. Counting	
1.1 *explain* the principles of counting;	*Factorial notation.* *Fundamental* principles of counting: *(a) addition;* *(b) multiplication;* *(c) permutations; and,* *(d) combinations.*
1.2 *determine the number of ways of arranging n objects;*	*Permutations* with and without repetitions. *Arranging objects:* *(a) in a line or a circle;* *(b) with restrictions;* *(c) without restrictions; and,* *(d) some of which are identical.*
1.3 *determine* the number of ways of choosing r distinct objects from a set of n distinct objects;	*Combinations, with or without restrictions.*

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
Counting (cont'd)	
1.4 use diagrams to *illustrate the sample space*;	*Construction of:* *(a) possibility space diagram;* *(b) venn diagram (no more than three sets); and,* *(c) tree diagram (no more than three branches at the first level and no more than three levels).*
1.5 identify the number of possible outcomes in a given sample space;	*Space diagrams.*
1.6 *define P(A);*	*Concept of probability.*
1.7 *use the fact that* $0 \leq P(A) \leq 1$;	*Basic probability rules and elementary applications.*
1.8 *use the property* $\sum P(x) = 1$ *for all;*	
1.9 *use the property that* $P(A') = 1 - P(A)$, *where* A' *is the event* A *does not occur;*	
1.10 *use the property* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ *for events A and B;*	
1.11 *use the property* $P(A \cap B) = 0$ *or* $P(A \cup B) = P(A) + P(B)$, *where* A *and* B *are mutually exclusive events;*	
1.12 *use the property* $P(A \cap B) = P(A) \times P(B)$, *where A and B are independent events; and,*	
1.13 *use the property* $P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$, *where* $P(B) \neq 0$.	

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
2. **Matrices and Systems of Linear Equations**	
2.1 *determine properties of $m \times n$ matrices, for $1 \leq m \leq 3$, and $1 \leq n \leq 3$;*	*Square matrix.* *Zero matrix.* *Identity matrix.* *Singular matrix.* *Non Singular matrix.* *Equal matrices.* *Associativity.* *Commutativity.* *Distributivity.*
2.2 *perform simple operations with conformable matrices;*	*Addition and subtraction.* *Transposing.* *Multiplication of a matrix by a scalar.* *Multiplication of a matrix by a matrix.*
2.3 *evaluate determinants;*	*Determinants of 2 x 2 or 3 x 3 matrices.* *Cofactors.*
2.4 *invert $n \times n$ matrices;*	*Square matrices.* *Singular and Non-singular matrices.* *Cofactors.* *Adjoint.* *Unit matrix.* *Multiplicative inverse.*

SPECIFIC OBJECTIVES	CONTENT
Students should be able to:	
Matrices and Systems of Linear Equations (cont'd)	
2.5 *express* a system of linear equations *in matrix* form;	*3 x 3 systems of linear equations.*
	Consistency of the systems (number of solutions).
2.6 *use the method of* row-reduction *to bring an* augmented matrix *with 2 or 3 unknowns to echelon form;*	*Solution by row-reduction to echelon form.*
2.7 determine whether the system *(matrix)* is consistent;	*Equivalence of the systems.*
2.8 *evaluate* all solutions of a consistent system; *and,*	*Solution by reduction to echelon form, n = 2, 3.*
2.9 solve a system of linear equations.	*Inverting a matrix.*
	Multiplication of matrices.
	Row reduction of an augmented matrix, *n* = 2, *n* = 3.
	Cramer's rule.
3. **Differential Equations and Modelling**	
3.1 solve first order linear differential equations; *and,*	Formulation and solution of differential equations of the form $y' - ky = f(x)$, where k is a real constant or a function of x, and f is a function.
	Particular solutions of first order linear differential equations.
	Using integrating factor and boundary conditions.

SPECIFIC OBJECTIVES

Students should be able to:

Differential Equations and Modelling (cont'd)

3.2 solve second order ordinary differential equations.

CONTENT

Second order ordinary differential equations with constant coefficients of the form:

1. $ay" + by' + cy = 0$
where $a, b, c \in \mathbb{R}$

The auxiliary equation may consist of:

(a) 2 real and distinct roots;

(b) 2 equal roots; or,

(c) 2 complex roots.

2. $ay" + by' + cy = f(x)$
where $f(x)$ *is:*

(a) a polynomial;

(b) an exponential function; or,

(c) a trigonometric function.

Solution consists of a complementary function (CF) and a particular integral (PI).

3. Using boundary conditions.

4. *Using given substitution to reduce to a suitable form.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

Systems of Linear Equations in Two Unknowns

1. *Students should be encouraged* to plot on graph paper the pair of straight lines represented by a given pair of linear equations in two unknowns, and to examine the relationship between the pair of straight lines in the cases where the system of equations has been shown to have:

(a) one solution;

(b) many solutions; *and,*

(c) no solutions.

2. *Teachers are encouraged to utilise explainer videos and/or PowerPoint presentations to demonstrate to students that g*iven a system of equations with a unique solution, there exist equivalent systems, obtained by row-reduction, having the same solution. *Teachers should then ask students to plot* on the same piece of graph paper all the straight lines represented by the successive pairs of linear equations which result from each of the row operations used to obtain the solution.

3. *Students should be encouraged to work in groups to respond to similar scenarios as seen in the following: 'A police department uses a computer imaging app to create digital photographs of alleged suspects of a crime from eyewitness accounts. One software package contains 180 hairlines, 90 sets of eyes and eyebrows, 75 noses, 100 mouths, and 70 chins and cheek structures.' Students should then be encouraged to provide responses to the following questions with explanations of how they arrived at their answers or solutions.*

(a) *How many different faces can the software package create?*

(b) *The suspect was wearing a face mask and as a result the eyewitness can only describe the hairline, eyes, and eyebrows of the suspect. How many different faces can be produced with this information?*

(c) *Calculate the probability that the digital photograph produced is the actual photograph of the suspect.*

4. *Teachers are encouraged to incorporate the use of real-world problems that require students to analyse data and propose solutions.*

RESOURCES

Bahall, Dipchand.	*Pure Mathematics Unit 2 for CAPE Examinations. Mcmillan Publishers Limited, 2013.*
Bolt, B. and Hobbs, D.	*101 Mathematical Projects: A Resource Book.* United Kingdom: Cambridge University Press, 1994.
Bostock, L. and Chandler, S.	*Core Mathematics for Advanced-Levels (4th Ed.).* United Kingdom: *Oxford University Press, 2015.*
Campbell, E.	*Pure Mathematics for CAPE, Vol. 2. Jamaica:* LMH Publishing Limited, 2007.
Crawshaw, J. and Chambers, J.	*A Concise Course in A-Level Statistics.* Cheltenham, United Kingdom: Stanley Thornes (Publishers) Limited, 1999.
Mann, P.S.	*Introductory Statistics (9th Ed.). New Jersey: Wiley, 2016.*
Toolsie, Raymond	*Pure Mathematics: A Complete Course for CAPE Unit 2. Trinidad West Indies: Caribbean Educational Publishers Ltd, 2003.*

Websites:

https://www.mathworks.com/help/matlab/examples/basic-matrix-operations.html

◆ OUTLINE OF ASSESSMENT

Each Unit of the syllabus is assessed separately. The scheme of assessment for each Unit is the same. A candidate's performance on each Unit is reported as an overall grade and a grade on each Module of the Unit. The assessment comprises two components, one external and one internal.

EXTERNAL ASSESSMENT **(80 per cent)**

The candidate is required to sit two written papers for a total of 4 hrs.

Paper 01 (1 hour 30 minutes)	This paper comprises forty-five, compulsory multiple-choice items, *15 from each module. Each item is worth 1 mark*.	**30 per cent**
Paper 02 (2 hours 30 minutes)	This paper comprises six, compulsory extended-response questions.	**50 per cent**

SCHOOL-BASED ASSESSMENT **(20 per cent)**

School-Based Assessment in respect of each Unit will contribute 20 percent to the total assessment of a candidate's performance on that Unit.

Paper 031 (20 per cent of the Total Assessment)

This paper is intended for candidates registered through schools or other approved institutions.

*The School-Based Assessment comprises a project designed and internally assessed by the teacher and externally moderated by **CXC®**. This paper comprises a single project requiring candidates to demonstrate the practical application of Mathematics in everyday life. In essence, it should allow candidates to probe, describe and explain a mathematical area of interest and communicate the findings using mathematical symbols, language and tools. The topic chosen may be from any module or combination of different modules of the syllabus.*

The project will primarily be theory based requiring the solution of a chosen problem, applying mathematical concepts and procedures from any Module in the syllabus in order to understand, describe or explain a real-world phenomenon. The candidate may include secondary data to support the theoretical concepts.

Paper 032 (Alternative to Paper 031) *(2 hours)*

This paper is an alternative to Paper 031, the School-Based Assessment and is intended for private candidates. This paper comprises three compulsory questions based on any of the modules or combination of different modules of the syllabus. The paper tests skills similar to those assessed in the School-Based Assessment.

MODERATION OF SCHOOL-BASED ASSESSMENT (PAPER *031*)

School-Based Assessment Record Sheets are available online via the **CXC®**'s website www.cxc.org.

All School-Based Assessment Record of marks must be submitted online using the SBA data capture module of the Online Registration System (ORS). *A*ssignments will be requested by **CXC®** for moderation purposes. These assignments will be reassessed by **CXC®** Examiners who moderate the School-Based Assessment. Teachers' marks may be adjusted as a result of moderation. The Examiners' comments will be sent to schools. All *assignments* must be *submitted* by the stipulated deadlines.

Copies of the students' assignments that are not submitted must be retained by the school until three months after publication by **CXC®** of the examination results.

ASSESSMENT DETAILS FOR EACH UNIT

External Assessment by Written Papers (80 per cent of Total Assessment)

Paper 01 (1 hour 30 minutes – 30 per cent of Total Assessment)

1. Composition of the Paper

(a) This paper consists of forty-five multiple-choice items, with fifteen items based on each Module.

(b) All items are compulsory.

2. Syllabus Coverage

(a) Knowledge of the entire syllabus is required.

(b) The paper is designed to test a candidate's knowledge across the breadth of the syllabus.

3. Question Type

Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

(a) Each item is allocated 1 mark.

(b) Each Module is allocated 15 marks.

(c) The total marks available for this paper is 45.

(d) This paper contributes 30 per cent towards the final assessment.

5. Award of Marks

Marks will be awarded for conceptual knowledge, algorithmic knowledge and reasoning.

Conceptual Knowledge: Recall or selection of facts or principles.

Algorithmic Knowledge: Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner; *computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.*

Reasoning: Selection of appropriate strategy, evidence of clear thinking, explanation and/or logical argument.

6. Use of Calculators

(a) Each candidate is required to have a silent, non-programmable calculator for the duration of the examination, and is entirely responsible for its functioning.

(b) The use of calculators with graphical displays will not be permitted.

(c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.

(d) Calculators must not be shared during the examination.

7. Use of Mathematical Tables

A booklet of mathematical formulae *and tables* will be provided.

Paper 02 (2 hours 30 minutes – 50 per cent of Total Assessment)

1. Composition of Paper

(a) The paper consists of six questions. Two questions are based on each Module (Module 1, Module 2 and Module 3).

(b) All questions are compulsory.

2. Syllabus Coverage

(a) Each question may be based on one or more than one topic in the Module from which the question is taken.

(b) Each question may develop a single theme or unconnected themes.

3. Question Type

(a) Questions may require an extended response.

(b) Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

(a) Each question is worth 25 marks.

(b) The number of marks allocated to each sub-question will appear in brackets on the examination paper.

(c) Each Module is allocated 50 marks.

(d) The total marks available for this paper is 150.

(e) This paper contributes 50 per cent towards the final assessment.

5. Award of Marks

(a) Marks will be awarded for conceptual knowledge, algorithmic knowledge and reasoning.

Conceptual Knowledge:	Recall *and understand* facts or principles.
Algorithmic Knowledge:	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner; *computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.*
Reasoning:	Selection of appropriate strategy, evidence of clear thinking, explanation and/or logical argument.

(b) Full marks will be awarded for **correct** answers and presence of **appropriate working**.

(c) Where an incorrect answer is given, credit may be awarded for correct method provided that the working is shown.

(d) If an incorrect answer in a previous question or part-question is used later in a section or a question, then marks may be awarded in the latter part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.

(e) A correct answer given with no indication of the method used (in the form of written working) will receive no marks. Candidates are, therefore, advised to show all relevant working.

6. Use of Calculators

(a) Each candidate is required to have a silent, non-programmable calculator for the duration of the examination, and is responsible for its functioning.

(b) The use of calculators with graphical displays will not be permitted.

(c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.

(d) Calculators must not be shared during the examination.

7. **Use of Mathematical Tables**

A booklet of mathematical formulae *and tables* will be provided.

SCHOOL-BASED ASSESSMENT

School-Based Assessment is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills, and attitudes that are associated with the subject. The activities for the School-Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus. Group work should be encouraged.

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their School-Based Assessment assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of School-Based Assessment. The guidelines provided for the assessment of these assignments are intended to assist teachers in awarding marks that are reliable estimates of the achievement of students in the School-Based Assessment component of the course. In order to ensure that the scores awarded by teachers are in line with the ***CXC®*** *standards, the Council undertakes the moderation of a sample of the School-Based Assessment assignments marked by each teacher.*

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the student at various stages of the experience. This helps to build the self- confidence of students as they proceed with their studies. School-Based Assessment also facilitates the development of the critical skills and abilities emphasised by this ***CAPE®*** *subject and enhance the validity of the examination on which candidate performance is reported. School-Based Assessment, therefore, makes a significant and unique contribution to both the development of relevant skills and the testing and rewarding of students for the development of those skills.*

The Caribbean Examinations Council seeks to ensure that the School-Based Assessment scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

Paper 031 (20 per cent of the Total Assessment)

This paper is intended for candidates registered through schools or other approved institutions.

This paper comprises a project requiring candidates to demonstrate the practical application of Mathematics in everyday life. In essence, it should allow candidates to probe, describe and explain a mathematical area of interest and communicate the findings using mathematical symbols, language and tools.

The topic chosen will be determined largely by the candidate in consultation with the teacher and may be from any module or combination of different modules of the syllabus.

The project is based on applying mathematical concepts, skills and procedures from any topic(s) in order to understand, describe or explain a real world phenomenon. The project is *theory* based and *no data collection is required. However, secondary* data *may be presented to support the theoretical concepts*.

CRITERIA FOR THE SCHOOL-BASED ASSESSMENT (SBA) (Paper 031)

This paper is compulsory and consists of a project. Candidates have the option to work in small groups (maximum 5 members) to complete their SBAs.

1. The aims of the project are to:

(a) promote self-learning;

(b) allow teachers the opportunity to engage in formative assessment of their students;

(c) provide opportunities for all candidates to show, with confidence, that they have mastered the syllabus;

(d) enable candidates to use the methods and procedures of statistical analysis to describe or explain real-life phenomena; and,

(e) foster the development of critical thinking skills among students.

2. Requirements of the project

(a) The project will be presented in the form of a report and should include the following:

(i) a project title;

(ii) a problem statement which provides the purpose of the project;

(iii) identification of important elements of the problem;

(iv) the solution of the problem; and,

(v) discussion of findings.

(b) Teachers are expected to guide candidates in choosing appropriate projects that relate to their interests and mathematical expertise.

(c) Candidates should make use of mathematical skills and theories from any of the Modules.

3. Integration of Project into the Course

(a) The activities related to project work should be integrated into the course so as to enable candidates to learn and practise the skills of undertaking a successful project.

(b) Some time in class should be allocated for general discussion of project work. For example, discussion of which specific objectives should be selected, how the

information should be presented and analysed.

(c) *Class time should also be allocated for discussion between teacher and student, and student and student.*

4. *Management of Project*

(a) ***Planning***

An early start to planning project work is highly recommended and the schedule of the dates for submission should be developed by teachers and candidates.

(b) ***Length***

The project must not exceed 1,500 words. The word count does not include: Tables, References, Table of contents, Appendices and Figures. Two marks will be deducted for exceeding the word limit by more than 200 words.

(c) ***Guidance***

Each candidate should know the requirements of the project and its assessment process.

Although candidates may consult with resource persons besides the teacher the candidates' submission should be their own work.

Candidates are not expected to work on their own. The teacher is expected to give appropriate guidance at all stages of project work, for example, chapters to read, alternative procedures to follow and other sources of information.

(d) ***Authenticity***

Teachers are required to ensure that all projects are the candidates' work.

The recommended procedures are to:

(i) *engage candidates in discussion;*

(ii) *ask candidates to describe procedures used and summarise findings either orally or written; and,*

(iii) *ask candidates to explain specific aspects of the analysis.*

◆ ASSESSMENT *CRITERIA*

The project will be presented in the form of a report and will be assessed on the following aspects:

(a) *project title;*
(b) *problem statement;*
(c) *mathematical formulation;*
(d) *the solution of the problem;*
(e) *discussion of findings; and,*
(f) *overall presentation.*

	Project Descriptors	*Marks*	*Total*
1.	***Project Title***		
	• *Title is clear and concise, and relates to real world problem* *Award **1 mark** for titles presented and not related to real world problems.*	*2*	
			2
2.	***Problem Statement***		
	• *Problem is clearly stated and is appropriate in level of difficulty*	*1*	
	• *Purpose is clearly stated and relates to a real-world problem* *Award **1 mark** for purpose presented and not related to real world problems.*	*2*	
			3
3.	***Mathematical Formulation***		
	• *Identifies all the important elements of the problem and shows a complete understanding of the relationships between them*	*2*	
	• *Shows understanding of the problem's mathematical concepts and principles*	*2*	
	• *Uses appropriate mathematical terminology and notations to model the problem mathematically*	*2*	
	• *Uses appropriate Mathematical methods for the problem/task*	*2*	
			8
4.	***The Problem Solution***		
	• *Assumptions are clearly identified and explained*	*1*	
	• *Proofs are clearly stated*	*1*	
	• *Diagrams are appropriate and clearly labelled*	*2*	
	• *Explanations are sufficient and clearly expressed*	*2*	
	• *Theorems are appropriate and/or Formulae are relevant to the solutions and are correctly applied*	*2*	
	• *Calculations are-accurate without errors*	*2*	
	• *Solutions are clearly stated*	*2*	
	• *Interpretation of results is appropriate given the purpose*	*2*	
	• *Applies the solution or proof to the given real-world problem*	*2*	
			16
5.	***Discussion of Findings***		
	• *Discussion is coherent, concise and relates to the purpose of the project*	*2*	
	• *Recommendations are relevant and practical*	*2*	
	• *Conclusion is succinct, fully reflects the objectives and is supported by evidence*	*2*	
			6
6.	***Overall Presentation***		
	• *Communicates information in a logical way using correct grammar, and appropriate mathematical jargon all the time.*	*2*	
	• *Appropriate citing of sources*	*1*	
	• *Inclusion of bibliography*	*2*	
			5
	Total marks		***40***

Procedures for Reporting and Submitting School-Based Assessment

(i) *Teachers are required to record the mark awarded to each candidate on the mark sheet provided by **CXC®**. The completed mark sheets should be submitted to **CXC®** no later than April 30 of the year of the examination.*

Note: The school is advised to keep a copy of the project for each candidate as well as copies of the mark sheets.

(ii) *Teachers will be required to submit to **CXC®** the projects of candidates according to the guidelines provided. The projects will be re-marked by **CXC®** for moderation purposes.*

Moderation of School Based Assessment
*The candidate's performance on the project will be moderated. The standard and range of marks awarded by the teacher will be adjusted where appropriate. However, the rank order assigned by the teacher will be adjusted only in special circumstances and then only after consideration of the data provided by the sample of marked projects submitted by the teacher and re-marked by **CXC®**.*

Paper 032 (20 per cent of Total Assessment)

1. **Composition of Paper**

   (a) This paper consists of three questions, based on *any of the modules or combination of different modules of the syllabus.*

   (b) All questions are compulsory.

2. **Question Type**

   (a) Each question may require an extended response.

   (b) A part of or an entire question may focus on mathematical modelling.

   (c) A question may be presented using words, symbols, tables, diagrams or a combination of these.

3. **Mark Allocation**

   (a) Each question carries a maximum of 20 marks.

   (b) The Paper carries a maximum of 60 marks.

   (c) For each question, marks should be allocated for the skills outlined on page 3 of this Syllabus.

4. **Award of Marks**

   (a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

For each *question*, the 20 marks *will* be awarded as follows:

Conceptual Knowledge:	Recall *and understand* facts or principles.
Algorithmic Knowledge:	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner. *Computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.*
Reasoning:	Selection of appropriate strategy, evidence of clear reasoning, explanation and/or logical argument.

(b) *Full marks will be awarded for correct answers and presence of appropriate working.*

(c) *Where an incorrect answer is given, credit may be awarded for correct method provided that the working is shown.*

(d) If an incorrect answer in a previous question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.

(e) A correct answer given with no indication of the method used (in the form of written working) will receive no marks. Candidates should be advised to show all relevant working.

5. Use of Calculators

Each candidate is required to have a silent, non-programmable calculator for the duration of the examination, and is responsible for its functioning.

(a) The use of calculators with graphical displays will not be permitted.

(b) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.

(c) Calculators must not be shared during the examination.

(d) Cell phones should not be used as an alternative to calculators.

6. Use of Mathematical Tables

A booklet of mathematical formulae and tables will be provided.

GENERAL GUIDELINES FOR TEACHERS

1. Teachers should note that the reliability of marks awarded is a significant factor in the *School-Based Assessment*, and has far-reaching implications for the candidate's final grade.

2. Candidates who do not fulfil the requirements of the *School-Based Assessment* will be considered absent from the whole examination.

3. Teachers are asked to note the following:

   (a) the relationship between the marks for the assignment and those submitted to ***CXC®*** on the *School-Based Assessment* form should be clearly shown;

   (b) *marks for the Project will be allocated across Modules in the ratio 1:1:1. The project will be marked out of a total of 40 marks. The marks earned by a student are assigned to each Module. For example, if a student earns 35 out of 40 for his School-Based Assessment, 35 marks will be assigned to Module 1, 35 marks to Module 2 and 35 marks to Module 3. The total score will be 35+35+35= 105 out of 120; and,*

   (c) the standard of marking should be consistent.

◆ REGULATIONS FOR PRIVATE CANDIDATES

Candidates who are registered privately will be required to sit Paper 01, Paper 02 and Paper 032. Paper 032 *takes the form of a written examination and* will be *2* hours' duration and will consist of three questions, each worth 20 marks. Each question will be based on the objectives and content of one of the three Modules of the Unit. Paper 032 will contribute 20 per cent of the total assessment of a candidate's performance on that Unit *and* will test the same skills as the School-Based Assessment.

Paper 032 (*2* hour*s*)

The paper consists of three questions. Each question *based on any of the modules or combination of different modules of the syllabus* and tests candidates' skills and abilities to:

1. recall, select and use appropriate facts, concepts and principles in a variety of contexts;

2. manipulate mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences;

3. select and use a simple mathematical model to describe a real-world situation;

4. simplify and solve mathematical models; and,

5. interpret mathematical results and their application in a real-world problem.

◆ REGULATIONS FOR RESIT CANDIDATES

CAPE® *candidates may reuse any moderated SBA score within a two-year period. In order to assist candidates in making decisions about whether or not to reuse a moderated SBA score, the Council will continue to indicate on the preliminary results if a candidate's moderated SBA score is less than 50 per cent in a particular Unit. Candidates re-using SBA scores should register as "Re-sit candidates" and must provide the previous candidate number when registering.*

◆ ASSESSMENT GRID

The Assessment Grid for each Unit contains marks assigned to papers and to Modules and percentage contributions of each paper to total scores.

Units 1 and 2

Papers	Module 1	Module 2	Module 3	Total	(%)
External Assessment Paper 01 (1 hour 30 minutes)	15 (30 weighted)	15 (30 weighted)	15 (30 weighted)	45 (90 weighted)	(30)
Paper 02 (2 hours 30 minutes)	50	50	50	150	(50)
School-Based Assessment Paper 031 or Paper 032 (*2 hours*)	*40* *(20 weighted)*	*40* *(20 weighted)*	*40* *(20 weighted)*	*120* *(60 weighted)*	(20)
Total	100	100	100	300	(100)

◆ MATHEMATICAL NOTATION

Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x: ...\}$	the set of all x such that ...
$n(A)$	the number of elements in set A
$\varnothing$	the empty set
U	the universal set
A'	the complement of the set A
$\mathbb{W}$	the set of whole numbers {0, 1, 2, 3, ...}
$\mathbb{N}$	the set of natural numbers {1, 2, 3, ...}
$\mathbb{Z}$	the set of integers
$\mathbb{Q}$	the set of rational numbers
$\overline{\mathbb{Q}}$	*the set of irrational numbers*
$\mathbb{R}$	the set of real numbers
$\mathbb{C}$	the set of complex numbers
$\subset$	is a *proper* subset of
$\not\subset$	is not a proper subset of
$\subseteq$	*is a subset of*
$\not\subseteq$	is not a subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$

Logic

$\wedge$	conjunction
$\vee$	(inclusive) injunction
$\underline{\vee}$	exclusive disjunction
$\neg$	negation
$\rightarrow$	conditionality
$\leftrightarrow$	bi-conditionality
$\Rightarrow$	implication
$\Leftrightarrow$	equivalence

Miscellaneous Symbols

$\equiv$	is identical to
$\approx$	is approximately equal to
$\propto$	is proportional to
∞	infinity

The following list summarises the notation used in the Mathematics papers of the Caribbean Advanced Proficiency Examination.

Operations

$\sum_{i=1}^{n} x_i$	$x_1 + x_2 + \ldots + x_n$
$\sqrt{x}$	the positive square root of the real number x
$\lvert x \rvert$	the modulus of the real number x
$n!$	n factorial, 1 x 2 x ... x n for $n \in$ ▪ (0! = 1)
${}^nC_r \binom{n}{r}$	the binomial coefficient, $\frac{n!}{(n-r)!r!}$, for $n, r \in$ □ , $0 \leq r \leq n$
${}^nP_r$	$\frac{n!}{(n-r)!}$

Functions

F	the function f
f(x)	the value of the function f at x
f: $A \rightarrow B$	the function f under which each element of the set A has an image in the set B
f: $x \rightarrow y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
Fg	the composite function f(g(x))
$\lim_{x \to a} f(x)$	the limit of f(x) as x tends to a
Δx, δx	an increment of x
$\frac{dy}{dx}, y'$	the *first* derivative of y with respect to x
$\frac{d^n y}{dx^n}, y^{(n)}$	the nth derivative of y with respect to x
$f'(x), f''(x), \cdots, f^{(n)}(x)$	the first, second, ..., nth derivatives of f(x) with respect to x
$\dot{x}, \ddot{x}$	the first and second derivatives of x with respect to time t
E	the exponential constant
ln x	the natural logarithm of x (to base e)
lg x	the logarithm of x to base 10

Complex Numbers

I	$\sqrt{-1}$
Z	a complex number, $z = x + y\text{i}$ where $x, y \in$ R
Re z	the real part of z

Im z	the imaginary part of z
$\|z\|$	the modulus of z
arg z	the argument of z, where $-\pi <$ arg $z \le \pi$
$\bar{z}$, z^*	the complex conjugate of z

Vectors

<u>a</u>, **a**, $\overrightarrow{AB}$	vectors
â	a unit vector in the direction of the vector **a**
$\|a\|$	the magnitude of the vector **a**
a.b	the scalar product of the vectors **a** and **b**
i, j, k	unit vectors in the directions of the positive Cartesian coordinate axes
$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	x**i** + y**j** + z**k**

Probability

S	the sample space
$A, B, \ldots$	the events $A, B, \ldots$
$\mathrm{P}(A')$	the probability that the event A *does not occur*

Matrices

M	a matrix M
(M^{-1})	inverse of the non-singular square matrix M
$\mathrm{M}^T, \mathrm{M}_T$	transpose of the matrix M
det M, $\|\mathrm{M}\|$	determinant of the square matrix M

◆ GLOSSARY OF EXAMINATION TERMS

WORD	DEFINITION	NOTES
Analyse	examine in detail	
Annotate	add a brief note to a label	Simple phrase or a few words only.
Apply	use knowledge/principles to solve problems	Make inferences/conclusions.
Assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
Calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
Classify	divide into groups according to observable characteristics	
Comment	state opinion or view with supporting reasons	
Compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
Construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.
Deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	

WORD	DEFINITION	NOTES
Define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
Demonstrate	show; direct attention to...	
Derive	to deduce, determine or extract from data by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
Describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
Determine	find the value of a physical quantity	
Design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analysed and presented.
Develop	expand or elaborate an idea or argument with supporting reasons	
Diagram	simplified representation showing the relationship between components	
Differentiate/Distinguish (between/among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
Discuss	present reasoned argument; consider points both for and against; explain the relative merits of a case	
Draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.

WORD	DEFINITION	NOTES
Estimate	make an approximate quantitative judgement	
Evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.
Explain	give reasons based on recall; account for	
Find	locate a feature or obtain as from a graph	
Formulate	devise a hypothesis	
Identify	name or point out specific components or features	
Illustrate	show clearly by using appropriate examples or diagrams, sketches	
Interpret	explain the meaning of	
Investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
Justify	explain the correctness of	
Label	add names to identify structures or parts indicated by pointers	
List	itemise without detail	
Measure	take accurate quantitative readings using appropriate instruments	
Name	give only the name of	No additional information is required.
Note	write down observations	

WORD	DEFINITION	NOTES
Observe	pay attention to details which characterise a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
Outline	give basic steps only	
Plan	prepare to conduct an investigation	
Predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	
Record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs, histograms or tables.
Relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
Sketch	make a simple freehand diagram showing relevant proportions and any important details	
State	provide factual information in concise terms outlining explanations	
Suggest	offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalisation which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
Use	apply knowledge/principles to solve problems	Make inferences/conclusions.

◆ GLOSSARY OF MATHEMATICAL TERMS

WORDS	MEANING
Absolute Value	The absolute value of a real number x, denoted by $\lvert x\rvert$, is defined by $\lvert x\rvert = x$ if $x > 0$ and $\lvert x\rvert = -x$ if $x < 0$. For example, $\lvert -4\rvert = 4$.
Algorithm	A process consisting of a specific sequence of operations to solve a certain types of problems. See **Heuristic**.
Argand Diagram	An Argand diagram is a rectangular coordinate system where the complex number $x + \mathrm{i}y$ is represented by the point whose coordinates are x and y. The x-axis is called the real axis and the y-axis is called the imaginary axis.
Argument of a Complex Number	The angle, $\theta = tan^{-1}\left(\frac{y}{x}\right)$, is called the argument of a complex number $z = x + \mathrm{i}y$.
Arithmetic Mean	The average of a set of values found by dividing the sum of the values by the amount of values.
Arithmetic Progression	An arithmetic progression is a sequence of elements, $a_1, a_2, a_3, \ldots,$ such that there is a common difference of successive terms. For example, the sequence {2, 5, 8, 11, 14, ...} has common difference, $d = 3$.
Asymptotes	A straight line is said to be an asymptote of a curve if the curve has the property of becoming and staying arbitrarily close to the line as the distance from the origin increases to infinity.
Augmented Matrix	If a system of linear equations is written in matrix form $Ax = b$, then the matrix $[A\lvert b]$ is called the augmented matrix.
Average	The average of a set of values is the number which represents the usual or typical value in that set. Average is synonymous with measures of central tendency. These include the mean, mode and median.
Axis of symmetry	A line that passes through a figure such that the portion of the figure on one side of the line is the mirror image of the portion on the other side of the line.
Bar Chart	A bar chart is a diagram which is used to represent the frequency of each category of a set of data in such a way that the height of each bar if proportionate to the frequency of the category it represents. Equal space should be left between consecutive bars to indicate it is not a histogram.

WORDS	MEANING
Base	In the equation $y = log_a x$, the quantity a is called the base. The base of a polygon is one of its sides; for example, a side of a triangle. The base of a solid is one of its faces; for example, the flat face of a cylinder. The base of a number system is the number of digits it contains; for example, the base of the binary system is two.
Bias	Bias is systematically misestimating the characteristics of a population (parameters) with the corresponding characteristics of the sample (statistics).
Biased Sample	A biased sample is a sample produced by methods which ensures that the statistics is systematically different from the corresponding parameters.
Bijective	A function is bijective if it is both injective and surjective; that is, both one-to-one, into and unto.
Bimodal	Bimodal refers to a set of data with two equally common modes.
Binomial	An algebraic expression consisting of the sum or difference of two terms. For example, $(ax + b)$ is a binomial.
Binomial Coefficients	The coefficients of the expansion $(x + y)^n$ are called binomial coefficients. For example, the coefficients of $(x + y)^3$ are 1, 3, 3 and 1.
Box-and-whiskers Plot	A box-and-whiskers plot is a diagram which displays the distribution of a set of data using the five number summary. Lines perpendicular to the axis are used to represent the five number summary. Single lines parallel to the *x*-axis are used to connect the lowest and highest values to the first and third quartiles respectively and double lines parallel to the *x*-axis form a box with the inner three values.
Categorical Variable	A categorical variable is a variable measured in terms possession of quality and not in terms of quantity.
Class Intervals	Non-overlapping intervals, which together contain every piece of data in a survey.
Closed Interval	A closed interval is an interval that contains its end points; it is denoted with square brackets $[a, b]$. For example, the interval $[-1,2]$ contains -1 and 2. For contrast see **open interval**.

WORDS	MEANING
Composite Function	A function consisting of two or more functions such that the output of one function is the input of the other function. For example, in the composite function $f(g(x))$ the input of f is g.
Compound Interest	A system of calculating interest on the sum of the initial amount invested together with the interest previously awarded; if A is the initial sum invested in an account and r is the rate of interest per period invested, then the total after n periods is $A(1+r)^n$.
Combinations	The term combinations refers to the number of possible ways of selecting r objects chosen from a total sample of size n if you don't care about the order in which the objects are arranged. Combinations are calculated using the formula ${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$. See **factorial**.
Complex Numbers	A complex number is formed by adding a pure imaginary number to a real number. The general form of a complex number is $z = x + \mathrm{i}y$, where x and y are both real numbers and i is the imaginary unit: $\mathrm{i}^2 = -1$. The number x is called the real part of the complex number, while the number y is called the imaginary part of the complex number.
Conditional Probability	The conditional probability is the probability of the occurrence of one event affecting another event. The conditional probability of event A occurring given that even B has occurred is denoted $\mathrm{P}(\mathrm{A}\mid\mathrm{B})$ (read "probability of A given B"). The formula for conditional probability is $\mathrm{P}(\mathrm{A}\mid\mathrm{B}) = \frac{\mathrm{P}(\mathrm{A}\cap\mathrm{B})}{\mathrm{P}(\mathrm{B})}$
Conjugate of a Complex Number	The conjugate of a complex number $z = x + \mathrm{i}y$ is the complex number $\bar{z} = x - \mathrm{i}y$, found by changing the sign of the imaginary part. For example, if $z = 3 - 4\mathrm{i}$, then $\bar{z} = 3 + 4\mathrm{i}$.
Continuous	The graph of $y = f(x)$ is continuous at a point a if: 1. $f(a)$ exists, 2. $\lim_{x \to a} f(x)$ exists, and 3. $\lim_{x \to a} f(x) = f(a)$. A function is said to be continuous in an interval if it is continuous at each point in the interval.
Continuous Random Variable	A continuous random variable is a random variable that can take on any real number value within a specified range. For contrast, see **Discrete Random Variable.**

WORDS	MEANING
Coterminal	Two angles are said to be coterminal if they have the same initial and terminal arms. For example, $\theta = 30^o$ is coterminal with $\alpha = 390^o$.
Critical Point	A critical point of a function $f(x)$ is the point $P(x, y)$ where the first derivative, $f'(x)$ is zero. See also **stationary points.**
Data	Data (plural of datum) are the facts about something. For example, the height of a building.
Degree	1. The degree is a unit of measure for angles. One degree is $\frac{1}{360}$ of a complete rotation. See also **Radian.** 2. The degree of a polynomial is the highest power of the variable that appears in the polynomial. For example, the polynomial $p(x) = 2 + 3x - x^2 + 7x^3$ has degree 3.
Delta	The Greek capital letter delta, which has the shape of a triangle: Δ, is used to represent "change in". For example Δx represents "change in x".
Dependent Events	In Statistics, two events A and B are said to be dependent if the occurrence of one event affects the probability of the occurrence of the other event. For contrast, see **Independent Events.**
Derivative	The derivative of a function $y = f(x)$ is the rate of change of that function. The notations used for derivative include: $y' = f'(x) = \frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$.
Descriptive Statistics	Descriptive statistics refers to a variety of techniques that allows for general description of the characteristics of the data collected. It also refers to the study of ways to describe data. For example, the mean, median, variance and standard deviation are descriptive statistics. For contrast, see **Inferential Statistics.**
Determinant	The determinant of a matrix is a number that is useful for describing the characteristics of the matrix. For example if the determinant is zero then the matrix has no inverse.
Differentiable	A continuous function is said to be differentiable over an interval if its derivative exists for every point in that interval. That means that the graph of the function is smooth with no kinks, cusps or breaks.
Differential Equation	A differential equation is an equation involving the derivatives of a function of one or more variables. For example, the equation $\frac{\mathrm{d}y}{\mathrm{d}x} - y = 0$ is a differential equation.

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Differentiation	Differentiation is the process of finding the derivative.
Discrete	A set of values are said to be discrete if they are all distinct and separated from each other. For example the set of shoe sizes where the elements of this set can only take on a limited and distinct set of values. See **Discrete Random Variables.**
Discrete Random Variable	A discrete random variable is a random variable that can only take on values from a discrete list. For contrast, see **Continuous Random Variables.**
Estimate	The best guess for an unknown quantity arrived at after considering all the information given in a problem.
Even Function	A function $y = f(x)$ is said to be even if it satisfies the property that $f(x) = f(-x)$. For example, $f(x) = \cos x$ and $g(x) = x^2$ are even functions. For contrast, see Odd Function.
Event	In probability, an event is a set of outcomes of an experiment. For example, the *event* A may be defined as obtaining two heads from tossing a coin twice.
Expected Value	The average amount that is predicted if an experiment is repeated many times. The expected value of a random variable *X* is denoted by $E[X]$. The expected value of a discrete random variable is found by taking the sum of the product of each outcome and its associated probability. In short, $$E[X] = \sum_{i=1}^{n} x_i p(x_i).$$
Experimental Probability	Experimental probability is the *chance* of something happening, based on repeated testing and observing results. It is the ratio of the number of times an event occurred to the number of times tested. For example, to find the experimental probability of winning a game, one must play the game many times, then *divide the number* of games won by the total number of games played.
Exponent	An exponent is a symbol or a number written above and to the right of another number. It indicates the operation of repeated multiplication.
Exponential Function	A function that has the form $y = a^x$, where a is any real number and *a is* called the base.
Extrapolation	An extrapolation is a predicted value that is outside the range of previously observed values. For contrast, see **Interpolation**.
Factor	A factor is one of two or more expressions which are multiplied together. A prime factor is an indecomposable factor. For example, the

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	factors of $(x^2-4)(x+3)$ include (x^2-4) and $(x+3)$, where $(x+3)$ is prime but (x^2-4) is not prime as it can be further decomposed into $(x-2)(x+2)$.
Factorial	The factorial of a positive integer n is the product of all the integers from 1 up to n and is denoted by $n!$, where $1! = 0! = 1$. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
Function	A correspondence in which each member of one set is mapped unto a member of another set.
Geometric Progression	A geometric progression is a sequence of terms obtained by multiplying the previous term by a fixed number which is called the common ratio. A geometric progression is of the form $a, ar, ar^2, ar^3, \ldots$.
Graph	A visual representation of data that displays the relationship among variables, usually cast along x and y axes.
Grouped Data	Grouped data refers to a range of values which are combined together so as to make trends in the data more apparent.
Heterogeneity	Heterogeneity is the state of being of incomparable magnitudes. For contrast, see **Homogeneity.**
Heuristic	A heuristic method of solving problems involve intelligent trial and error. For contrast, see **Algorithm.**
Histogram	A histogram is a bar graph with no spaces between the bars where the area of the bars is proportionate to the corresponding frequencies. If the bars have the same width then the heights are proportionate to the frequencies.
Homogeneity	Homogeneity is the state of being of comparable magnitudes. For contrast, see **Heterogeneity.**
Identity	1. An equation that is true for every possible value of the variables. For example $x^2 - 1 \equiv (x-1)(x+1)$ is an identity while $x^2 - 1 = 3$ is not, as it is only true for the values $x = \pm 2$. 2. The **identity element** of an operation is a number such that when operated on with any other number results in the other number. For example, the identity element under addition of real numbers is zero; the identity element under multiplication of 2×2 matrices is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

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Independent Events	In Statistics, two events are said to be independent if they do not affect each other. That is, the occurrence of one event does not depend on whether or not the other event occurred.
Inferential Statistics	Inferential Statistics is the branch of mathematics which deals with the generalisations of samples to the population of values.
Infinity	The symbol ∞ indicating a limitless quantity. For example, the result of a nonzero number divided by zero is infinity.
Integration	Integration is the process of finding the integral which is the antiderivative of a function.
Interpolation	An interpolation is an estimate of an unknown value which is within the range of previously observed values. For contrast, see **Extrapolation.**
Interval	An interval on a number line is a continuum of points bounded by two limits (end points). An **Open Interval** refers to an interval that excludes the end points and is denoted (a, b). For example, $(0,1)$. A **Closed Interval** in an interval which includes the end points and is denoted $[a, b]$. For example $[-1,3]$. A **Half-Open Interval** is an interval which includes one end point and excludes the other. For example, $[0, \infty)$.
Interval Scale	Interval scale refers to data where the difference between values can be quantified in absolute terms and any zero value is arbitrary. Finding a ratio of data values on this scale gives meaningless results. For example, temperature is measured on the interval scale: the difference between 19^oC and 38^oC is 19^oC, however, 38^oC is not twice as warm as 19^oC and a temperature of 0^oC does not mean there is no temperature. See also Nominal, Ordinal and Ratio scales.
Inverse	1. **The inverse of an element under an operation** is another element which when operated on with the first element results in the identity. For example, the inverse of a real number under addition is the negative of that number. 2. **The inverse of a function** $f(x)$ is another function denoted $f^{-1}(x)$, which is such that $f[f^{-1}(x)] = f^{-1}[f(x)] = x$.
Irrational Number	A number that cannot be represented as an exact ratio of two integers. For example, π or the square root of 2.
Limit	The limit of a function is the value which the dependent variable approaches as the independent variable approaches some fixed value.

WORDS	**MEANING**
Line of Best Fit	The line of best fit is the line that minimises the sum of the squares of the deviations between each point and the line.
Linear Expression	An expression of the form $ax + b$ where x is a variable and a and b are constants, or in more variables, an expression of the form $ax + by + c, ax + by + cz + d$ where a, b, c and d are constants.
Logarithm	A logarithm is the power of another number called the base that is required to make its value a third number. For example 3 is the logarithm which carries 2 to 8. In general, if y is the logarithm which carries a to x, then it is written as $y = \log_a x$ where a is called the base. There are two popular bases: base 10 and base e. 1. **The Common Logarithm (log)**: the equation $y = \log x$ is the shortened form for $y = \log_{10} x$. 2. **The Natural Logarithm (ln)**: The equation $y = \ln x$ is the shortened form for $y = \log_e x$.
Matrix	A rectangular arrangement of numbers in rows and columns.
Method	In Statistics, the research methods are the tools, techniques or processes that we use in our research. These might be, for example, surveys, interviews, or participant observation. Methods and how they are used are shaped by methodology.
Methodology	Methodology is the study of how research is done, how we find out about things, and how knowledge is gained. In other words, methodology is about the principles that guide our research practices. Methodology therefore explains why we're using certain methods or tools in our research.
Modulus	The modulus of a complex number $z = x + iy$ is the real number $\|z\| = \sqrt{x^2 + y^2}$. For example, the modulus of $z = -7 + 24i$ is $\|z\| = \sqrt{(-7)^2 + 24^2} = 25$
Mutually Exclusive Events	Two events are said to be mutually exclusive if they cannot occur simultaneously, in other words, if they have nothing in common. For example, the event "Head" is mutually exclusive to the event "Tail" when a coin is tossed.
Mutually Exhaustive Events	Two events are said to be mutually exhaustive if their union represents the sample space.
Nominal Scale	Nominal scale refers to data which names of the outcome of an experiment. For example, the country of origin of the members of the West Indies cricket team. See also Ordinal, Interval and Ratio scales.

WORDS	**MEANING**
Normal	The normal to a curve is a line which is perpendicular to the tangent to the curve at the point of contact.
Odd Function	A function is an odd function if it satisfies the property that $f(-x) = -f(x)$. For example, $f(x) = \sin x$ and $g(x) = x^3$ are odd functions. For contrast, see **Even Function**.
Ordinal Scale	Data is said be in the ordinal scale if they are names of outcomes where sequential values are assigned to each name. For example, if Daniel is ranked number 3 on the most prolific goal scorer at the Football World Cup, then it indicates that two other players scored more goals than Daniel. However, the difference between the 3rd ranked and the 10th ranked is not necessarily the same as the difference between the 23rd and 30th ranked players. See also **Nominal, Interval and Ratio scales**.
Outlier	An outlier is an observed value that is significantly different from the other observed values.
Parameter	In statistics, a parameter is a value that characterises a population.
Partial Derivative	The partial derivative of $y = f(x_1, x_2, x_3, \ldots, x_n)$ with respect to x_i is the derivative of y with respect to *x*, while all other independent variables are treated as constants. The patrial derivative is denoted by $\frac{\partial f}{\partial x}$. For example, if $f(x, y, z) = 2xy + x^2 z - \frac{3x^3 y}{z}$, then $\frac{\partial f}{\partial x} = 2y + 2xz - \frac{9x^2 y}{z}$
Pascal Triangle	The Pascal triangle is a triangular array of numbers such that each number is the sum of the two numbers above it (one left and one right). The numbers in the *n*th row of the triangle are the coefficients of the binomial expansion $(x + y)^n$.
Percentile	The *p*th *percentile of a list of numbers* is the smallest value such that *p*% of the numbers in the list is below that value. See also **Quartiles.**
Permutations	Permutations refers to the number of different ways of selecting a group of r objects from a set of n *objects* when the order of the elements in the group is of importance and the items are not replaced. If $r = n$ then the permutations is $n!$; if $r < n$ then the number of permutation is ${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$.
Piecewise Continuous Function	A function is said to be piecewise continuous if it can be broken into different segments where each segment is continuous.
Polynomial	A polynomial is an algebraic expression involving a sum of algebraic terms with nonnegative integer powers. *For example,* $2x^3 + 3x^2 - x + 6$ *is a polynomial in one variable.*

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Population	In statistics, a population is the set of all items under consideration.
Principal Root	The principal root of a number is the positive root. For example, the principal square root of 36 is 6 (not -6).
Principal Value	The principal value of the arcsin and arctan functions lies on the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. The principal value of the arcos function lies on the interval $0 \leq x \leq \pi$.
Probability	1. The probability of an event is a measure of how likely it is for an event to occur. The probability of an event is always a number between zero and 1. 2. Probability is the study of chance occurrences.
Probability Distribution	A probability distribution is a table or function that gives all the possible values of a random variable together with their respective probabilities.
Probability Space	The probability space is the set of all outcomes of a probability experiment.
Proportion	1. A relationship between two ratios in which the first ratio is always equal to the second. Usually of the form $\frac{a}{b} = \frac{c}{d}$. 2. The fraction of a part and the whole. If two parts of a whole are in the ratio 2:7, then the corresponding proportions are $\frac{2}{9}$ and $\frac{7}{9}$ respectively.
Pythagorean Triple	A Pythagorean triple refers to three numbers, a, b & c, satisfying the property that $a^2 + b^2 = c^2$.
Quadrant	The four parts of the coordinate plane divided by the *x*- and *y*-axes are called quadrants. Each of these quadrants has a number designation. First quadrant – contains all the points with positive *x* and positive *y* coordinates. Second quadrant – contains all the points with negative *x* and positive *y* coordinates. The third quadrant contains all the points with both coordinates negative. Fourth quadrant – contains all the points with positive *x* and negative y coordinates.
Quadrantal Angles	Quadrantal Angles are the angles measuring $0^o, 90^o, 180^o$ & 270^o and all angles coterminal with these. See **Coterminal.**
Quartic	A quartic equation is a polynomial of degree 4.
Quartiles	Consider a set of numbers arranged in ascending or descending order. The quartiles are the three numbers which divide the set into four parts of equal amount of numbers. The first quartile in a list of

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	numbers is the number such that a quarter of the numbers is below it. The second quartile is the median. The third quartile is the number such that three quarters of the numbers are below it. See also **Percentile.**
Quintic	A quintic equation is a polynomial of degree 5.
Radian	The radian is a unit of measure for angles, where one radian is $\frac{1}{2\pi}$ of a complete rotation. One radian is the angle in a circle subtended by an arc of length equal to that of the radius of the circle. See also **Degrees**.
Radical	The radical symbol (√) is used to indicate the taking of a root of a number. $\sqrt[q]{x}$ means the q^{th} root of x; if $q = 2$ then it is usually written as $\sqrt{x}$. For example $\sqrt[5]{243} = 3$, $\sqrt[4]{16} = 2$. The radical always means to take the positive value. For example, both 5 and -5 satisfy the equation $x^2 = 25$, but $\sqrt{25} = 5$.
Random Variable	A random variable is a variable that takes on a particular value when a random event occurs.
Ratio Scale	Data are said to be on the ratio scale if they can be ranked, the distance between two values can be measured and the zero is absolute, that is, zero means "absence of". See also Nominal, Ordinal and Interval Scales.
Regression	Regression is a statistical technique used for determining the relationship between two quantities.
Residual	In linear regression, the residual refers to the difference between the actual point and the point predicted by the regression line. That is the vertical distance between the two points.
Root	1. The root of an equation is the same as the solution of that equation. For example, if $y = f(x)$, then the roots are the values of x for which $y = 0$. Graphically, the roots are the x-intercepts of the graph. 2. The n^{th} root of a real number x is a number which, when multiplied by itself n times, gives x. If n is odd then there is one root for every value of x; if n is even then there are two roots (one positive and one negative) for positive values of x and no real roots for negative values of x. The positive root is called the **Principal root** and is represented by the radical sign (√). For example, the principal square root of 9 is written as $\sqrt{9} = 3$ but the square roots of 9 are $\pm\sqrt{9} = \pm 3$.
Sample	A group of items chosen from a population.

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Sample Space	The set of all possible outcomes of a probability experiment. Also called probability space.
Sampling Frame	In statistics, the sampling frame refers to the list of cases from which a sample is to be taken.
Scientific Notation	A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (for example, $7000 = 7 \times 10^3$ or $0.0000019 = 1.9 \times 10^{-6}$).
Series	A series is an indicated sum of a sequence.
Sigma	1. The Greek capital letter sigma, Σ, denotes the summation of a set of values. 2. The corresponding lowercase letter sigma, σ, denotes the standard deviation.
Significant Digits	The amount of digits required for calculations or measurements to be close enough to the actual value. Some rules in determining the number of digits considered significant in a number: - The leftmost non-zero digit is the first significant digit. - Zeros between two non-zero digits are significant. - Trailing zeros to the right of the decimal point are considered significant.
Simple Event	A non-decomposable outcome of a probability experiment.
Skew	Skewness is a measure of the asymmetry of a distribution of data.
Square Matrix	A matrix with equal number of rows and columns.
Square Root	The square root of a positive real number n is the number m such that $m^2 = n$. For example, the square roots of 16 are 4 and -4.
Standard Deviation	The standard deviation of a set of numbers is a measure of the average deviation of the set of numbers from their mean.
Stationary Point	The stationary point of a function $f(x)$ is the point $P(x_o, y_o)$ where $f'(x) = 0$. There are three type of stationary points, these are: 1. Maximum point is the stationary point such that $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} < 0$; 2. Minimum point is the stationary point such that $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} > 0$; 3. Point of Inflexion is the stationary point where $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 0$ and the point is neither a maximum nor a minimum point.

WORDS	MEANING
Statistic	A statistic is a quantity calculated from among the set of items in a sample.
Statistical Inference	The process of estimating unobservable characteristics of a population by using information obtained from a sample.
Symmetry	Two points *A* and *B* are symmetric with respect to a line if the line is a perpendicular bisector of the segment *AB*.
Tangent	A line is a tangent to a curve at a point *A* if it just touches the curve at *A* in such a way that it remains on one side of the curve at *A*. A tangent to a circle intersects the circle only once.
Theoretical Probability	The chances of events happening as determined by calculating results that would occur under ideal circumstances. For example, the theoretical probability of rolling a 4 on a fair four-sided die is ¼ or 25%, because there is one chance in four to roll a 4, and under ideal circumstances one out of every four rolls would be a 4.
Trigonometry	The study of triangles. Three trigonometric functions defined for either acute angles in the right-angled triangle are: Sine of the angle *x* is the ratio of the side opposite the angle and the hypotenuse. In short, $\sin x = \frac{O}{H}$; Cosine of the angle *x* is the ratio of the short side adjacent to the angle and the hypotenuse. In short, $\cos x = \frac{A}{H}$; Tangent of the angle *x* is the ratio of the side opposite the angle and the short side adjacent to the angle. In short $\tan x = \frac{O}{A}$.
Z-Score	The *z*-score of a value x is the number of standard deviations it is away from the mean of the set of all values, $z_{score} = \frac{x - \bar{x}}{\sigma}$.

◆ ADDITIONAL NOTES

UNIT 1
MODULE 1: BASIC ALGEBRA AND FUNCTIONS

Proof by Mathematical Induction (MI)

Typical Question

Prove that some formula or statement P_n is true for all positive integers $n \in k$, where k is some positive integer; usually $k = 1$.

Procedure

Step 1: Verify that when $k = 1$: P_n is true for $n = k = 1$. This establishes that P_n is true for $n = 1$.

Step 2: Assume P_n is true for $n = k$, where k is a positive integer > 1. At this point, the statement k replaces n in the statement P_n and is taken as true.

Step 3: Show that P_n is true for $n = k + 1$ using the true statement in step 2 with n replaced by $k + 1$.

Step 4: At the end of step 3, it is stated that statement P_n is true for all positive integers $n \geq k$.

Summary

Proof by MI: For $k > 1$, verify Step 1 for k and proceed through to Step 4.

Observation

Most users of MI do not see how this proves that P_n is true. The reason for this is that there is a massive gap between Steps 3 and 4 which can only be filled by becoming aware that Step 4 only follows because Steps 1 to 3 are repeated an infinity of times to generate the set of all positive integers. The focal point is the few words "for all positive integers $n \geq k$" which points to the determination of the set S of all positive integers for which P is true.

Step 1 says that $1 \in$ S for $k = 1$.
Step 3 says that $k + 1 \in$ S whenever $k \in$ S, so immediately $2 \in$ S since $1 \in$ S.

Iterating on Step 3 says that $3 \in$ S since $2 \in$ S and so on, so that S = {1, 2, 3 ...}, that is, S is the set of all positive integers when $k = 1$ which brings us to Step 4.

When $k > 1$, the procedure starts at a different positive integer, but the execution of steps is the same. Thus, it is necessary to explain what happens between Steps 3 and 4 to obtain a full appreciation of the method.

Example 1: Use Mathematical Induction to prove that $n^3 – n$ is divisible by 3, whenever n is a positive integer.

Solution: Let P_n be the proposition that "$n^3 – n$ is divisible by 3".

Basic Step: P_1 is true, since $1^3 - 1 = 0$ which is divisible by 3.

Inductive Step: Assume P_k is true, $k \in N$, $k > 1$: that is, $k^3 – k$ is divisible by 3.

We must show that P_{k+1} is true, if P_k is true. That is, $(k + 1)^3 – (k + 1)$ is divisible by 3.

$$\text{Now, } (k+1)^3 - (k+1) = (k^3 + 3k^2 + 3k + 1) – (k + 1)$$
$$= (k^3 - k) + 3(k^2 + k)$$

Both terms are divisible by 3 since $(k^3 - k)$ is divisible by 3 by the assumption and $3(k^2 + k)$ is a multiple of 3. Hence, P_{k+1} is true whenever P_k is true. Thus, $n^3 – n$ is divisible by 3 whenever n is a positive integer.

Example 2:

Prove by Mathematical Induction that $7^n - 1$ is divisible by 3 for $\forall\ n \in N$.

Solution:

Let P_n be the statement " $7^n - 1$ is divisible by 3 for $\forall\ n \in N$

For $n = 1$, P is true, for $n = 2$, P is true. Hence $P_n = 3a$, for $a \in N$

Assuming that, P_k is true for $k = n$, where $n > 2$, $P_{k+1} = 3b$ for $b \in N$.

$$P_{k+1} - P_k = 7^{k+1} - 1 - \left(7^k - 1\right) = 7^k\left(7 - 1\right) = 3\left(2\times 7^k\right)$$

`

$$P_{k+1} = 3\left(b - a\right).$$

By the assumption for P_k for $k \in N$ then P_k is true $\forall\ n \in N$

Example 3: Prove by Mathematical Induction that the sum S_n of the first n odd positive integers is n^2.

Solution: Let P_n be the proposition that the sum S_n of the first n odd positive integers is n^2.

Basic Step: For $n = 1$ the first odd positive integer is 1, so $S_1 = 1$, that is: $S_1 = 1 = 1^2$, hence P_1 is true.

Inductive Step: Assume P_k is true, $k \in N$, $k > 1$. That is, $S_n = 1 + 3 + 5 + \ldots +$

$(2k - 1) = k^2$.

Now, $S_{k+1} = 1 + 3 + 5 + \ldots + (2k - 1) + (2k + 1)$
$= [1 + 3 + 5 + \ldots + (2k - 1)] + (2k + 1)$
$= k^2 + (2k + 1)$, by the assumption,
$= (k + 1)^2$

Thus, P_{n+1} is true whenever P_k is true.

Since P_1 is true and $P_{k+1} \rightarrow P_{n+1}$ is true, the proposition P_n is true for all positive integers n.

Example 4: Prove by Mathematical Induction that $\sum_{r=1}^{n} r(r+1) = \frac{n}{3}(n+1)(n+2)$.

Solution: Let the statement $\sum_{r=1}^{n} r(r+1) = \frac{n}{3}(n+1)(n+2)$ be P_n.

For $n = 1$ and $n = 2$, P_n is true.

Assuming that for $k = n$, $P_k = \sum_{r=1}^{k} r(r+1) = \frac{k}{3}(k+1)(k+2)$

$$P_{k+1=} \; P_k + (k+1)(k+2)$$

Adding the $(k + 1)^{\text{th}}$ term gives $= \frac{k}{3}(k+1)(k+2) + (k+1)(k+2)$

$$= \frac{k+1}{3}\left[(k+1)+1\right]\left[(k+2)+1\right]$$

By the assumption for $k = n$ then P_n is true $\forall\, n \in N$.

Functions (Injective, surjective, bijective, *inverse*)

Mathematical proof that a function is one-to-one (injective), onto (surjective) or (one-to-one and onto function) bijective should be introduced at this stage.

UNIT 1
MODULE 3: CALCULUS

Differential Equation

Example:

A particle moves along a path such that at time, *t* secs, it's velocity is given by $-3t^2 + 11t - 6 \text{ ms}^{-1}$.

(a) Find the times at which the particle is momentarily at rest after being observed.

(b) Sketch the graph of *v* on *t*.

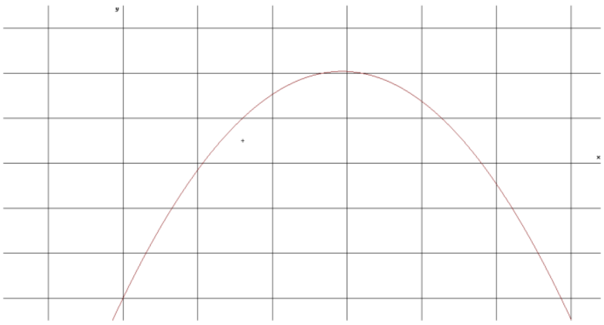
(c) At time 4 seconds after being observed the displacement, *s* metres, of the particle is 17.5 m. Find the displacement of the particle 7 seconds after being observed.

Solution:

(a) The particle is at momentary rest when *v* = 0......i.e. when (3*t* – 2) (*t* – 3) = 0.

$t = \frac{2}{3}\text{sec}, \ 3\text{secs}$

(b)



(c) Given $\frac{\mathrm{d}s}{\mathrm{d}t} = 3t^2 - 11t + 6$ separating the variables gives $\mathrm{d}s = 3t^2 - 11t + 6 \ \mathrm{d}t$

Integrating both sides with the initial conditions gives $\int_{\frac{35}{2}}^{s} \mathrm{d}s = \int_{5}^{7} \left(3t^2 - 11t + 6\right) \mathrm{d}t$

$s - \frac{35}{2} = \left(t^3 - \frac{11}{2}t^2 + 6t\right)_5^7 \qquad s = \frac{231}{2}\text{metres}$

UNIT 2
MODULE 1: COMPLEX NUMBERS AND CALCULUS II

Principal Argument of a Complex Number

The representation of the complex number $z = 1 + \mathrm{i}$ *on the Argand diagram may be used to introduce this topic. Encourage students to indicate and evaluate the argument of z. The students' answers should be displayed on the chalkboard.*

Indicate that the location of z on the Argand diagram is unique, and therefore only one value of the argument is needed to position z. That argument is called the principal argument, arg z, where:

$$-\pi < \textit{principal argument} \leq \pi.$$

De Moivre's theorem to evaluate $\int \mathrm{e}^{ax} \cos bx \, \mathrm{d}x \left(\int \mathrm{e}^{ax} \sin bx \, \mathrm{d}x \right)$ by expressing for example

$$\mathrm{Re} \int \mathrm{e}^{(a+b\mathrm{i})x} \, \mathrm{d}x \left(\mathrm{Im} \int \mathrm{e}^{(a+b\mathrm{i})x} \, \mathrm{d}x \right)$$

$$\mathrm{Re} \int \mathrm{e}^{(a+b\mathrm{i})x} \, \mathrm{d}x = \mathrm{Re}\left(\frac{1}{a+b\mathrm{i}} \mathrm{e}^{(a+b\mathrm{i})x} + C \right) = \frac{a - b\mathrm{i}}{a^2 + b^2} e^{ax} \left(\cos x + i \sin bx \right) + C$$

$$= \frac{a}{a^2 + b^2} - \frac{b\mathrm{i}}{a^2 + b^2} \mathrm{e}^{ax} \left(\cos bx + \mathrm{i} \sin bx \right) + C$$

$$= \frac{\mathrm{e}^{ax}}{a^2 + b^2} \left(a \cos bx - b \cos bx \right) + C$$

The same approach can be used for $\int \mathrm{e}^{ax} \sin bx \, \mathrm{d}x = \mathrm{Im} \int \mathrm{e}^{(a+b\mathrm{i})x} \, \mathrm{d}x$. This is very useful in reducing the repeated integration by parts of say $\int \mathrm{e}^{2x} \cos 4x \, \mathrm{d}x \left(\int \mathrm{e}^{2x} \sin 4x \, \mathrm{d}x \right)$

The Binomial Theorem

Students may be motivated to do this topic by having successive expansions of $(a + x)^n$ and then investigating the coefficients obtained when expansions are carried out.

$$(a + b)^1 = a + b$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + a^4$$

and so on.

By extracting the coefficients of each term made up of powers of a, x or a and x.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Encourage students to use the emerging pattern to generate further expansions of $(a + x)^n$. This can be done by generating the coefficients from Pascal's Triangle and then investigating other patterns. For example, by looking at the powers of a and x (powers of x increase from 0 to n, while powers of a decrease from n to 0; powers of a and x add up to n).

In discussing the need to find a more efficient method of doing the expansions, the Binomial Theorem may be introduced. However, this can only be done after the students are exposed to principles of counting, with particular reference to the process of selecting. In so doing, teachers will need to guide students through appropriate examples involving the selection of r objects, say, from a group of n unlike objects. This activity can lead to defining ${}^nC_r$ as the number of ways of selecting r objects from a group of n unlike objects.

In teaching this principle, enough examples should be presented before ${}^nC_r = \frac{n!}{(n-r)!r!}$ formula is developed.

The binomial theorem may then be established by using the expansion of $(1 + x)^n$ as a starting point. A suggested approach is given below:

Consider $(1 + x)^n$.

To expand, the student is expected to multiply $(1 + x)$ by itself n times, that is,
$(1 + x)^n = (1 + x)(1 + x)(1+ x) \ldots (1 + x)$.

The result of the expansion is found as given below:

The constant term is obtained by multiplying all the 1's. The result is therefore 1.

The term in x is obtained by multiplying $(n-1)$ 1's and one x. This x, however, may be chosen from any of the n brackets. That is, we need to choose one x out of n different brackets. This can be done in ${}^{n}C_1$ ways. Hence, the coefficient of x is ${}^{n}C_1$.

Similarly, the term in x^2 may be obtained by choosing two x's and $(n-2)$ 1's. The x's may be chosen from any two of the n brackets. This can be done in ${}^{n}C_2$ ways. The coefficient of x^2 is therefore ${}^{n}C_2$.

This process continues and the expansion is obtained:

$$(1+x)^n = 1 + {}^{n}C_1 x + {}^{n}C_2 x^2 + {}^{n}C_3 x^3 + \ldots + x^n$$

This is known as the binomial theorem. The theorem may be written as

$$(1+x)^n = \sum_{r=0}^{n} {}^{n}c_r x^r$$

The generalisation of this could be done as a class activity where students are asked to show that:

$$(a+b)^n = a^n + {}^{n}C_1 a^{n-1} b + {}^{n}C_2 a^{n-2} b^2 + {}^{n}C_3 a^{n-3} b^3 + \ldots + b^n$$

This is the binomial expansion of $(a+b)^n$ for positive integral values of n. The expansion terminates after $(n+1)$ terms.

For $n \in \mathbb{Q}$, it should be demonstrated to the student that the notation

$$n! = n(n-1)(n-2)(n-3) \ \ldots \times 1$$

Is applicable to the expansion of

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \frac{n(n-1)\ldots(n-r+1)}{r!} + \ldots$$

where $n \in \mathbb{Q}$.

Existence of Roots

Introduce the existence of the root of a continuous function $f(x)$ between given values a and b as an application of the Intermediate Value Theorem.

Emphasis should be placed on the fact that:

(a) f must be continuous between a and b;

(b) The product of $f(a)$ and $f(b)$ is less than zero, that is, $f(a)$ and $f(b)$ must have opposite signs.

Linear Interpolation

Engage students in the activity below.

Given the points (x_0, y_0) and (x_1, y_1) *on a continuous curve* $y = \mathrm{f}(x)$, *students can establish that for* $f(x_0)$ *and* $f(x_1)$ *with opposite signs and that* f *is continuous, then* $x_0 < x < x_1$, *for the equation* $\mathrm{f}(x) = 0$. *If* $|f(x_0)| < |f(x_1)|$ *say, students can be introduced to the concept of similar triangles to find successive approximations, holding* $f(x_1)$ *constant*. *This intuitive approach is formalised in* ***linear interpolation****, where the two points* (x_0, y_0) and (x_1, y_1) *can be joined by a straight line and the* x*-value of the point on this line is calculated. A first approximation for* x *can be found using*

$$\frac{x_0}{|f(x_0)|} = \frac{x_1}{|f(x_1)|} \; .$$

Successive approximations can be found with this approach until the same answer to the desired degree of accuracy is obtained.

UNIT 2
MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS

Counting

Allow students to consider the three scenarios given below.

(a) Throw two dice. Find the probability that the sum of the dots on the uppermost faces of the dice is 6.

(b) An insurance salesman visits a household. What is the probability that he will be successful in selling a policy?

(c) A hurricane is situated 500km east of Barbados. What is the probability that it will hit the island?

These three scenarios are very different for the calculation of probability. In *(a),* the probability is calculated as the number of successful outcomes divided by the total possible number of outcomes. In this classical approach, the probability assignments are based on equally likely outcomes and the entire sample space is known from the start.

The situation in *(b)* is no longer as well determined as in 'a'. It is necessary to obtain historical data for the salesman in question and estimate the required probability by dividing the number of successful sales by the total number of households visited. This frequency approach still relies on the existence of data and its applications are more realistic than those of the classical methodology.

For *(c)* it is very unclear that a probability can be assigned. Historical data is most likely unavailable or insufficient for the frequency approach. The statistician might have to revert to informed educated guesses. This is quite permissible and reflects the analyst's prior opinion. This approach lends itself to a Bayesian methodology.

One should note that the rules and results of probability theory remain exactly the same regardless of the method used to estimate the probability of events.

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ALGEBRA, GEOMETRY AND CALCULUS

PURE MATHEMATICS

UNIT 1 – Paper 01

90 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 90 minutes to answer them.
2. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
3. Look at the simple item below.

   Sample Item

   The lines $2y - 3x - 13 = 0$ and $y + x + 1 = 0$ intersect at the point

   Sample Answer

   (A) (–3, –2)
   (B) (–3, 2)
   (C) (3, –2)
   (D) (3, 2)

   Ⓐ Ⓑ ● Ⓓ

   The best answer to this item is "(–3, 2)", so answer space (B) has been shaded.
4. You may do any rough work in this booklet.
5. The use of silent, non-programmable calculators is allowed

   Examination Materials Permitted

   A list of mathematical formulae and tables (provided) – **Revised 2010**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Let **p**, **q** and **r** be the propositions
p: Students have a driving licence,
q: Students have a passport,
r: Students have an identification card.

The compund proposition, Students have driving licence or identification card (but not both) together with a passport is expressed a

(A) $((\mathbf{p} \wedge \mathbf{r}) \vee \sim (\mathbf{p} \wedge \mathbf{r})) \wedge \mathbf{q}$
(B) $((\mathbf{p} \vee \mathbf{r}) \vee \sim (\mathbf{p} \wedge \mathbf{r})) \vee \mathbf{q}$
(C) $((\mathbf{p} \vee \mathbf{r}) \vee \sim (\mathbf{p} \vee \mathbf{r})) \wedge \mathbf{q}$
(D) $((\mathbf{p} \vee \mathbf{r}) \wedge \sim (\mathbf{p} \wedge \mathbf{r})) \wedge \mathbf{q}$

2. The counpund proposition $\mathbf{p} \wedge \mathbf{q}$ is true can be illustrated by the truth table

(A)

p	q	p ∧ q
0	0	0
0	1	0
1	0	0
1	1	1

(B)

p	q	p ∧ q
0	0	1
0	1	0
1	0	0
1	1	1

(C)

p	q	p ∧ q
0	0	0
0	1	0
1	0	1
1	1	1

(D)

p	q	p ∧ q
0	0	1
0	1	0
1	0	1
1	1	0

3. The contrapositive for the conditional proposition $\mathbf{p} \rightarrow \mathbf{q}$ is

(A) $\mathbf{q} \rightarrow \mathbf{p}$
(B) $\sim \mathbf{p} \rightarrow \mathbf{q}$
(C) $\sim \mathbf{q} \rightarrow \sim \mathbf{p}$
(D) $\mathbf{p} \rightarrow \sim \mathbf{q}$

4. The proposition $\mathbf{q} \rightarrow \mathbf{p}$ is logically equivalent to

(A) $\sim \mathbf{p} \wedge \sim \mathbf{q}$
(B) $\mathbf{p} \vee \sim \mathbf{q}$
(C) $\sim \mathbf{q} \wedge \mathbf{p}$
(D) $\mathbf{q} \wedge \sim \mathbf{p}$

5. If a remainder of 7 is obtained when $x^3 - 3x + \text{k}$ is divided by $x - 3$, then k equals

(A) -11
(B) -1
(C) 1
(D) 11

6. Give that $x = 3^y$, $y > 0$ then $log_x 3$ is equal to

(A) y

(B) $3y$

(C) $\frac{1}{y}$

(D) $\frac{3}{y}$

GO ON TO THE NEXT PAGE

7. Given that $f(x) = 2 - e^{2x}$, the inverse function, $f^{-1}(x)$, for $x < 2$ *is*

(A) $\ln(2 - x)$

(B) $\ln(2 - 2x)$

(C) $2\ln(2 - x)$

(D) $\frac{1}{2}\ln(2 - x)$

8. Given that $fg(x) = x$, where $g(x) = \frac{2x+1}{3}$, $f(x) =$

(A) $\frac{3x-1}{2}$

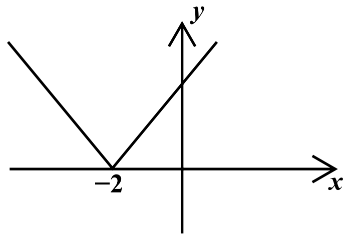
(B) $\frac{3}{2x+1}$

(C) $\frac{2}{3x+1}$

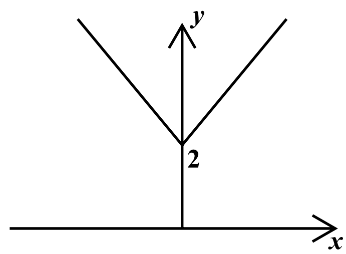
(D) $\frac{3x}{2x+1}$

9. If $f(x) = |x|$, which of the diagrams below represents the graph of $y = f(x) + 2$?

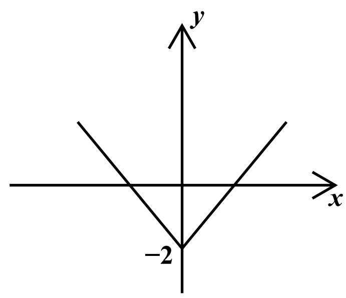
(A)



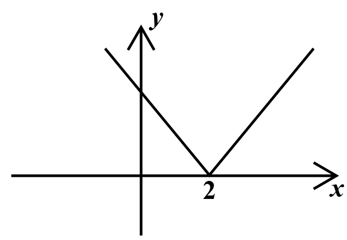
(B)



(C)



(D)



10. If $3^{2x+1} - 4(3^x) + 1 = 0$ then which of the statements below is true?

I. $x = -1$
II. $x = 1$
III. $x = 0$

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only

11. $\dfrac{2\sqrt{3}+3\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ can be simplified correctly to

(A) $\sqrt{6}$

(B) $2\sqrt{5}$

(C) $12+5\sqrt{6}$

(D) $\dfrac{12+5\sqrt{6}}{5}$

12. The cubic equation $2x^3 + x^2 - 22x + 24 = 0$ has roots α, β and y. The value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{y}$ is

(A) $-\dfrac{1}{12}$

(B) $-\dfrac{1}{11}$

(C) $-\dfrac{1}{2}$

(D) $\dfrac{11}{12}$

13. The range (s) of values of x for which $\dfrac{3x+2}{x-1} > 0$ are

(A) $x > -\dfrac{2}{3}, x > 1$

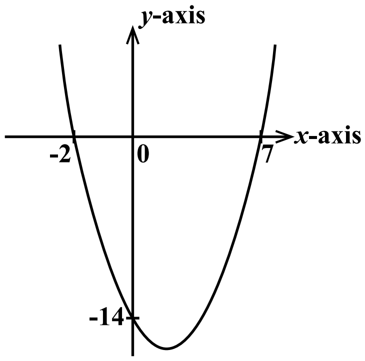
(B) $-\dfrac{2}{3} < x < 1$

(C) $x < \dfrac{2}{3}, x > 1$

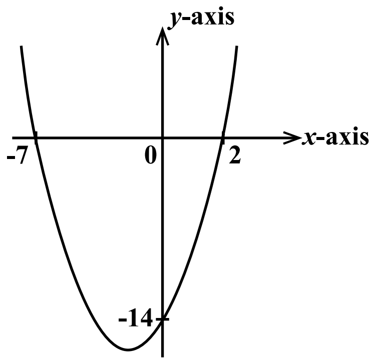
(D) $x < -\dfrac{2}{3}, x > 1$

14. Which one of the graphs below best represents the equation $y = x^2 - 5x - 14$?

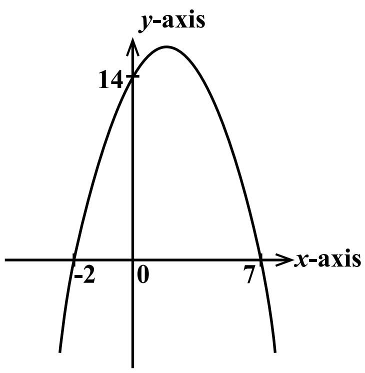
(A)



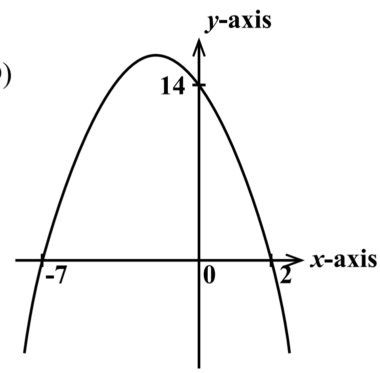
(B)



(C)



(D)



15. The values of x for which $|x + 5| > 3$ are

(A) $x < -8, x < -2$
(B) $x > 0, x < 1$
(C) $x > -2, x < -8$
(D) $x > -2, x > 8$

16. $\dfrac{1+\cot^2\theta}{\sec\theta\operatorname{cosec}\theta} =$

(A) $\tan\theta$
(B) $\cos\theta$
(C) $\cot\theta$
(D) $\operatorname{cosec}\theta$

17. The general solution of the equation $\cos 2\theta = 1$ is

(A) $n\pi + \dfrac{\pi}{4}$

(B) $n\pi$

(C) $n\pi + \dfrac{\pi}{2}$

(D) $\dfrac{(2n+1)\pi}{4}$

18. $\cos\theta + 3\sin\theta = 2$ can be expressed as

(A) $4\cos\left(\theta - \tan^{-1}\left(\dfrac{1}{3}\right)\right) = 2$

(B) $2\cos\left(\theta + \tan^{-1}(3)\right) = 2$

(C) $\sqrt{10}\cos\left(\theta - \tan^{-1}(3)\right) = 2$

(D) $\sqrt{10}\cos\left(\theta + \tan^{-1}\left(\dfrac{1}{3}\right)\right) = 2$

19. If $\cos A = \frac{3}{5}$ and A is acute, then $\sin 2A$ is equal to

(A) $\frac{6}{25}$

(B) $\frac{8}{25}$

(C) $\frac{12}{25}$

(D) $\frac{24}{25}$

20. The maximum value of $\dfrac{1}{2\cos\left(\theta+\frac{\pi}{4}\right)}$ is

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 2

21. A curve C_1 is given by the equation $y = x^2 + 1$, and a curve C_2 is given by the equation $\frac{16}{x^2}+1, x \in \mathbb{R}, x > 0$. The value of x for which $C_1 = C_2$ is

(A) -4
(B) 2
(C) -2
(D) 4

22. The tangent to the circle, C, with equation $x^2 + y^2 + 4x - 10y - 5 = 0$ at the point $P(3, 2)$ has equation

(A) $3x + 5y - 19 = 0$
(B) $5x + 3y + 19 = 0$
(C) $3x - 5y + 19 = 0$
(D) $5x - 3y - 9 = 0$

23. The centre of the circle $(x-3)^2+(y+2)^2=25$ is

(A) $(3, -2)$
(B) $(2, -3)$
(C) $(-3, 2)$
(D) $(-2, 3)$

24. The Cartesian equation of the curve C given by the parametric equations $x = 3\sin\theta - 2$, $y = 4\cos\theta + 3$ is

(A) $x^2 + y^2 = 309$
(B) $9x^2 + 16y^2 = 13$
(C) $(x-3)^2 + 4(y-4)^2 = 36$
(D) $16(x+2)^2 + 9(y-3)^2 = 144$

25. Relative to a fixed origin, O, the position vector of A is $\overrightarrow{\mathbf{OA}} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and the position vector of B is $\overrightarrow{\mathbf{OB}} = 9\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ $\left|\overrightarrow{\mathbf{AB}}\right|$ is

(A) 1 unit

(B) 7 units

(C) $3\sqrt{21}$ units

(D) 49 units

26. Relative to a fixed origin, O, ths point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$, and point B has position vector $(-5\mathbf{i} + 9\mathbf{j} - 5\mathbf{k})$. The line, 1, passes through the points A and B. A vector equation for the line 1 is given by

(A) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-7\mathbf{i} + 6\mathbf{j} - \mathbf{k})$
(B) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-5\mathbf{i} + 9\mathbf{j} - 5\mathbf{k})$
(C) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-3\mathbf{i} + 12\mathbf{j} - 9\mathbf{k})$
(D) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-10\mathbf{i} + 27\mathbf{j} + 20\mathbf{k})$

27. Relative to a fixed origin, O, the line, P_1, has position vector $\begin{pmatrix} 8 \\ 13 \\ -12 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$,

and the line, P_2, has position vector

$\begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$ where λ and μ are scalars.

The cosine of the acute angle between P_1 and P_2 is given by

(A) $\cos\theta = -\frac{2}{3}$

(B) $\cos\theta = \frac{2}{3}$

(C) $\cos\theta = \frac{8}{27}$

(D) $\cos\theta = \frac{2}{27}$

28. Relative to a fixed origin, O, the point A has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point B has position vector $(5\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$. Given that a vector $\mathbf{v}$ is of magnitude $3\sqrt{6}$ units in the direction of $\left|\overline{\mathbf{AB}}\right|$, then $\mathbf{v}$ =

(A) $3\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$
(B) $-3\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$
(C) $-3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$
(D) $3\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$

29. The value that θ, $0 \leq \theta \leq \pi$, which satisfies the equation $2\cos^2\theta + 3\cos\theta - 2 = 0$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

30. If the length of the vector $x = 5\mathbf{i} - (k - 2)\mathbf{j}$ is $\sqrt{34}$ and k is real, then k =

I. 5
II. −1
III. −5

(A) III only
(B) I and II only
(C) II and III only
(D) I, II and III

31. $\lim_{x \to 3} \frac{2x^2 - 5x - 3}{x^2 - 2x - 3}$ is

(A) 0

(B) 1

(C) $\frac{7}{4}$

(D) ∞

32. $\lim_{x \to 0} \frac{\sin x}{\frac{x}{2}}$ is

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

33. Given that $\lim_{x \to -1} \{3f(x) + 2\} = 11$, where *f (x)* is real and continuous, the $\lim_{x \to -1} \{2f(x) + 5x\}$ is

(A) -11
(B) 1
(C) 4
(D) 13

34. Given that $f(x) = (6x + 4) \sin x$, then $f'(x)$ is

(A) $6 \cos x$
(B) $2(3x + 2) \cos x + 6 \sin x$
(C) $6x \cos x + 6 \sin x$
(D) $3 + 2 \sin x + (3x + 2) \cos x$

35. The derivative by first principles of the function $f(x) = \frac{1}{x^2}$ is given by

(A) $\lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

(B) $\lim_{h \to 0} \frac{\frac{1}{(x^2+h)} - \frac{1}{x^2}}{h}$

(C) $\lim_{h \to 0} \frac{\frac{1}{x^2} - \frac{1}{(x^2+h)}}{h}$

(D) $\lim_{h \to 0} \frac{1}{(x+h)^2} - \frac{1}{x^2}$

36. If $y = \cos x - \sin x$, then $\frac{d^2y}{dx^2}$ is

(A) $\cos x + \sin x$
(B) $\cos x - \sin x$
(C) $-\cos x + \sin x$
(D) $-\cos x - \sin x$

37. At $x = 2$, the function $2x^3 - 6x^2 + 5x$

(A) is decreasing
(B) is increasing
(C) has a minimum value
(D) has a maximum value

38. Given $f(x) = (x - 2)(x^3 + 5)$, $f'(x)$ is?

(A) $3x^2$
(B) $-2(3x^2 + 5)$
(C) $4x^3 - 6x^2 + 5$
(D) $x^4 - 2x^3 + 5x - 10$

39. The curve, C, with equation $y = x^3 - 6x^2 + 9x$ has stationary points at $P(3, 0)$ and $Q(1, 4)$. The nature of these stationary points is

(A) $(3, 0)_{\text{max}}$ $(1, 4)_{\text{max}}$
(B) $(3, 0)_{\text{infl}}$ $(1, 4)_{\text{min}}$
(C) $(3, 0)_{\text{infl}}$ $(1, 4)_{\text{max}}$
(D) $(3, 0)_{\text{min}}$ $(1, 4)_{\text{max}}$

40. Given that $\frac{d}{dx}\frac{2x-1}{3x+2} = \frac{7}{(3x+2)^2}$ then

$$\int_1^2 \frac{21}{(3x+2)^2}\,dx =$$

(A) $\left.\frac{3(7)}{(3x+2)^2}\right|_1^2$

(B) $\left.\frac{2x-1}{(3x+2)}\right|_1^2$

(C) $\left.\frac{3(2x-1)}{(3x+2)}\right|_1^2$

(D) $\left.\frac{-21}{(3x+2)}\right|_1^2$

41. The area of the finite region, R, enclosed by the curve $y = x - \frac{1}{\sqrt{x}}$, the lines $x = 1$ and $x = 4$ is

(A) $\frac{9}{2}$

(B) $\frac{11}{2}$

(C) $\frac{27}{4}$

(D) $\frac{19}{2}$

42. The volume (in units^3) generated when the region bounded by the graphs of $y^2 = x + 3$, $x = 0$ and $x = 3$ is rotated through 2π radians about the x-axis is

(A) 63

(B) $\frac{27}{2}$

(C) 63π

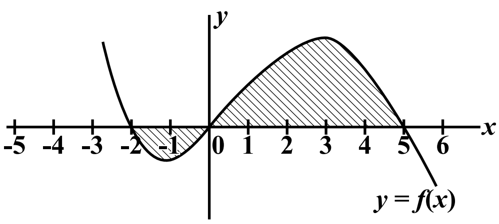
(D) $\frac{27}{2}\pi$

43. Given that $\int_{1}^{3} f(x)\,dx = 8$, then

$\int_{1}^{3} \left[2f(x) - 5\right] dx =$

(A) 6
(B) 11
(C) 13
(D) 21

44. The total shaded area in the diagram below is given by



(A) $\int_{-2}^{5} f(x)\,dx$

(B) $-\int_{-2}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx$

(C) $\int_{-5}^{-2} f(x)\,dx$

(D) $\int_{-2}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx$

45. Given that $\int_{-2}^{0} f(x)\,dx = \frac{16}{3}$ and water is pumped into a large tank at a rate that is proportional to its volume, V, $\int_{-2}^{2} f(x)\,dx = \frac{32}{3}$ where $f(x)$ is a real continuous function in the closed interval $[-2, 2]$, then $\int_{0}^{2} f(x)\,dx =$

(A) $\frac{16}{3}$
(B) 16
(C) $\frac{64}{3}$
(D) 32

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

UNIT 1

PAPER 01

Item	Key	Specific Obj.
1	D	1.1, 1.2
2	A	1.1, 1.2
3	C	1.3
4	B	1.4
5	A	3.2
6	C	5.1
7	D	5.6
8	A	4.2
9	B	6.4
10	C	5.6
11	A	2.3
12	D	7.1
13	D	4.1
14	A	4.5
15	C	6.5

Item	Key	Specific Obj.
16	C	1.1
17	B	1.8
18	C	1.10
19	D	1.5
20	C	1.12
21	B	2.4
22	D	2.2
23	A	2.1
24	D	2.5
25	B	3.3
26	A	3.7
27	A	3.6
28	B	3.4
29	C	1.11
30	B	3.3

Item	Key	Specific Obj.
31	D	1.4
32	D	1.5
33	B	1.4
34	B	2.5
35	A	2.3
36	C	2.10
37	B	2.8
38	C	2.5
39	D	2.12
40	C	3.1
41	B	3.9
42	D	3.10
43	A	3.8
44	B	3.9
45	A	3.8

TEST CODE **02134020**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1

ALGEBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

PAPER 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

<u>Examination Materials Permitted</u>

Mathematical formulae and tables (provided) – **Revised 2022**
Electronic calculator
Ruler and graph paper

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"*"Barcode Area"*"
Sequential Bar Code

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Let **p** and **q** be any two propositions. Complete the truth table below.

p	**q**	**p ∧ q**	**~(p ∧ q)**	**~p**	**~q**	**~p ∨ ~q**
T	T					
T	F					
F	T					
F	F					

[4 marks]

(ii) Hence, state whether the statements **~(p ∧ q)** and **~p ∨ ~q** are logically equivalent. Justify your response.

[2 marks]

(b) Solve the equation $|x^2 - x - 4| = 2$ for all real values of x.

[6 marks]

GO ON TO THE NEXT PAGE

(c) (i) Determine the value(s) of the constants p and q such that $x - 2$ is a common factor of $x^3 - x^2 - 2px + 3q$ and $qx^3 - px^2 + x + 2$.

[10 marks]

(ii) Hence, determine all roots of the equation $x^3 - x^2 - 2px + 3q$.

[3 marks]

Total 25 marks

2. (a) A function f is defined as $f(x) = log_3\,(x + 3)$.

(i) Determine

a) the domain of the function

b) the range of the function

[2 marks]

(ii) Determine the inverse function $f^{-1}(x)$.

[3 marks]

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(iii) On the grid provided below, sketch the graph of $f(x)$ and $f^{-1}(x)$, clearly indicating the intercepts.

[7 marks]

(b) Prove by mathematical induction that $\sum_{r}^{n} = 2\frac{1}{(r-1)r} = 1 - \frac{1}{n}$ is true for all values $n \geq 2$.

[6 marks]

(c) The cubic equation $2x^3 - 4x^2 + 5x - 3 = 0$ has roots α, β and γ.

State the value of:

(i) $\sum \alpha$

[1 mark]

(ii) $\sum \alpha\beta$

[1 mark]

(iii) $\alpha\beta\gamma$

[1 mark]

(iv) Hence, determine the value of $\frac{1}{\alpha^3\beta^3} + \frac{1}{\beta^3\gamma^3} + \frac{1}{\alpha^3\gamma^3}$

[4 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION B

Module 2

Answer BOTH questions.

3. (a) Prove the identity $\dfrac{\sin 3\theta + \sin 5\theta}{\cos 3\theta + \cos 5\theta} = \tan 4\theta$.

[4 marks]

(b) (i) Show that the equation $2 \sin \theta - \cot\theta = 0$ may be written as $2 \cos^2 \theta + \cos \theta - 2 = 0$

[3 marks]

(ii) Hence, determine the general solution for θ.

[4 marks]

(c) (i) Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $R\sin(\theta + a)$ where $R > 0$ and $0 < a < \frac{\pi}{2}$.

[3 marks]

(ii) Hence, determine the maximum value of $f(\theta) = \dfrac{1}{\sqrt{3}\sin\theta + \cos\theta + 3}$.

[5 marks]

(d) Determine whether the lines $l_1 : r = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$ and $l_2 : r = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$ intersect.

[6 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) Prove that the parametric equations $x = 3 \sin \theta + 5$, $y = 2 \cos \theta + 6$ represent the equation of an ellipse.

[3 marks]

(b) (i) B is the point (3, 4). A point P moves so that the fixed distance from point B is $\sqrt{5}$ units. Show that the locus of P is a circle, C, with centre (3, 4).

[3 marks]

(ii) Determine the equation of the tangent and normal to the circle, C, at the point (5, 5).

[4 marks]

(iii) Determine the coordinates of the points of intersection of the circle C and the line $3y = -x + 10$.

[4 marks]

(c) Let $A = (-1, 1, -1)$, $B = (-2, -1, -4)$ and $C = (-4, 2, -5)$.

(i) Show that the vector $\begin{pmatrix} 11 \\ 5 \\ -7 \end{pmatrix}$ is perpendicular to the plane A, B and C.

[5 marks]

(ii) Hence, determine the Cartesian equation of the plane, π_1, through A, B and C.

[3 marks]

(iii) If the angle between two planes is the angle between their normal vectors, determine the angle between π_1 and π_2: $2x - y + 2z = 3$.

[3 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION C

Module 3

Answer BOTH questions.

5. (a) (i) The function *f(x)* is defined as $f(x)=\left(\dfrac{2x^2-4x+5}{5x}+\dfrac{5x-4}{x}\right)$.

Determine $\lim_{x\to 1} f(x)$

[3 marks]

(ii) Differentiate $f(x) = x^3 - 6x$ from first principles.

[5 marks]

GO ON TO THE NEXT PAGE

(b) The function $g(x)$ takes the form $ax^4 + bx^2 + c$. Given that $g''(x) = 36x^2 - 10$, $g(1) = 4$ and $g'(0) = 0$, determine the values of a, b and c.

[7 marks]

(c) A spherical balloon is being inflated by an air pump. The rate of increase of the radius of the balloon is 1.5 cm/s.

(i) Determine the rate at which the volume of the balloon is increasing when the radius is 5 cm, giving your answer in terms of π.

[5 marks]

GO ON TO THE NEXT PAGE

(ii) Given that the maximum volume of the balloon is 2500 cm^3, determine

a) the time it takes to completely inflate the balloon.

[3 marks]

b) the radius of the balloon when it is fully inflated.
Use $\pi = 3.14$

[The volume, *V*, of a sphere of radius *r* is given by $V = \frac{4}{3}\pi r^3$**]**

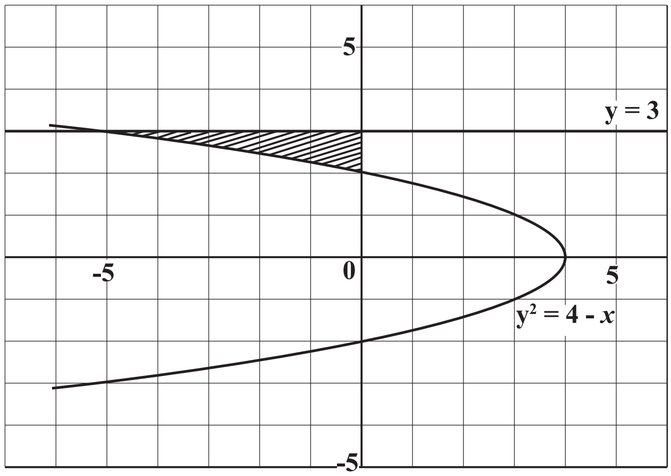
[2 marks]

Total 25 marks

6. (a) Differentiate $y = (2x^3 + 3)\cos(x^2)$ with respect to x.

[4 marks]

(b) The diagram below shows the region bounded by the curve $y^2 = 4 - x$, the line $y = 3$ and the y-axis.



(i) Calculate the area of the shaded region.

[6 marks]

GO ON TO THE NEXT PAGE

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(ii) Calculate the volume of the solid generated when the shaded region is rotated completely about the line $y = 3$, giving your answer in terms of π.

[5 marks]

(c) The function $f(x)$ is defined by $f(x) = x^3 - \frac{7}{2}x^2 + 2x$.

Determine the coordinates and the nature of the stationary points of $f(x)$.

[10 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No. $\boxed{}$

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EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATIONS®

PURE MATHEMATICS

UNIT 1 — Paper 02

KEY AND MARK SCHEME

MAY/JUNE 2022

SPECIMEN PAPER

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

<table>
<tr><th></th><th>Question</th><th>Solutions</th><th>Total</th></tr>
<tr><td>1.</td><td>(a) (i)</td><td>
<table>
<tr><th>p</th><th>q</th><th>p ∧ q</th><th>~(p ∧ q)</th><th>~p</th><th>~q</th><th>~p∨~q</th></tr>
<tr><td>T</td><td>T</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td></tr>
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(1) (1) (1) (1)
either column</td><td>4</td></tr>
<tr><td></td><td>(ii)</td><td>Since ~(p ∧ q) and ~p ∨ ~q have the same values in the truth table, they are logically equivalent.
[2 marks]</td><td>2</td></tr>
<tr><td></td><td>(b)</td><td>$\left|x^2 - x - 4\right| = 2$
Either $x^2 - x - 4 = 2$ OR $x^2 - x - 4 = -2$
[2 marks]

$x^2 - x - 4 = 2$
$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$x = -2,\ 3$ [2 marks]

OR

$x^2 - x - 4 = -2$
$x^2 - x - 6 = -2$
$x^2 - x - 2 = 0$
$(x+1)(x-2) = 0$
$x = -1,\ 2$ [2 marks]</td><td>6</td></tr>
</table>

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total

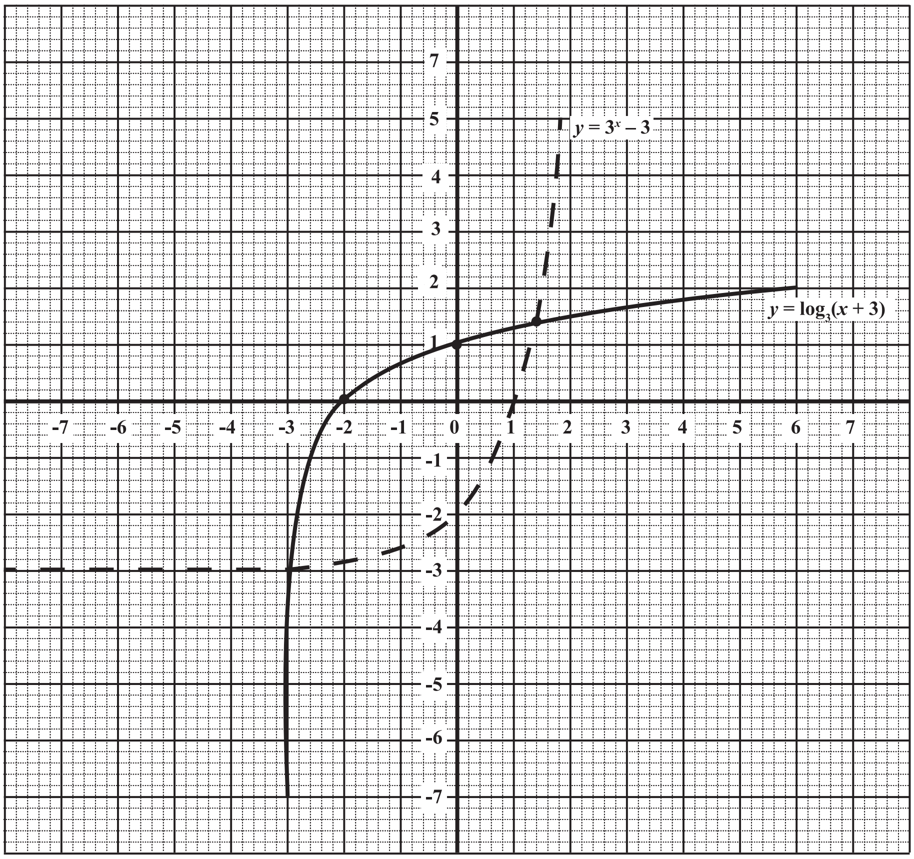
	(c) (i)	If $x-2$ is a factor of $x^3-x^2-2px+3q$ then
		$f(2)=0$ **[1 mark]**
		i.e. $4p-3q=4$ **Equation 1 [2 marks]**
		
		If $x-2$ is a factor of qx^3-px^2+x+2 then
		$g(2)=0$ **[1 mark]**
		i.e. $4p-8q=4$ **Equation 2 [2 marks]**
		
		$4p-3q=4$
		$4p-8q=4$
		
		Recognizing that equations can be solved simultaneously **[1 mark] SOI**
		
		Attempt to solve simultaneous equations **[1 mark]**
		
		Solution
		$p=1,\ q=0$
		[2 marks]
			10

	(ii)	$x-2\overline{)x^3-2x=x^2+x}$ $\therefore x^3-x^2-2x=x(x+1)(x-2)$ $(x+1)(x-2)=0$ $x=-1,\ 0,\ 2$ **[3 marks]**	3
		TOTAL	25
		Specific Objectives: 1.1, 1., 1.4, 3.1, 3.3, 6.1, 6.2, 6.5	

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
2.	(a) (i)	Domain: $x > -3$ or $(-3, \infty)$ **[1 mark]** Range: $(-\infty, \infty)$ **[1 mark]**	2
	(ii)	$f(x) = \log_3(x+3)$ $y = \log_3(x+3)$ $x = \log_3(y+3)$ $3^x = y+3$ $y = 3^x - 3 = f^{-1}(x)$ **[3 marks]**	3
	(iii)	$y = \log_3(x+3)$ When $x = 0$, $y = \log_3 3 = 1$ $y-$intercept $(0,1)$ **[1 mark]** When $y = 0$, $0 = \log_3(x+3)$ $1 = x+3$ $x = -2$ $x-$intercept $(-2,0)$ **[1 mark]** $y = 3^x - 3$ When $x = 0$ $y = 3^0 - 3 = -2$ $y-$intercept $(0,-2)$ **[1 mark]** When $y = 0$ $0 = 3^x - 3$ $3 = 3^x$ $x = 1$ $x-$intercept $(1,0)$ **[1 mark]**	7

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME



Graph showing $y = \log_3(x+3)$ **[1 mark]**

Graph showing $y = 3^x - 3$ **[1 mark]**

Showing all 4 intercepts **[1 mark]**

PURE MATHEMATICS
UNIT 1 — Paper 02
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	Question	Solutions	Total
	(b)	Verifying statement true for n = 1 **[1 mark]** Assume statement true for n = k **[1 mark]** $\sum_{r=2}^{n} \frac{1}{(r-1)r} = 1 - \frac{1}{k}$ $\sum_{r=2}^{k+1} \frac{1}{(r-1)r} = \sum_{r=2}^{k} \frac{1}{(r-1)r} + (k+1)^{th}\ term$ **[2 marks]** $\sum_{r=2}^{k+1} \frac{1}{(r-1)r} = \left(1 - \frac{1}{k}\right) + \frac{1}{k(k+1)}$ **[1 mark]** Proving the statement true for $n = k + 1$ $\sum_{r=2}^{k+1} \frac{1}{(r-1)r} = 1 - \frac{1}{k+1}$ Hence by the Principle of Mathematical Induction, the statement is true for all values $n \geq 2$. **[1 mark]**	**6**
	(c) (i)	$2x^3 - 4x^2 + 5x - 3 = 0$ $(\div 2)\, x^3 - 2x^2 + \frac{5}{2}x - \frac{3}{2} = 0$ **[1 mark]** $\sum \alpha = -(-2) = 2$	**1**
	(ii)	$\sum \alpha\beta = \frac{5}{2}$ **[1 mark]**	**1**
	(iii)	$\alpha\beta\gamma = \frac{3}{2}$ **[1 mark]**	**1**

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(iv)	$\frac{1}{\alpha^3\beta^3}+\frac{1}{\beta^3\gamma^3}+\frac{1}{\alpha^3\gamma^3}$ **[1 mark]** $=\frac{\gamma^3+\alpha^3+\beta^3}{\alpha^3\beta^3\gamma^3}$ $=\frac{\sum\alpha^3}{(\alpha\beta\gamma)^3}$ $\sum\alpha^3=\left(\sum\alpha\right)^3-3\left(\sum\alpha\right)\left(\sum\alpha\beta\right)+3\alpha\beta\gamma$ **[1 mark]** $=2^3-3(2)\left(\frac{5}{2}\right)+3\left(\frac{3}{2}\right)$ $=-\frac{5}{2}$ **[1 mark]** $\frac{1}{\alpha^3\beta^3}+\frac{1}{\beta^3\gamma^3}+\frac{1}{\alpha^3\gamma^3}=-\frac{\frac{5}{2}}{\left(\frac{3}{2}\right)^3}=-\frac{10}{27}$ **[1 mark]**	4
		TOTAL	25
		Specific Objectives: 2.4, 2.5, 4.1, 4.5, 5.4, 5.5	

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
3.	(a)	$\frac{\sin 3\theta + \sin 5\theta}{\cos 3\theta + \cos 5\theta}$ $= \frac{2\sin\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right)}{2\cos\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right)}$ **[1 mark]** $= \frac{\sin 4\theta}{\cos 4\theta}$ **[1 mark]** **[1 mark]** $= \tan 4\theta$ **[1 mark]**	**4**
	(b) (i)	$2\sin\theta - \cos\theta = 0$ $\cos\theta = \frac{\cos\theta}{\sin\theta}$ **[1 mark]** $2\sin\theta - \frac{\cos\theta}{\sin\theta} = 0$ $(x\sin\theta)$ **[1 mark]** $2\sin^2\theta - \cos\theta = 0$ $2\left(1-\cos^2\theta\right) - \cos\theta = 0$ **[1 mark]** $2\cos^2\theta + \cos\theta - 2 = 0$	**3**
	(ii)	$2\sin^2\theta + \cos\theta - 2 = 0$ $(2\cos\theta - 1)(\cos\theta + 2) = 0$ **[1 mark]** $\cos\theta = \frac{1}{2}$ or $\cos\theta = -2$ $\theta = 60^o, \ldots$ **[1 mark]** θ has no solution **[1 mark]** $\theta = 360n \pm 60^o$ **[1 mark]**	**4**

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(c) (i)	$\sqrt{3}\sin\theta+\cos\theta=R\sin(\theta+\alpha)$ Equating coefficients: **[1 mark]** $\cos\theta: R\sin\alpha=1$ $\sin\theta: R\cos\alpha=\sqrt{3}$ $\tan\alpha=\frac{1}{\sqrt{3}}$ $\alpha=\frac{\pi}{6}$ **[1 mark]** $R^2=\sqrt{\left(\sqrt{3}^2\right)+1^2=2}$ **[1 mark]** $\sqrt{3}\sin\theta+\cos\theta=2\sin\left(\theta+\frac{\pi}{6}\right)$	3
	(ii)	$-1\le\sin\left(\theta+\frac{\pi}{6}\right)\le1$ **[1 mark]** $(\times2)-2\le2\sin\left(\theta+\frac{\pi}{6}\right)\le2$ **[1 mark]** $(+3)1\le2\sin\left(\theta+\frac{\pi}{6}\right)+3\le5$ **[1 mark]** Inverting: $\frac{1}{5}\le\frac{1}{2\sin\left(\theta+\frac{\pi}{6}\right)+3}\le1$ **[1 mark]** Hence the maximum value of $f(\theta)=1$ **[1 mark]**	5

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(d)	$l_1 : r = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$ $l_2 : r = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$ Expressing in parametric form: **[1 mark]** $1 - \lambda = 3 + 6\mu$ **(1)** $3\lambda = 3 + 2\mu$ **(2)** $5 - \lambda = 4 + 3\mu$ **(3)** **(3)-(2)** $4 = 1 - 3\mu$ **[1 mark]** $\mu = -1$ Substitute into **(1)**: $\lambda = 4$ **[1 mark]** Substituting for λ and μ into equation **(2)** **[1 mark]** $3(4) = 3 + 2(-1)$ **[1 mark]** $12 \neq 1$ Since there are no unique values of λ and μ which satisfy all three equations, the lines l_1 and l_2 do not intersect. **[1 mark]**	**6**
		TOTAL	
		Specific Objectives: 1.1, 1.5, 1.6, 1.7, 1.8, 1.9, 1.11, 3.7, 3.8	

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	**Solutions**	**Total**
4.	**(a)**	$x = 3\sin\theta + 5$ $\dfrac{x-5}{3} = \sin\theta$ **[1 mark]** $y = 2\cos\theta + 6$ $\dfrac{y-6}{2} = \cos\theta$ **[1 mark]** $\left(\dfrac{x-5}{3}\right)^2 + \left(\dfrac{y-6}{2}\right)^2 = 1$ $\left(\cos^2\theta + \sin^2\theta = 1\right)$ **[1 mark]**	**3**
	(b) (i)	Let *P* have coordinates (*x*, *y*). $\dfrac{\lvert BP\rvert = \sqrt{5}}{\sqrt{(x-3)^2+(y-4)^2}} = \sqrt{5}$ **[1 mark]** Squaring both sides $(x-3)^2 + (y-4)^2 = 5$ **[1 mark]** The locus of *P* is a circle with centre (3, 4) and radius $\sqrt{5}$ units. **[1 mark]**	**3**
	(ii)	Gradient of normal = $\dfrac{5-4}{5-3} = \dfrac{1}{2}$ **[1 mark]** Equation of normal: $y - 5 = \dfrac{1}{2}(x-5)$ **[1 mark]** $y = \dfrac{1}{2}x + \dfrac{5}{2}$ Gradient of tangent = -2 **[1 mark]** Equation of tangent: $y - 5 = -2(x-5)$ **[1 mark]** $y = -2x + 15$	**4**

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	**Solutions**	**Total**
	(iii)	Let $(x-3)^3+(y-4)^2=5$ **(1)** $3y=-x+10$ **(2)** From (2) $x=10-3y$ Substitute into **(1)** $(10-3y-3)^2+(y-4)^2=5$ $(7-3y)^2+(y-4)^2=5$ $10y^2+50y+60=0$ $y^2+5y+6=0$ **[1 mark]** $(y-3)+(y-2)=0$ $y=2\ or\ 3$ **[1 mark]** When $y=2,\ x=4$ **[1 mark]** When $y=3,\ x=1$ Coordinate of the points of intersection (1,3) and (4,2) **[1 mark]**	**4**

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(c) (i)	$\overrightarrow{OA}=\begin{pmatrix}-1\\1\\-1\end{pmatrix}\overrightarrow{OB}=\begin{pmatrix}-2\\-1\\-4\end{pmatrix}\overrightarrow{OC}=\begin{pmatrix}-4\\2\\-5\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ **[1 mark]** $\overrightarrow{CA}=\begin{pmatrix}3\\-1\\4\end{pmatrix}$ **[1 mark]** $\begin{pmatrix}1\\2\\3\end{pmatrix}\begin{pmatrix}11\\5\\-7\end{pmatrix}=11+10-21=0$ **[1 mark]** $\begin{pmatrix}3\\-1\\4\end{pmatrix}\begin{pmatrix}11\\5\\-7\end{pmatrix}=33-5-28=0$ **[1 mark]** Since both dot products = 0 and both vectors $\overrightarrow{BA}$ *and* $\overrightarrow{CA}$ lie on the plant, $\begin{pmatrix}11\\5\\-7\end{pmatrix}$ is perpendicular to the plane. **[1 mark]**	**5**
	(ii)	Using r.n = a.n **[1 mark]** $r\begin{pmatrix}11\\5\\-7\end{pmatrix}=\begin{pmatrix}-1\\1\\-1\end{pmatrix}\begin{pmatrix}11\\5\\-7\end{pmatrix}$ **[1 mark]** Hence the cartesian equation of the plane is $11x+5y-7z=1.$ **[1 mark]**	**3**

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(iii)	Let θ be the angle between the planes π_1 and π_2. $\cos\theta = \frac{n_1.n_2}{\lvert n_1\rvert\lvert n_2\rvert}$ **[1 mark]** $= \frac{\begin{pmatrix}11\\5\\-7\end{pmatrix}\begin{pmatrix}2\\-1\\2\end{pmatrix}}{\sqrt{195}\sqrt{9}}$ **[1 mark]** $\theta = 87.3^o$ **[1 mark]**	3
		TOTAL	25
		Specific Objectives: 2.3, 2.4, 2.5, 3.1, 3.9	

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
5.	(a) (i)	$\lim_{x \to 1} f(x) = \lim_{x \to 1}\left(\frac{2x^2-4x+5}{5x}+\frac{5x-4}{x}\right)$ **[1 mark]** $=\left(\frac{2(1)^2-4(1)+5}{5(1)}+\frac{5(1)-4}{1}\right)$ **[1 mark]** $=\frac{8}{5}$ **[1 mark]**	3
	(ii)	First principle: **[1 mark]** $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ **[1 mark]** $f(x) = x^3 - 6x$ $f(x+h) = (x+h)^3 - 6(x+h)$ **[1 mark]** $f'(x) = \lim_{h \to 0} \frac{(x+h)^3-6(x+h)-(x^3-6x)}{h}$ **[1 mark]** Expand and simplify $f'(x) = \lim_{h \to 0} \frac{3x^2+3xh^2+h^2-6h}{h}$ **[1 mark]** Divide through by h: $f'(x) = \lim_{h \to 0} (3x^2+3xh+h^2-6)$ **[1 mark]** With limit applied $f'(x) = 3x^2 - 6$ **[1 mark]**	5

	Question	Solutions	Total
	(b)	Integrate: $g''(x) = 36x^2 - 10$ $g''(x) = 12x^2 - 10x + c$ **[1 mark]** Using $g'(0),\ c = 0$ **[1 mark]** Therefore, $g'(x) = 12x^3 - 10x$ Integrate $g'(x) \Rightarrow g(x) = 3x^4 - 5x^2 + c$ **[1 mark]** Using $g(1) = 4$ $3(1)^4 - 5(1)^2 + c = 4$ **[1 mark]** $c = 6$ **[1 mark]** $g(x) = 3x^4 - 5x^2 + 6$ **[1 mark]** Hence: $a = 3,\ b = -5$ **[1 mark]**	7
	(c) (i)	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ **[1 mark]** Since $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r^2$ **[1 mark]** $\frac{dV}{dt} = d\pi r^2 \times 1.5 = \frac{6\pi r^2 cm^3}{s}$ **[1 mark]** **[1 mark]** At r = 5, $\frac{dV}{dt} = 6\pi(5)^2 = 150\pi \frac{cm^2}{s}$ **[1 mark]**	5

	Question	Solutions	Total
	(ii)	$\int_0^{2500} dv = \int_0^t 150\pi \ dt$ **[1 mark] [1 mark]** When $2500 = 150\pi t \Rightarrow t = 5.3s.$ **[1 mark]** $r = \sqrt[3]{\frac{2500 \times 3}{4\pi}} = 8.43\,cm$ **[1 mark]** **[1 mark]**	5
		TOTAL	25
		Specific Objectives: 1.4, 2.2, 2.4, 2.7, 2.10	

PURE MATHEMATICS
UNIT 1 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
6.	(a) (i)	Product rule: $(fg)' = f'g + fg'$ $f = 2x^3 + 3$ $f' = 6x^2$ **[1 mark]** $g = \cos\left(x^2\right)$ Apply Chain rule Let $u = x^2$ and $g = \cos(u)$ **[1 mark]** $g'(u) = -\sin(u)$ $u' = 2x$ $g' = -2x\sin\left(x^2\right)$ **[1 mark]** Apply Product rule: $(fg)' = 6x^2\cos\left(x^2\right) - 2x\left(2x^3+3\right)\sin\left(x^2\right)$ **[1 mark]**	4
	(b) (i)	Area of rectangle region between x = -5 and x = -5 and x = 0 under the line y = 3: A = LW = 5 × 3 = 15 squared units **[1 mark]** Area under curve: $\int_{-5}^{0}\sqrt{4-x}\,dx$ **[1 mark]** Apply substitution: U = 4 - x Boundaries: U = 4 and U = 9 **[1 mark]** Integrating: $-\int_4^9 \sqrt{u}\,du = \left[\frac{3}{2}u^{3/2}\right]_4^9 = \frac{38}{2}$ squared units **[2 marks]** Area of shaded region $= 15 - \frac{38}{3} = \frac{7}{3}$ squared units **[1 mark]** **[The answer should be given in squared units]**	6

PURE MATHEMATICS
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KEY AND MARK SCHEME

	Question	Solutions	Total
	(ii)	Volume of revolution: $\int_{-5}^{0} \pi\left(\sqrt{4-x}-3\right)^2 dx = \int_{-5}^{0} \pi\left[-x-6\sqrt{4-x}+13\right]dx$ **[1 mark]** $=\pi\left[-\frac{x^2}{2}+4(4-x)^{3/2}+13x\right]_{-5}^{0}$ **[2 marks]** $=\pi\left[-\frac{25}{2}-76+65\right]$ **[1 mark]** Volume = $\frac{3}{2}\pi$ cubic units **[1 mark]** **[The answer should be given in cubic units]**	5
	(c)	$\frac{d}{dx}\left(f(x)\right)=3x^2-7x+2$ **[1 mark]** $3x^2-7x+2=0$ **[1 mark]** $x=-\frac{-(-7)\pm\sqrt{(-7)^2-4(3)(2)}}{2(3)}$ $x=2,\ x=\frac{1}{3}$ **[1 mark]** When $x=2,\ f(x)=-2\Rightarrow(2,-2)$ **[1 mark]** When $x=\frac{1}{3},\ f(x)=\frac{17}{54}=0.315\Rightarrow\left(\frac{1}{3},\frac{17}{54}\right)$ **[1 mark]** $\frac{d}{dx}\left(3x^2-7x+2\right)=6x-7$ **[1 mark]** At $x=2;\frac{d^2}{dx^2}\left(f(x)\right)>0\left(2,-2\right)$ is a minimum point **[2 marks]** At $x=\frac{1}{3};\frac{d^2}{dx^2}\left(f(x)\right)<0\left(\frac{1}{3},\frac{17}{54}\right)$ is a maxmum point **[2 marks]**	10
		TOTAL	25
		Specific Objectives: 2.9, 2.12, 2.15, 3.9, 3.10	

TEST CODE **02134032**

SPECIMEN 2022

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PUREMATHEMATICS

UNIT 1

ALGERBA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

PAPER 032

2 hours

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE questions.
2. Answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
6. If you need to rewrite any answer and ther is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer**.

***The questions on this paper may be based on Specific Objectives taken from ANY Module in the Unit.**

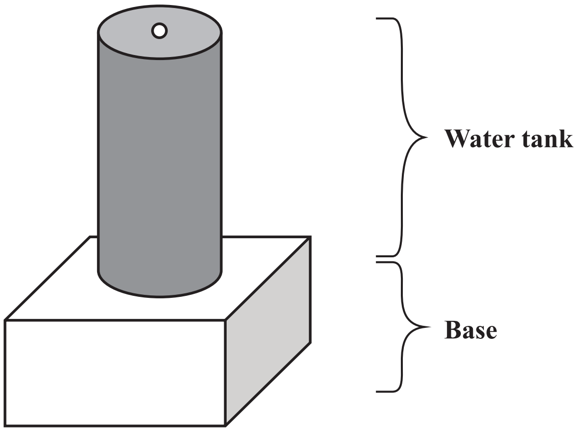
<u>Examination Materials Permitted</u>

Mathematical formulae and tables (provided) − **Revised 2022**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

"*"Barcode Area"*"
Sequential Bar Code

1. A closed cylindrical water tank is to be installed in a community to provide the residents with a constant supply of water. The diagram below, **not drawn to scale**, shows the water tank mounted on a concrete base.



Use $\pi = 3.14$

The population of the community is 400 persons with each person using an average of 0.05 m^3 of water each day

(a) Calculate, in cubic metres, the minimum volume of water required in the tank daily.

[1 mark]

The manufacturer of the tank can produce a closed cylindrical tank with a volume of 30 m^3 and radius, *xm*.

(b) Write an expression, in terms of x, for the height, h, of the tank.

[1 mark]

GO ON TO THE NEXT PAGE

(c) Write an expression, in terms of x, for the total surface area of the tank.

[2 marks]

(d) Determine the value of x that would **minimize** the surface area of the rank. Verify that this value of x produces a minimum surface area.

[7 marks]

(e) The **volume** of water in the tank is being consumed at a rate of 1 m^3h^{-1}. At what rate is the level of the water in the tank decreasing?

[4 marks]

The rate at which the **height** of the water in the tank decreases is given by $\frac{dh}{dt} = -\frac{1}{h^2}$.

(f) Determine an expression for the height of the water in the tank in terms of time (hours).

[2 marks]

(g) Initially, the water in the tank is filled to a height of 4 m. Calculate the time it takes for the water level to drop by 2 m.

[3 marks]

Total 20 marks

2. Tropical Cruises is a company which operates a fleet of yachts and is interested in offering party cruises between the islands of Barbados, Grenada and St Vincent.

It was determined that the total revenue that could be generated from ticket sales is given by the equation $R(x) = 196x - 3x^2$ where x represents the number of passengers (the level of demand).

(a) Show that the marginal revenue is given by $R'(x) = 196 - 6x$.

[1 mark]

(b) Hence, determine the marginal revenue in (hundreds of dollars) when the level of demand, x is 30.

[1 mark]

The total cost function for Tropical Cruises is modelled by $C(x) = 14 + 4x$.

(c) Show that the profit function for this venture is given by $P(x) = -3x^2 + 192x - 14$.

[3 marks]

(d) Determine an expression for the marginal profit.

[2 marks]

(e) Compute the marginal profit when the level of demand, x, is 14.

[1 mark]

(f) Determine whether the profit is increasing or decreasing when the level of demand is 14.

[1 mark]

(g) Determine the level of demand that maximizes the profit, that is, the value of x for which the marginal profit is equal to zero.

[2 marks]

(h) Verify that the level of demand stated in (g) is a maximum.

[2 marks]

GO ON TO THE NEXT PAGE

The marginal cost function $C'(x)$ of a rival tourist entity, Wadadli Cruises, is given by $C'(x) = 16x + 10$.

(i) Write an expression for the total cost function $C(x)$ for Wadadli Cruises.

[3 marks]

(j) Hence, calculate the value of the constant of integration (overhead cost) for the total cost in (i), given the total cost $C(x) = 32$ when $x = 0$.

[1 mark]

(k) Which of the two companies, Tropical Cruises or Wadadli, would have the lower overhead cost? **Justify your response**.

[3 marks]

Total 20 marks

3. The table below shows the ports visited by Tropical Cruises in the respective islands.

Island	**Port**	**Location**	**Height Above Sea Level (m)**
Barbados	Bridgetown (*B*)	13.1° North, 59.6° West	1
Grenada	Prickly Bay (*P*)	12° North, 61.8° West	21
St Vincent and the Grenadines	Kingstown (*K*)	13.2° North, 61.2° West	14

*To the nearest metre

(a) If $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ represents a port located at x° north and y° west and z metres above sea level, express the location of the ports B, P and K as position vectors.

[3 marks]

(b) In the space provided below, sketch the relative positions of the three ports B, P and K as would be seen on a map.

[3 marks]

GO ON TO THE NEXT PAGE

“*”Barcode Area”*”
Sequential Bar Code

(c) Determine the vector equation of the line joining EACH of the following ports:

(i) B and P

[2 marks]

(ii) B and K

[2 marks]

(d) Calculate the acute angle between the lines BP and BK.

[4 marks]

(e) Show that the vector $11\mathbf{i} + 15.4\mathbf{j} - 1.98\mathbf{k}$ is perpendicular to both vectors, $\overrightarrow{BP}$ and $\overrightarrow{BK}$.

[3 marks]

(f) Hence, detemine the Cartesian equation of the plane containing the ports Prickly Bay, Bridgetown and Kingstown.

[3 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

EXTRA SPACE

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Question No.

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“*”Barcode Area”*”
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EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATIONS®

PURE MATHEMATICS

UNIT 1 — Paper 032

KEY AND MARK SCHEME

MAY/JUNE 2022

SPECIMEN PAPER

PURE MATHEMATICS
UNIT 1 — Paper 032
KEY AND MARK SCHEME

	Questions	Solutions	Total
1.	**(a)**	Minimum volume of water $= 400 \times 0.05$ $= 20m^3$ **[1 mark]**	**1**
	(b)	$V = \pi r^2 h$ $h = \dfrac{V}{\pi r^2}$ Substituting V = 30 and r = x $h = \dfrac{30}{\pi x^2}$ making h the subject of the formula. **[1 mark]**	**1**
	(c)	The surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$ Substituting $h = \dfrac{30}{\pi x^2}$ and r = x $A = 2\pi r^2 + 2\pi x\left(\dfrac{30}{\pi x^2}\right)$ Substituting into formula $= 2\pi r^2 + \dfrac{60}{x}$ simplifying expression. **[2 marks]**	**2**

PURE MATHEMATICS
UNIT 1 — Paper 032
KEY AND MARK SCHEME

	Questions	Solutions	Total
	(d)	$A = 2\pi x^2 + \frac{60}{x}$ $\frac{dA}{dx} = 4\pi x - \frac{60}{x^2}$ Differentiating **[1 mark]** Stationary points occur when $\frac{dA}{dx} = 0$ i.e. $0 = 4\pi x - \frac{60}{x^2}$ placing $\frac{dA}{dx} = 0$. $\frac{60}{x^2} = 4\pi x$ **[1 mark]** $\frac{15}{\pi} = x^3$ **[1 mark]** $\sqrt[3]{\frac{15}{\pi}} = x$ **[1 mark]** find x $x = 1.683$ **[1 mark]** $\approx 1.68m^3 (3sf)$ units $\frac{d^2A}{dx^2} = 4\pi + \frac{120}{x^2}$ finding the 2nd derivative **[1 mark]** When $x = 1.68$ $\frac{d^2A}{dx^2} > 0$ 2nd derivative sign test. Hence the area is a minimum. **[1 mark]**	7

PURE MATHEMATICS
UNIT 1 — Paper 032
KEY AND MARK SCHEME

	Questions	Solutions	Total
	(e)	$V = \pi r^2 h$ $\frac{dV}{dh} = \pi r^2$ **[1 mark]** Differentiation $= \pi(1.68)^2$ $= 8.86\ (3\ s.f.)$ **[1 mark]** Substituting and evaluating $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ Correct use of chain rule **[1 mark]** $= \frac{1}{8.86}$ **(1)** $= 0.1129$ $\approx 0.13\ mh^{-1}$ Obtaining correct value of $\frac{dh}{dt}$. **[1 mark]**	**4**
	(f)	The rate at which the height of the water in the tank decreases is given by $\frac{dh}{dt} = -\frac{1}{h^2}$ $h^2 dh = -dt$ Separating variables **[1 mark]** The variables are separable and the solution can be found by integration. $-\int h^2 dh = \int dt$ $-\frac{h^2}{3} = t + c$ Integrating correctly **[1 mark]** $h = -\sqrt[3]{3(t+c)}$	**2**

PURE MATHEMATICS
UNIT 1 — Paper 032
KEY AND MARK SCHEME

	Questions	Solutions	Total
	(g)	When, t = 0, h = 4 substituting the initial conditions. $-\frac{4^3}{3} = 0 + c$ $-\frac{64}{3} = c$ Obtaining the value of the constant **[1 mark]** When h = 2 $-\frac{2^3}{3} = t - \frac{64}{3}$ **[1 mark]** $\frac{64}{3} - \frac{8}{3} = t$ $t = \frac{56}{3}$ hours Obtaining the time t. **[1 mark]**	3
		Specific Objective Module 2.2, 2.3, 2.4, 2.7, 2.9; 2.10; 2.11; 2.12, 3.2; 3.4; 3.12; 3.13	

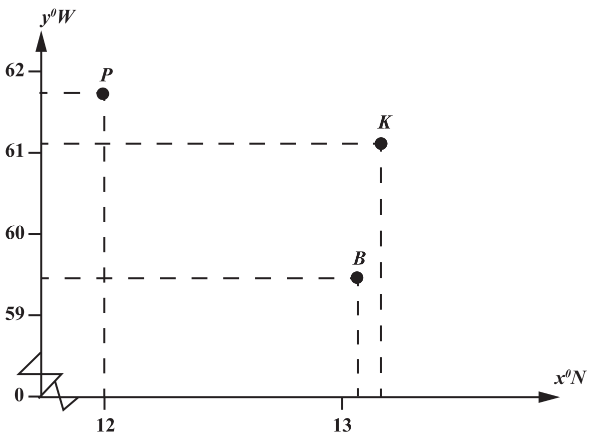
PURE MATHEMATICS
UNIT 1 — Paper 032
KEY AND MARK SCHEME

	Questions	**Solutions**	**Total**
2.	**(a)**	$R(x)=196x-3x^2$ $R'(x)=196-2(3x)$ $=196-6x$ **[1 mark]**	**1**
	(b)	When demand level x = 30 Marginal revenue = 196 - 6(30) = 196 - 180 = 16(hundred dollars) **[1 mark]**	**1**
	(c)	$C(x)=14+4x$ $P(x)=R(x)-C(x)$ **[1 mark]** $=196x-3x^2-(14+4x)$ **[1 mark]** $=196x-3x^2-14-4x$ $P(x)=-3x^2+192x-14$ **[1 mark]**	**3**
	(d)	Marginal profit function $P'(x)=2(-3x)+192$ **[1 mark]** $P'(x)=-6x+192$ **[1 mark]**	**2**
	(e)	When demand $(x=14)$ Marginal Profit $=-6(14)+192$ $=-84+192$ $=108(\mathit{hundred\ dollars})$ **[1 mark]**	**1**
	(f)	Since Marginal profit >0 then profit is increasing **[1 mark]**	**1**
	(g)	$=-6x+192=0$ **[1 mark]** $=-6x=-192$ $x=32$ **[1 mark]** The level of demand is 32 passengers	**2**

PURE MATHEMATICS
UNIT 1 — Paper 032
KEY AND MARK SCHEME

	Questions	Solutions	Total
	(h)	$P''(x) = -6 < 0$ **[1 mark]** Hence at $x = 32$, $P(x)$ is maximum. **[1 mark]**	**2**
	(i)	$C(x) = \int (16x+10)dx$ **[1 mark]** $= \frac{1}{1+1}(16x1+1)+10x+c$ **[1 mark]** $= \frac{1}{2}(16x^2)+10x+c$ $= 8x^2+10x+c$ **[1 mark]**	**3**
	(j)	$C(x) = 32$ when $x = 0.$ $8x^2+10x+c = 32$ When $x = 0.$ $8(0)^2+10(0)+c = 32$ $0+0+c = 32$ $c = 32$ **[1 mark]**	**1**
	(k)	By comparison, Total Cost function for Tropical Cruises is $C(x) = 14+4x$. While the total cost function for Wadadli Cruises is $C(x) = 8x^2+10x+c$. The overhead cost (when x = 0) for Tropical Cruises is $C(x) = 14+4(0) = 14$ and the overhead cost (when x = 0) for Wadadli Cruises is C = 32 from part (x). Hence, by comparison, Tropical Cruises would have the lower overhead cost. **[1 mark]**	**3**
		Specific Objective Module: 2.2, 2.3, 2.4, 2.8; 2.9; 2.10; 3,1; 3.2; 3.3; 3.4	

PURE MATHEMATICS
UNIT 1 — Paper 032
KEY AND MARK SCHEME

	Questions	Solutions	Total
3.	(a)	$\overrightarrow{OB} = 13.1i + 59.6j + k$ **[1 mark]** $\overrightarrow{OP} = 12i + 61.8j + 21k$ **[1 mark]** $\overrightarrow{OK} = 13.2i + 61.2j + 14k$ **[1 mark]**	3
	(b)	 **1 mark each for plotting the points B, K and P**	3
	(c) (i)	$\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB}$ **[1 mark]** $= 1.1i + 2.2j + 20k$ The equation of the line, $BP,\ r = \begin{pmatrix} 13.1 \\ 59.6 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1.1 \\ 2.2 \\ 11 \end{pmatrix}$ **[1 mark]**	2
	(ii)	$\overrightarrow{BK} = \overrightarrow{OK} - \overrightarrow{OB}$ $= 0.1i + 1.6j + 13k$ **[1 mark]** $\begin{pmatrix} 13.1 \\ 59.6 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 0.1 \\ 1.6 \\ 13 \end{pmatrix}$ **[1 mark]**	2

PURE MATHEMATICS
UNIT 1 — Paper 032
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	Question	Solutions	Total
	(d)	Let θ represent the angle between the lines BP and BK. $\cos\theta = \dfrac{\begin{pmatrix}-1.1\\2.2\\11\end{pmatrix}\begin{pmatrix}0.1\\1.6\\13\end{pmatrix}}{\sqrt{(-1.1)^2+(2.2)^2+(11)^2}\sqrt{(0.1)^2+(1.6)^2+(13)^2}}$ $= \dfrac{146.63}{147.64}$ $\theta = \cos^{-1}\left(\dfrac{146.41}{147.64}\right)$ $= 7.40^0$ **[1 mark]** Correct formula **[1 mark]** Correct numerator **[1 mark]** Correct denominator **[1 mark]**	4
	(e)	$\begin{pmatrix}-1.1\\2.2\\11\end{pmatrix}\begin{pmatrix}11\\15.4\\-1.98\end{pmatrix}=0$ Showing dot product $=0$ **[1 mark]** $\begin{pmatrix}0.1\\1.6\\13\end{pmatrix}\begin{pmatrix}11\\15.4\\-1.98\end{pmatrix}=0$ Showing dot product $=0$ **[1 mark]** Since dot product $=0$ in both cases, the vector $11i+15.4j-1.98k$ is prependicular to both vectors $\overrightarrow{BP}$ and $\overrightarrow{BK}$. Stating reason for being perpendicular. **[1 mark]**	3

PURE MATHEMATICS
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KEY AND MARK SCHEME

	Question	Solutions	Total
	(f)	Since $\begin{pmatrix} 11 \\ 15.4 \\ -1.98 \end{pmatrix}$ is perpendicular to both $\overrightarrow{BP}$ and $\overrightarrow{BK}$, then this vector is perpendicular to the plane containing all 3 ports. The equation of the plane containing the ports B, P and K is given by r.n = a.n Correct formula $r\begin{pmatrix} 11 \\ 15.4 \\ -1.98 \end{pmatrix} = \begin{pmatrix} 13.1 \\ 59.6 \\ 1 \end{pmatrix}\begin{pmatrix} 11 \\ 15.4 \\ -1.98 \end{pmatrix}$ **[1 mark]** Correct substitution **[1 mark]** $11x + 15.4y - 1.98z$ $= (13.1)(11) + (59.6)(15.4) + (1)(-1.98)$ $11x + 15.4y - 1.98z = 1059.96$ Stating the Cartesian equation **[1 mark]**	**3**
		Specific Objective Module: 3.1; 3.2; 3.3; 3.6; 3.7; 3.9	

TEST CODE **02234010**

SPECIMEN 2022

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

COMPLEX NUMBERS, ANALYSIS AND MATRICES

UNIT 2 – Paper 01

90 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 90 minutes to answer them.
2. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
3. Look at the sample item below.

   <u>Sample Item</u>

   If the function $f(x)$ is defined by $f(x)$ cos x then $f(x)$ is

   <u>Sample Answer</u>

   (A) (B) (C) ●

   (A) $-\frac{1}{2\sqrt{x}}\sin\sqrt{x}$

   (B) $-\frac{1}{2}\sin\sqrt{x}$

   (C) $\frac{1}{\sqrt{x}}\sin\sqrt{x}$

   (D) $\frac{1}{2\sqrt{x}}\sin\sqrt{x}$

   The best answer to this item is "$\frac{1}{2\sqrt{x}}\sin\sqrt{x}$", so answer space (D) has been shaded.

4. You may do any rough work in this booklet.
5. The use of silent, non-programmable scientific calculators is allowed.

   <u>Examination Materials Permitted</u>

   A list of mathematical formulae and tables (provided) — **Revised 2012**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1. If z and z^* are two conjugate complex numbers, where $z = x + \text{i}y$, $x, y \in \mathbb{R}$, then $z\,z^* =$

(A) $x^2 + y^2$
(B) $x^2 - y^2 - 2xy\text{i}$
(C) $x^2 - y^2$
(D) $x^2 + y^2 - 2xy\text{i}$

2. If $|\,z + \mathbf{i}\,| = |\,z + 1|$, where z is a complex number, then the locus of z is

(A) $y = 0$

(B) $y = 1$

(C) $y = x$

(D) $y = \dfrac{x}{2}$

3. Given that $z + 3z^* = 12 + 8\text{i}$, then $z =$

(A) $-3 - 4\text{i}$
(B) $3 - 4\text{i}$
(C) $3 + 4\text{i}$
(D) $-3 + 4\text{i}$

4. The gradient of the normal to the curve with the equation $xy^3 + y^2 + 1 = 0$ at the point $(2, -1)$ is

(A) -4

(B) $-\dfrac{1}{4}$

(C) $\dfrac{1}{4}$

(D) 4

5. If $\dfrac{\text{d}y}{\text{d}x} = \dfrac{5x}{y}$, then $\dfrac{\text{d}^2y}{\text{d}x^2} =$

(A) $\dfrac{5}{y^3} - \dfrac{25x^2}{y^2}$

(B) $\dfrac{5}{y} - \dfrac{25x^2}{y^3}$

(C) $\dfrac{1}{y} - \dfrac{25}{xy}$

(D) $\dfrac{5}{xy} - \dfrac{25}{x^3y^3}$

6. Given $y = a\ arc\ cos(ax)$, where a is a constant, $\dfrac{\text{d}y}{\text{d}x} =$

(A) $\dfrac{a^2}{\sqrt{(1-a^2x^2)}}$

(B) $-\dfrac{1}{\sqrt{(1-a^2x^2)}}$

(C) $-\dfrac{a^2}{\sqrt{(1-a^2x^2)}}$

(D) $\dfrac{1}{\sqrt{(1-a^2x^2)}}$

7. Given that $f(x, y, z) = x^2y + y^2z - z^2x$ then $\dfrac{\partial f}{\partial y} =$

(A) $x^2y + 2yz$
(B) $x^2 + 2yz$
(C) $x^2 + y^2$
(D) $x^2 + y^2 + z^2$

8. $\int\left(x+3+\dfrac{M}{x^2-3x+2}\right)dx$ may be expressed as

(A) $\int\left(\dfrac{Px+Q}{x^2-3x+2}\right)dx$

(B) $\int\left(\dfrac{P}{x-1}+\dfrac{Q}{x-2}\right)dx$

(C) $\int\left(x+3+\dfrac{Px+Q}{x^2-3x+2}\right)dx$

(D) $\int\left(x+3+\dfrac{P}{x-1}+\dfrac{Q}{x-2}\right)dx$

9. Given $I_n=\int \tan^n x\,dx$, for $n>2$, $I_n=$

(A) $\dfrac{1}{n-1}\tan^{n-1}x+I_{n-2}$

(B) $\dfrac{1}{n-1}\tan^{n-1}x\sec^2 x-I_{n-2}$

(C) $\tan^{n-1}x-I_{n-2}$

(D) $\dfrac{1}{n-1}\tan^{n-1}x-I_{n-2}$

10. $\dfrac{d}{dx}(e^{3x^2+2x+1})$ is

(A) $(6x+2)\,e^{6x+2}$
(B) $(6x+2)\,e^{3x^2+2x+1}$
(C) $(3x^2+2x+1)\,e^{6x+2}$
(D) $(3x^2+2x+1)\,e^{3x^2+2x+1}$

11. A curve is defined parametrically by the equations $x=t^2$, $y=t(1-t^2)$. The gradient of the curve, in terms of t, is

(A) $\dfrac{1-3t^2}{2t}$

(B) $\dfrac{2t}{1-3t^2}$

(C) $2t(1-2t)$

(D) $2t(1+2t)$

12. Given $y=\ln(2x+3)^3$, then $\dfrac{dy}{dx}$ is

(A) $\dfrac{2x}{2x+3}$

(B) $\dfrac{2}{2x+3}$

(C) $\dfrac{6x}{2x+3}$

(D) $\dfrac{6}{2x+3}$

13. $\int xe^{2x}\,dx$ may be expressed as

(A) $2xe^{2x}+e^{2x}+c$

(B) $2xe^{2x}-4e^{2x}+c$

(C) $\dfrac{1}{2}xe^{2x}-\dfrac{1}{4}e^{2x}+c$

(D) $\dfrac{1}{2}x^2e^{2x}+\dfrac{1}{2}xe^{2x}+c$

14. Which of the following functions, when integrated with respect to x, gives the result $x-\ln x^2+K$?

(A) $\dfrac{x-2}{x}$

(B) $\dfrac{1-2x}{x^2}$

(C) $\dfrac{1}{1-x^2}$

(D) $1-\dfrac{2}{x^2}$

15. Written as partial fractions $\dfrac{5}{(x+2)(x-3)}$ is

(A) $\dfrac{1}{(x+2)} + \dfrac{1}{x-3}$

(B) $\dfrac{-1}{(x+2)} + \dfrac{1}{x-3}$

(C) $\dfrac{1}{(x+2)} + \dfrac{-1}{x-3}$

(D) $\dfrac{-1}{(x+2)} + \dfrac{-1}{x-3}$

16. Given that a sequence of positive integers $\{U_n\}$ is defined by $U_1 = 2$ and $U_{n+1} = 3U_n + 2$, then $U_n =$

(A) $3n - 1$
(B) $3^n + 1$
(C) $3^n - 1$
(D) $3n + 2$

17. The sequence $a_n = \dfrac{3n^2 - n + 4}{2n^2 + 1}$

(A) converges
(B) diverges
(C) is periodic
(D) is alternating

18. The nth term of a sequence is given by $u_n = 9 - 4\left(\dfrac{1}{2}\right)^{n-1}$. The 5th term of the sequence is

(A) $\dfrac{9}{4}$

(B) $\dfrac{35}{4}$

(C) $\dfrac{37}{4}$

(D) $\dfrac{71}{8}$

19. Given that $\sum\limits_{r=1}^{n} u_n = 5n + 2n^2$, then $u_n =$

(A) $4n + 3$
(B) $5n + 2$
(C) $2n^2 + n - 3$
(D) $4n^2 + 4n + 7$

20. $\sum\limits_{r=1}^{m-1} 3\left(\dfrac{1}{2}\right)^r =$

(A) $3 - 3 \times 2^{-m}$
(B) $3 - 3 \times 2^{(1-m)}$
(C) $6 - 3 \times 2^{(m-1)}$
(D) $6 - 3 \times 2^{(1-m)}$

21. The Maclaurin's series expansion for $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \dfrac{x^9}{9!} - \ldots$ has a general term BEST defined as

(A) $(-1)^n \dfrac{x^{2n+1}}{(2n+1)!}$

(B) $(-1)^{n+1} \dfrac{x^{2n+1}}{(2n+1)!}$

(C) $\dfrac{37}{4}$

(D) $\dfrac{71}{8}$

22. The first 2 non-zero terms of the expansion of $\sin\left(x + \dfrac{\pi}{6}\right)$ are

(A) $\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}x$

(B) $\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}x$

(C) $\dfrac{1}{2} + \dfrac{1}{2}x$

(D) $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}x$

GO ON TO THE NEXT PAGE

23. If $\left(2x^2 - \frac{2}{x}\right)^6 = ... + k + ...$, where k is independent of x, then $k =$

(A) –960
(B) –480
(C) 480
(D) 960

24. $f(x) = x^3 - x^2 - 6$. Given that $f(x) = 0$ has a real root α in the interval [2.2, 2.3], applying linear interpolation once on this interval an approximation to α, correct to 3 decimal places, is

(A) 2.216
(B) 2.217
(C) 2.218
(D) 2.219

25. Taking 1.6 as a first approximation to α, where the equation $4 \cos x + e^{-x} = 0$ has a real root α in the interval (1.6, 1.7), using the Newton-Raphson method a second approximation to α (correct to 3 decimal places) ≈

(A) 1.602
(B) 1.620
(C) 1.622
(D) 1.635

26. $f(x) = 3x^3 - 2x - 6$. Given that $f(x) = 0$ has a real root, α, between $x = 1.4$ and $x = 1.45$, starting with $x_0 = 1.43$ and using the iteration $x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$, the value of x_1 correct to 4 decimal places is

(A) 1.4369
(B) 1.4370
(C) 1.4371
(D) 1.4372

27. Given $E \equiv \sum_{n=19}^{30} \frac{3^n}{n}$. The number of terms in the expansion of E is

(A) 10
(B) 11
(C) 12
(D) 13

28. The second and fifth terms of a convergent geometric series with first term $\frac{81}{2}$ are 27 and 8, respectively. The sum to infinity of this series is

(A) $\frac{2}{3}$

(B) $\frac{3}{2}$

(C) $\frac{81}{2}$

(D) $\frac{243}{2}$

29. The first term of an AP is ‘a’ and its common difference is –1. The sum of the first 10 terms is equal to

(A) $5(2a - 9)$
(B) $5(2a + 9)$
(C) $10(2a + 11)$
(D) $10(2a - 11)$

30. The coefficient of x^2 in the expansion of $(2 - 3x)^5$ is

(A) –720
(B) –240
(C) 240
(D) 720

31. On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively. The probability that on a randomly chosen day Bill travels by foot and is late is

(A) $\frac{1}{30}$
(B) $\frac{1}{10}$
(C) $\frac{3}{10}$
(D) $\frac{13}{30}$

32. Ten cards, each of a different colour, and consisting of a red card and a blue card, are to be arranged in a line. The number of different arrangements in which the red card is not next to the blue card is

(A) $9! - 2 \times 2!$
(B) $10! - 9! \times 2!$
(C) $10! - 2! \times 2!$
(D) $8! - 2! \times 2!$

33. The number of ways in which all 10 letters of the word **STANISLAUS** can be arranged if the **S**s must all be together is

(A) $\frac{8! \times 3!}{2!}$
(B) $8! \times 3!$
(C) $\frac{8!}{2!}$
(D) $\frac{8!}{3!}$

34. A committee of 4 is to be chosen from 4 teachers and 4 students. The number of different committees that can be chosen if there must be at least 2 teachers is

(A) 36
(B) 45
(C) 53
(D) 192

35. A and B are two events such that

$P(A) = p$ and $P(B) = \frac{1}{3}$.

The probability that neither occurs is $\frac{1}{2}$.

If A and B are mutually exclusive events then $p =$

(A) $\frac{5}{6}$

(B) $\frac{2}{3}$

(C) $\frac{1}{5}$

(D) $\frac{1}{6}$

<u>Items **36–37**</u> refer to the matrix.

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix}.$$

36. The cofactor of the circled element, −2, is

(A) −2
(B) −1
(C) 0
(D) 2

37. The determinant of the given matrix is

(A) −5
(B) −3
(C) 3
(D) 5

38. Given $\begin{vmatrix} 6 & 0 & 1 \\ 7 & 7 & 0 \\ 0 & -12 & x \end{vmatrix} = 0$, the value of x is

(A) −2
(B) 2
(C) 7
(D) 12

39. Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$ and

$B = \begin{bmatrix} -4 & 0 & 2 \\ 0 & 6 & -4 \\ 2 & -4 & 2 \end{bmatrix}$, by considering

AB, then $A^{-1} =$

(A) $2B$

(B) B

(C) $\frac{1}{2}B$

(D) $\frac{1}{2}AB$

40. If $A = \begin{bmatrix} 2 & -7 & 8 \\ 3 & -6 & -5 \\ 4 & 0 & -1 \end{bmatrix}$, the transpose of matrix, A, results in $|A|$ being

(A) zero
(B) squared
(C) negative
(D) unchanged

41. The general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2x^2 + x - 1$ is

(A) $y = e^{2x}(A + Bx) + 2x^2 + x - 1$

(B) $y = e^{-2x}(A + B) + 2x^2 - x + 1$

(C) $y = e^{2x}(A + Bx) + \frac{x^2}{2} + \frac{5}{4}x + \frac{3}{4}$

(D) $y = e^{2x}(A - Bx) + \frac{x^2}{2} + \frac{5}{4}x + \frac{3}{4}$

42. The general solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0 \text{ is}$$

(A) $y = Ae^{x} + Be^{-2x}$
(B) $y = Ae^{x} + Be^{2x}$
(C) $y = e^{x}(C\cos 2x + D\sin 2x)$
(D) $y = e^{2x}(C\cos x + D\sin x)$

43. The general solution of the differential equation $\sin x\frac{dy}{dx} - y\cos x = \sin 2x \sin x$ is found by evaluating

(A) $\int \frac{d}{dx} y \sin x \, dx = \int 2\cos x \, dx$

(B) $\int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int 2\cos x \, dx$

(C) $\int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int \sin 2x \, dx$

(D) $\int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int \cos x \, dx$

44. A particular integral of the differential equation $\frac{d^2y}{dx^2} + 25y = 3\cos 5x$ is of the form $y = \lambda x \sin 5x$. The general solution of the differential equation is

(A) $y = A\cos 5x - B\sin 5x - \lambda x\sin 5x$
(B) $y = A\cos 5x + B\sin 5x + \lambda x\sin 5x$
(C) $y = A\cos 5x + B\sin 5x - \lambda x\sin 5x$
(D) $y = A\cos 5x - B\sin 5x + \lambda x\sin 5x$

45. The general solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 3e^{x}$ is of the form

(A) $y = Ae^{x} + Be^{2x} + ke^{x}$
(B) $y = Ae^{x} + Be^{2x} - 3e^{x}$
(C) $y = Ae^{-x} + Be^{-2x} + kxe^{x}$
(D) $y = Ae^{x} + Be^{2x} + kxe^{x}$

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

UNIT 2

PAPER 01

Item	Key	Specific Obj.
1	A	1.4
2	C	1.10
3	B	1.4
4	A	2.4
5	B	2.4
6	C	2.5
7	B	2.8
8	D	3.1
9	D	3.9
10	B	2.1
11	A	2.3
12	D	2.2
13	C	3.7
14	A	3.5
15	B	3.1

Item	Key	Specific Obj.
16	C	1.2
17	A	1.3
18	B	1.2
19	A	2.3
20	B	2.3
21	A	2.7
22	A	2.8
23	D	3.2
24	C	4.3
25	B	4.5
26	C	4.6
27	C	2.2
28	D	2.5
29	A	2.2
30	D	3.2

Item	Key	Specific Obj.
31	A	1.12
32	B	1.2
33	D	1.2
34	C	1.3
35	D	1.11
36	A	2.4
37	C	2.3
38	B	2.3
39	C	2.4
40	D	2.2
41	C	3.2
42	C	3.2
43	B	3.1
44	B	3.2
45	D	3.2

TEST CODE **02234020**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2

COMPLEX NUMBERS, ANALYSIS AND MATRICES

SPECIMEN PAPER

PAPER 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials

Mathematical formulae and tables (provided) – **Revised 2022**
Electronic calculator
Ruler and graph paper

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

“*”Barcode Area”*”
Sequential Bar Code

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Express the complex number $\frac{3-4i}{2+i}$ in the form $a + bi$ where a and b are real numbers.

[4 marks]

(ii) Hence, calculate the principal argument of $\frac{3-4i}{2+i}$.

[3 marks]

(b) (i) Given that $u^2 = -3 + 4i$, derive the complex numbers of the form $u = x + iy$, where $x, y \in \mathbb{R}$.

[6 marks]

(ii) Hence solve the equation $z^2 - z + (1 - i) = 0$.

[4 marks]

(c) The complex number $z = x + iy$ satisfies the equation $|z - 3 + 2i| = 2|z - 1 - 4i|$.

The complex number z is represented by the point **P** on an Argand diagram.

(i) Show that the locus of **P** is a circle.

[4 marks]

(ii) State the centre and radius of the circle.

[2 marks]

(iii) Use the grid provided below to sketch the circle on an Argand diagram.

[2 marks]

Total 25 marks

2. (a) Evaluate $\int e^{2x} \cos 3x \, dx$.

[7 marks]

(b) (i) Given that $x^2 + 4y^2 = 1$, write an expression for $\dfrac{dy}{dx}$.

[3 marks]

(ii) Hence, show that $\dfrac{d^2 y}{dx^2} = \dfrac{-1}{16y^3}$.

[4 marks]

(c) Use the substitution $u = 1 + x^2$ to evaluate $\int_0^1 \frac{x^3}{(1+x^2)^4}\,dx$.

[7 marks]

(d) Let $f(x, y) = 3x^2 - xy - 3y^2$

(i) Write expressions for $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

[2 marks]

(ii) Hence, determine the value of $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$.

[2 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Determine the value of the constants A and B such that

$$\frac{1}{(1-2r)(1+2r)} = \frac{A}{1-2r} + \frac{B}{1+2r}.$$

[5 marks]

(ii) Hence, determine the value of S where $S = \sum_{r=1}^{n} \frac{1}{(1-2r)+(1+2r)}$.

[5 marks]

(iii) Deduce the sum to infinity of S.

[3 marks]

GO ON TO THE NEXT PAGE

(b) (i) Write an expression for the r^{th} term of the series $1(8) + 2(20) + 3(32) + \ldots$

[2 marks]

(ii) Prove, by mathematical induction, that the sum to n terms of the series in (b) (i) is $2n^2\,(2n + 2)$.

[10 marks

Total 25 marks

4. (a) Given the series $\frac{1}{3} + \frac{1}{3^4} + \frac{1}{3^7} + \frac{1}{3^{10}} + \cdots$

(i) show that the series is a geometric series.

[3 marks]

(ii) determine the sum of the first n terms of the series.

[4 marks]

(b) Use Maclaurin's Theorem to find the first THREE **non-zero** terms in the power series expansion of sin $2x$.

[7 marks]

GO ON TO THE NEXT PAGE

(c) (i) Expand $\sqrt{\left(\frac{1+x}{1-x}\right)}$ up to and including the term in x^3 stating the values of x for which the expression is valid.

[7 marks]

(ii) By taking $x = 0.04$, determine an approximation for $\sqrt{\frac{52}{3}}$, correct to 3 decimal places.

[4 marks]

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) Two fair coins and one fair die are tossed at the same time.

(i) Calculate the total number of outcomes in the sample space.

[3 marks]

(ii) Determine the probability of obtaining exactly one head and a 6.

[2 marks]

(iii) Calculate the probability of obtaining at least one head and an even number on the die, on a particular attempt.

[4 marks]

(b) A record is kept of the books borrowed from a library. The table below shows the number of overdue books outstanding at the end of the week.

	Adult		**Child**	
	Male	**Female**	**Male**	**Female**
Fiction	17	10	14	20
Non-fiction	9	14	5	11

One of these books has been requested by another reader. Determine the probability that

(i) it is non-fiction, borrowed by a male child

[2 marks]

(ii) it is fiction, borrowed by a female

[3 marks]

(iii) it was borrowed by an adult male, given that it is fiction.

[3 marks]

(c) On a particular weekend, 100 customers purchased tools (T), fertilizer (F) or Seeds (S) at Green Thumb Garden Supply Store. Of these,

38 purchased tools
56 purchased fertilizer
40 purchased seeds
30 purchased seeds and fertilizer
17 purchased seeds and tools
20 purchased tools and fertilizer
14 purchased tools, seeds and fertilizer

(i) Represent the information on a Venn diagram.

[3 marks]

(ii) Using your diagram determine,

a) the probability that a customer does not purchase seeds or tools

[2 marks]

b) whether the purchase of seeds and fertilizer are independent.

[3 marks]

Total 25 marks

6. (a) A matrix **A** is given as $\begin{pmatrix} 2 & 1 & -1 \\ 0 & 4 & 3 \\ -1 & 6 & 0 \end{pmatrix}$

(i) Calculate the determinant of **A**.

[5 marks

(ii) Hence, determine $\mathbf{A}^{-1}$, the inverse of **A.**

[5 marks]

GO ON TO THE NEXT PAGE

(b) The amount of salt, y kg, that dissolves in a tank of water at time t minutes satisfies the first order differential equation $\frac{dy}{dt}+\frac{2y}{t+10}=3$. Using a suitable integrating factor, show that the general solution of this differential equation is $y=t+10+\frac{c}{(t+10)^2}$, where c is an arbitrary constant.

[7 marks]

(c) Determine the general solution of the differential equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}-4y=8x^2$.

[8 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

EXTRA SPACE

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Question No. $\boxed{}$

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATIONS®

PURE MATHEMATICS

UNIT 2 — Paper 02

KEY AND MARK SCHEME

MAY/JUNE 2022

SPECIMEN PAPER

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
1.	(a) (i)	$\frac{3-4i}{2+i} = \frac{(3-4i)(2-i)}{(2+i)(2-i)}$ **[1 mark]** $= \frac{6-8i-3i-4}{4+1}$ **[1 mark]** Simplification $= \frac{2-11i}{5}$ **[1 mark]** Numerator & denominator correct $= \frac{2}{5} - \frac{11}{5}i$ **[1 mark]** "his" answer in the form a + ib	4
	(ii)	Principal argument: $Arg\ \theta = \tan^{-1} \frac{-11/5}{2/5}$ **[1 mark]** Correct form of argument $\theta = \tan^{-1} \frac{-11}{2}$ simplification **[1 mark]** $\theta = -1.39$ rad CAO **[1 mark]**	3

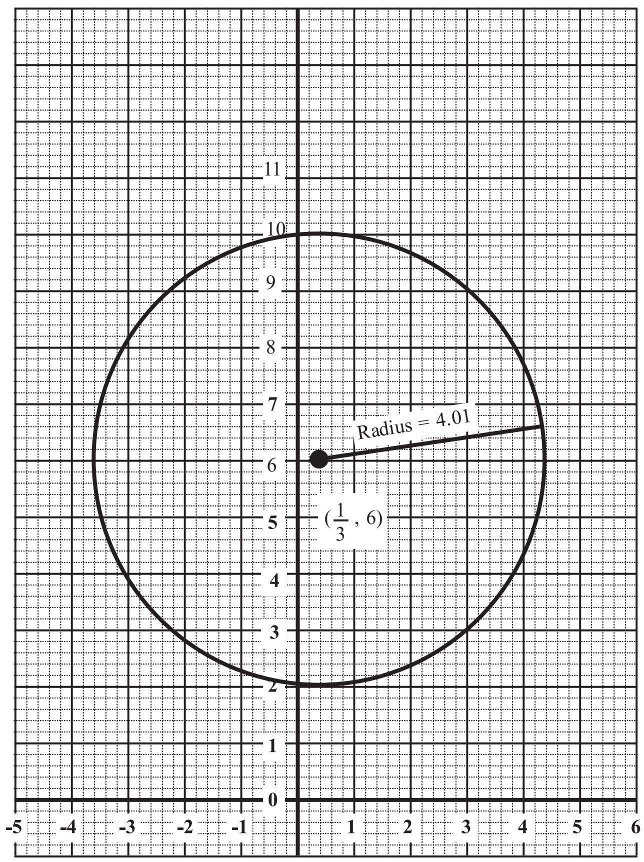
PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(b) (i)	Derive the complex number of the form $u = x + iy$, where $x,\ y \in R$, and $u^2 = -3 + 4i$ $(x+iy)^2 = -3+4i$ $\Rightarrow x^2 + 2ixy - y^2 = -3 + 4i$ expansion of u^2 **[1 mark]** $\Rightarrow x^2 - y^2 = -3$ equating real and imaginary parts $2xy = 4$ **[1 mark]** $\Rightarrow xy = 2$ $\Rightarrow y = \frac{2}{x}$ Simplifying for y **[1 mark]** $\Rightarrow x^2 - \left(\frac{2}{x}\right)^2 = -3$ $\Rightarrow \left(x^2\right)^2 - 3x^2 - 4 = 0$ Simplifying equation **[1 mark]** $\Rightarrow \left(x^2-4\right)\left(x^2+1\right) = 0$ $\therefore x^2 - 4 = 0$ *or* $x^2 = -1$ (inadmissible) solving **[1 mark]** $\Rightarrow x = \pm 2$ and $y = \mp 2$ Required solution: $2-2i, -2+2i$ **[1 mark]** at least one solution	**6**

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(ii)	Hence solve the equation $z^2 - z + (1-i) = 0$ $z = \frac{-i \pm \sqrt{-3+4i}}{2}$ **[1 mark]** $z = \frac{-i \pm (2-2i)}{2}$ OR $\frac{-i \pm (2-2i)}{2}$ **[1 mark]** correct values for $\sqrt{-3+4i}$ $z = \frac{2-3i}{2}$ OR $z = \frac{-2+i}{2}$ **[1 mark]** correct simplification of 'his' values $z = 1 - \frac{3}{2}i$ OR $z = -1 + \frac{1}{2}i$ in the form $a+bi$ **[1 mark]**	4
	(c) (i)	The complex number $z = x+iy$ satisfies the equation $\lvert z-3+2i\rvert = 2\lvert z-1-4i\rvert$ The complex number z is represented by the point P in an Argand diagram. Show that the locus of P is a circle. $\lvert x+iy-3+2i\rvert = 2\lvert x+iy-1-4i\rvert$ $\lvert (x-3)+(y+2)i\rvert = 2\lvert (x-1)+(y-4)i\rvert$ **[1 mark]** Substitution $(x-3)^2 + (y+2)^2 = 4(x-1)^2 + 4(y-4)^2$ Modulus $x^2 - 6x + 9 + y^2 + 4y + 4 = 4x^2 - 8x + 4 + 4y^2 - 32y + 64$ **[1 mark]** $3x^2 + 3y^2 - 2x - 36y + 60 = 0$ **[1 mark]** (1 mark) Simplification $x^2 + y^2 - \frac{2}{3}x - 12y + 20 = 0$ **[1 mark]** Correct equation	4

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(c)(ii)	State the centre and radius of the circle. Centre = $\left(\frac{1}{3}, 6\right)$ **[1 mark]** Radius = 4.01 **[1 mark]**	**2**
	(iii)	 Drawing of circle **[1 mark]** Displaying the centre and radius **[1 mark]**	**2**
		TOTAL	**25**
		Specific Objectives: 1.1, 1.2, 1.3, 1.4, 1.5, 1.7, 1.8, 1.10	

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
2.	(a)	Let I = $\int e^{2x}\cos 3x\,dx$ $=\frac{1}{2}e^{2x}\cos 3x+\int\frac{1}{2}e^{2x}3\sin 3x\,dx$ Integration by parts correct process **[1 mark]** $=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{2}\int e^{2x}\sin 3x\,dx$ **[1 mark]** at least 1 part correct $=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{2}\left[\frac{1}{2}e^{2x}\sin 3x-\int\frac{1}{2}e^{2x}3\cos 3x\,dx\right]$ **[1 mark]** Repeat integration $=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{4}e^{2x}\sin 3x-\frac{9}{4}\int e^{2x}\cos 3x\,dx$ $I=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{4}e^{2x}\sin 3x-\frac{9}{4}I$ **[2 marks]** $\frac{13}{4}I=\frac{1}{2}e^{2x}\cos 3x+\frac{3}{4}e^{2x}\sin 3x$ $I=\frac{4}{13}\times\frac{1}{4}e^{2x}\left[2\cos 3x+3\sin 3x\right]$ **[1 mark]** Simplification $I=\frac{1}{13}e^{2x}\left(2\cos 3x+3\sin 3x\right)+c$ **[1 mark]** **<u>Alternate Solution</u>** $I=\mathrm{Re}\int e^{2x}e^{3ix}dx$ Real part$=\int e^{(2+3i)x}dx$ exponential form $\Rightarrow\mathrm{Re}\left[\frac{e(2+3i)x}{2+3i}\right]$+constant integration $=\mathrm{Re}\frac{2-3i}{(2+3i)(2+3i)}e^{2x}\left(\cos 3x+i\sin 3x\right)$ conjugation $=\mathrm{Re}\frac{e2x}{13}(2-3i)(\cos 3x+i\sin 3x)$ simplification $\Rightarrow\int e^{2x}\cos 3x\,dx=\frac{e^{2x}}{13}(2\cos 3x+3\sin 3x)$ real part $\frac{e^{2x}}{13}(3\sin 2x-2\cos 2x)$+const.+constant	7

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(b) (i)	$x^2+4y^2=1$ $2x+8y\dfrac{dy}{dx}=0 \qquad (1\,mark)$ **[1 mark]** differentiating x^2 differentiating $4y^2$ $8y\dfrac{dy}{dx}=-2x \qquad (1\,mark)$ **[1 mark]** $\dfrac{dy}{dx}=\dfrac{-x}{4y} \qquad (1\,mark)$ **[1 mark]**	**3**
	(ii)	$\dfrac{d^2y}{dx^2}=\dfrac{-4y+4x\dfrac{dy}{dx}}{16y^2}$ correct formula $=\dfrac{-y+x\dfrac{dy}{dx}}{4y^2}$ **[1 mark]** correct substitution $=\dfrac{-y-\dfrac{x^2}{4y}}{4y^2}$ **[1 mark]** $=\dfrac{-4y^2-x^2}{16y^3}$ **[1 mark]** simplification $=\dfrac{-1}{16y^3}$ **[1 mark]**	**4**

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(c)	$u = 1 + x^2$ $du = 2xdx$ **[1 mark]** Differentiating When $x = 0,\ u = 1$ When $x = 1, u = 2$ **[1 mark]** $\int_0^1 \frac{x^3}{\left(1+x^2\right)^4} dx = \int_1^2 \frac{1}{2}\left(\frac{(u-1)}{u^4}\right) du$ substituting $= \frac{1}{2}\int_1^2 \left(\frac{1}{u^3} - \frac{1}{u^4}\right) du = \frac{1}{2}\int_1^2 \left(u^{-3} - u^{-4}\right) du$ **[1 mark]** $= \frac{1}{2}\left[\frac{-1}{2}u^{-2} + \frac{1}{3}u^{-3}\right]_1^2$ **[1 mark]** integration $= \frac{1}{2}\left[\frac{-1}{8} + \frac{1}{24} + \frac{1}{2} - \frac{1}{3}\right]$ **[2 marks]** evaluation $= \frac{1}{24}$ **[1 mark]**	7
	(d) (i)	$\frac{\partial f}{\partial x} = 6x - y$ **[1 mark]** $\frac{\partial f}{\partial y} = -x - 6y$ **[1 mark]**	2
	(ii)	Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ **[1 mark]** $6 - 6 = 0$ **[1 mark]**	2
		TOTAL	25
		Specific Objectives: 2.4, 2.7, 2.8, 3.6, 3.7	

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
3.	(a) (i)	$\Rightarrow \frac{1}{(1-2r)(1+2r)} = \frac{A}{1-2r} + \frac{B}{1+2r}$ **[1 mark]** $\Rightarrow 1 = A(1+2r) + B(1-2r)$ **[1 mark]** $\Rightarrow 0 = 2A - 2B$ and $A + B = 1$ **[1 mark]** $A = \frac{1}{2}$ and $B = \frac{1}{2}$ **[2 marks]**	5
	(ii)	$s = \sum_{r-1}^{n} \frac{1}{(1-2r)(1+2r)} = \sum_{r-1}^{n} \frac{1}{2}\left(\frac{1}{1-2r} + \frac{1}{1+2r}\right)$ **[1 mark]** $= \frac{1}{2}\left(-\frac{1}{1} + \frac{1}{3}\right) + \frac{1}{2}\left(-\frac{1}{3} + \frac{1}{5}\right) + \frac{1}{2}\left(-\frac{1}{5} + \frac{1}{7}\right) + \ldots + \frac{1}{2}\left(-\frac{1}{1-2n} + \frac{1}{1+2n}\right)$ **[3 marks]** Any three brackets, 1 mark each $= \frac{1}{2}\left(-1 + \frac{1}{1+2n}\right)$ **[1 mark]**	5
	(iii)	As $n \to \infty,\ \frac{1}{1+2n} \to 0$ **[2 marks]** Hence $S_\infty = -\frac{1}{2}$ **[1 mark]**	3
	(b) (i)	S = 1(8) + 2(20) + 3(32) + … **[1 mark]** In each term, 1[st] factor is in the natural sequence and the second factor differs by 12 $\Rightarrow$ the r[th] term is $r(12r-4)$ **[1 mark]**	2

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(ii)	$S_n = \sum_{r-1}^{n} r(12r-4)$ for $n=1$ $S_1 = \sum_{r-1}^{1} r(12r-4) = 1\times 8 = 8$ and $2(1)^2[2(1)+2] = 2\times 4 = 8$ hence, $S_n = 2n^2(2n+2)$ **[1 mark]** is true for $n=1$ **[1 mark]** Assume $S_n = 2n^2(2n+2)$ for $\in n = k \in$ □ that is, $S_k = 2k^2(2k+2)$ **[1 mark]** Then, $S_{k+1} = \sum_{r-1}^{k+1}(12r-4) = S_k(k+1)(12k+8)$ **[1 mark]** $= 2k^2(2k+2)+(k+1)(12k+8)$ **[1 mark]** $= 4k^2(k+1)+4(k+1)(3k+2)$ **[1 mark]** $= 4(k+1)+[k^2+3k+2]$ **[1 mark]** $\Rightarrow S_{k+1} = 4(k+1)[(k+1)+(k+2)]$ $= 4(k+1)^2(k+2)$ **[1 mark]** $= 2(k+1)^2 \times 2[(k+1)+1]$ $= 2(k+1)^2 \times [2(k+1)+2]$ **[1 mark]** $\Rightarrow$ true for $n=k+1$ whenever it is assumed true for $n=k$, $\Rightarrow$ true for all $n \in N$ $\Rightarrow S_n = 2n^2(2n+2)\, n \in N$ **[1 mark]**	10
		TOTAL	25
		Specific Objectives: 2.2, 2.4, 2.5, 2.6, 3.1	

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
4.	(a) (i)	$s=\frac{1}{3}+\frac{1}{3^4}+\frac{1}{3^{\rightarrow}}+\frac{1}{3^{10}}+\ldots$ $\frac{\frac{1}{3^4}}{\frac{1}{3}}=\frac{\frac{1}{3^7}}{\frac{1}{3^4}}$ **[1 mark]** Let $=\frac{1}{3^3}$ **[1 mark]** $\therefore$S is geometric with common ratio **[1 mark]** $r=\frac{1}{3^3}\,or\,\frac{1}{27}$	3
	(ii)	$S_n\,\frac{\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^{3n}\right]}{1-\left(\frac{1}{3}\right)^{3}}$ **[1 mark]** $=\frac{\frac{1}{3}\left[1-\frac{1}{3^{3n}}\right]}{1-\frac{1}{27}}$ **[1 mark]** $=\frac{1}{3}\times\frac{27}{26}\left[1-\frac{1}{3^{3n}}\right]$ **[1 mark]** $=\frac{9}{26}\left[1-\frac{1}{3^{3n}}\right]or\,\frac{9}{26}\left[1-\frac{1}{27^n}\right]$ **[1 mark]**	4

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(b)	$f(x)=\sin 2x$ $\Rightarrow \quad f'(x)=2\cos 2x$ **[1 mark]** $\Rightarrow \quad f''(x)=-4\sin 2x$ **[1 mark]** $\Rightarrow \quad f'''(x)=-8\cos 2x$ **[1 mark]** $\Rightarrow \quad f^{iv}(x)=16\sin 2x$ **[1 mark]** So, $f(0)=0,\ f'(0)=2,\ f''(0)=0, f'''(0)=-8,\ f^{iv}(0)=0$ **[1 mark]** Hence, by Maclaurin's Theorem, $\sin 2x = 2x-\frac{8x^3}{3i}+\frac{32x^5}{5i}+...$ **[1 mark]** $=2x-\frac{4}{3}x^3+\frac{4}{15}x^5$ **[1 mark]**	7
	(c) (i)	$\sqrt{\left(\frac{1+x}{1-x}\right)}$ $=(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ **[1 mark]** $=\left(1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3...\right)\left(1+\frac{1}{2}x-\frac{3}{8}x^2+\frac{5}{16}x^3...\right)$ **[3 marks]** $=1+x+\frac{1}{2}x^2+\frac{1}{2}x^3$ **[2 marks]** for $-1<x<1$ **[1 mark]**	7
	(ii)	$\sqrt{\frac{1.04}{0.96}}=\sqrt{\frac{104}{96}}=\frac{1}{4}\sqrt{\frac{52}{3}}$ **[1 mark]** $\sqrt{\frac{52}{3}}=4\sqrt{\frac{1+x}{1-x}}$ **[1 mark]** Where $x=0.04$ $\Rightarrow \sqrt{\frac{52}{3}}=4\left\{1+0.04+\frac{1}{2}(0.04)^2+\frac{1}{2}(0.04)^3\right\}$ **[1 mark]** $=4.163 \quad$ (3 d.p.) **[1 mark]**	4
		TOTAL	**25**
		Specific Objectives: 2.2, 2.3, 2.7, 3.2	

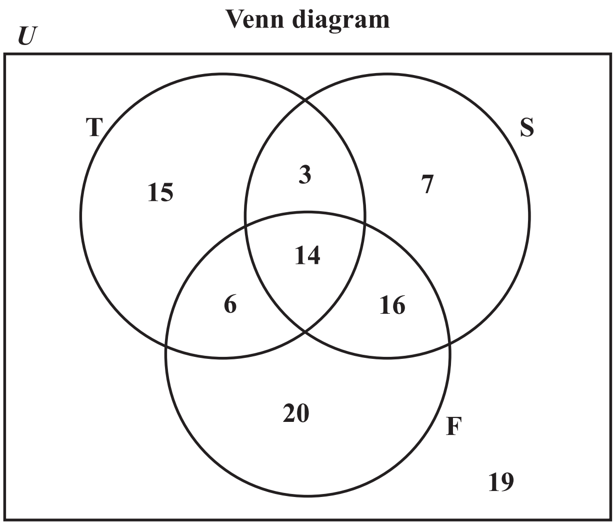
PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
5.	**(a) (i)**	2 fair coins and 1 fair die are tossed at the same time. The number of outcomes for the coins are HH, HT, TH, TT so we have 4 outcomes and the number of outcomes for the die is 6. Therefore, there are $4\times 6=4$ outcomes. Number of outcomes on the die **[1 mark]** Number of outcomes on the 2 coins **[1 mark]** Computation of all possible outcomes **[1 mark]**	**3**
	(ii)	There are 2 outcomes with 1 head. Therefore, we have $2\times 6=12$ outcomes with exactly 1 head. The probability of obtaining exactly 1 head is given by $\frac{12}{24}=0.5$. Recognition that this can be obtained 2 possible ways (HT, TH) with any of the 6 possibilities on the die **[1 mark]** Computation of probability based on the space **[1 mark]**	**2**

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(iii)	The number of outcomes with at least 1 head is 3 and the number of even numbers is 3. Therefore there are $3\times3=9$ outcomes with at least one head and an even number. The probability of obtaining at least 1 head and an even number is given by $\frac{9}{24}=0.375$ Recognition that there are 3 cases with at least 1 head. **[1 mark]** Recognition of number of outcomes on the die **[1 mark]** Calculation of the number of possibilities **[1 mark]** Calculation of probability **[1 mark]**	4
	(b) (i)	The probability that it is non-fiction, borrowed by a boy is given by $\frac{5}{100}=\frac{1}{20}$ Recognition that the cell intersecting non-fiction and boy is 5 **[1 mark]** Total possible outcomes = 100 **[1 mark]**	2
	(ii)	The probability that it is fiction, borrowed by a female is given by $\frac{30}{100}=\frac{3}{10}$ Recognition that the cells women and girl intersecting fiction total 30 **[1 mark]** Total possible outcomes = 100 **[1 mark]** Correct computation **[1 mark]**	3

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(iii)	The probability that it was borrowed by a man (M), given that it is fiction(F) is given by $P(M/F)=\dfrac{P(M\cap F)}{P(F)}=\dfrac{17/100}{61/100}=\dfrac{17}{61}$ Recognition that it is conditional probability **[1 mark]** Correct formula used **[1 mark]** Correct answer **[1 mark]**	3
	(c) (i)	Let T, S and F represent respectively the customers purchasing tools, seeds and fertilizer. One mark for each circle correct **[3 marks]** 	3

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(ii) a)	$P(T \cup S)' = \frac{39}{100}$ Numerator $(19+20=39)$ **[1 mark]** Correct answer $\frac{39}{100}$ **[1 mark]**	2
	b)	$P(S) = \frac{40}{100}$ (from candidate's diagram) $P(F) = \frac{56}{100}$ (from candidate's diagram) $P(S \cap F) = \frac{30}{100}$ (from candidate's diagram) $P(S) \times P(F) = \frac{28}{125}$ Since $P(S \cap F) \neq P(S) \times P(F)$ then S and F are not independent $P(S \cap F) = \frac{30}{100}$ **[1 mark]** $P(S) \times P(F) = \frac{28}{125}$ **[1 mark]** Conclusion (not independent or dependent) **[1 mark]**	3
		TOTAL	25
		Specific Objectives: 1.4, 1.5, 1.6, 1.7, 1.9, 1.12, 1.13	

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
6.	(a) (i)	Given $A=\begin{pmatrix}2&1&-1\\0&4&3\\-1&6&0\end{pmatrix}$ $\lvert A\rvert=2\begin{vmatrix}4&3\\6&0\end{vmatrix}-1\begin{vmatrix}0&3\\-1&0\end{vmatrix}+(-1)\begin{vmatrix}0&4\\-1&6\end{vmatrix}$ **[3 marks]** $\lvert A\rvert=2(-18)-1(3)-(4)=-43$ **[1 mark]** Determinant of the A **[1 mark]**	5
	(ii)	Cofactor of $A=\begin{pmatrix}-18&-3&4\\-6&-1&-13\\7&-6&8\end{pmatrix}$ Adjoint of $A=\begin{pmatrix}-18&-6&7\\-3&-1&-6\\4&-13&8\end{pmatrix}$ $A^{-1}=-\frac{1}{43}\begin{pmatrix}-18&-6&7\\-3&-1&-6\\4&-13&8\end{pmatrix}$ One mark for each correct row **[3 marks]** Transposing the matrix of cofactors **[1 mark]** Multiplying the reciprocal of the determinant by the Adjoint matrix **[1 mark]**	5

	Question	Solutions	Total
	(b)	$e^{\int \frac{1}{t+10}dt} = e^{2\ln(t+10)} = (t+10)^2$ $y(t+10)^2 = 3\int (t+10)^2$ $= 3\frac{(t+10)^3}{3} + c$ $= (t+10)^3 + c$ $y = \frac{(t+10)^3 + c}{(t+10)^2}$ $= t + 10 + \frac{c}{(t+10)^2}$ Integrating $\frac{1}{t+10}$ correctly **[1 mark]** Integrating factor (**[1 mark]** Multiplying the DE by the IF **[1 mark]** Integrating **[1 mark]** Dividing by IF **[1 mark]** Isolating y **[1 mark]** Expressing the answer in the correct form **[1 mark]**	**7**

PURE MATHEMATICS
UNIT 2 — Paper 02
KEY AND MARK SCHEME

	Question	Solutions	Total
	(c)	$m^2-3m-4=0$ $(m-4)(m+1)=0$ $m=4;\ m=-1$ $y_c=Ae^{4x}+Be^{-x}$ $y_p=Cx^2+Dx+E$ $y'_p=2Cx+D$ $y''_p=2C$ Sub into DE $2C-3(2Cx+D)-4(Cx^2+Dx+E)=8x^2$ $2C-6Cx-3D-4Cx^2-4Dx-4E=8x^2$ Equating coefficients $-4C=8\Rightarrow C=-2$ $-6C-4D=0$ $-6(-2)-4D=0\Rightarrow D=3$ $2C-3D-4E=0$ $2(-2)-3(3)-4E=0\Rightarrow E=-\frac{13}{4}$ $\therefore y=Ae^{4x}+Be^{-x}-2x^2+3x-\frac{13}{4}$ Auxiliary equation **[1 mark]** Complementary function **[1 mark]** Particular integral **[1 mark]** Derivative **[1 mark]** Values of C, D, E **[3 marks]** General solution **[1 mark]**	8
		TOTAL	25
		Specific Objectives: 2.3, 2.4, 3.1, 3.2	

TEST CODE **02234032**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2

COMPLEX NUMBERS, ANALYSIS AND MATRICES

SPECIMEN PAPER

PAPER 032

2 hours

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE questions.
2. Answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

***The questions on this paper may be based on Specific Objectives taken from ANY Module in the Unit.**

<u>Examination Materials Permitted</u>

Mathematical formulae and tables (provided) – **Revised 2022**
Mathematical instruments
Silent, non-programmable electronic calculator

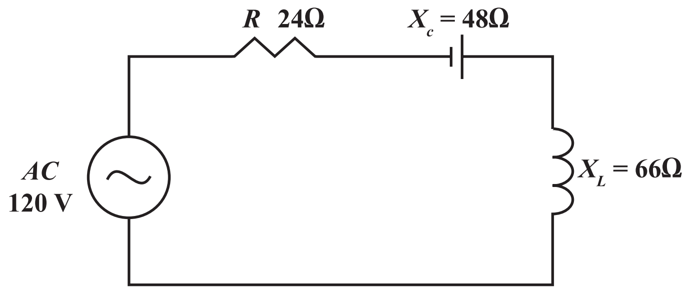
DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

"*"Barcode Area"*"
Sequential Bar Code

SECTION A

Module 1

1. The diagram below shows an electrical circuit consisting of a resistor (R), an inductor (L) and a capacitor (C).



(a) The impedance, Z, of the circuit may be represented by the equation $Z = R + i|X_L - X_C|$.

With the aid of a suitable diagram, express the impedance in polar form where $-\pi < \arg Z \le \pi$.

[7 marks]

GO ON TO THE NEXT PAGE

(b) Sugar is poured into a cylindrical tower. As the sugar is poured it forms a right circular cone. The base radius is increasing at the rate of 3m/s, while the perpendicular height of the cone is increasing at a rate of 1.2m/s.

[Volume of a right circular cone, $V=\frac{1}{3}\pi r^2 h$]

(i) Write an expression for the rate of change of the volume with respect to

a) the radius

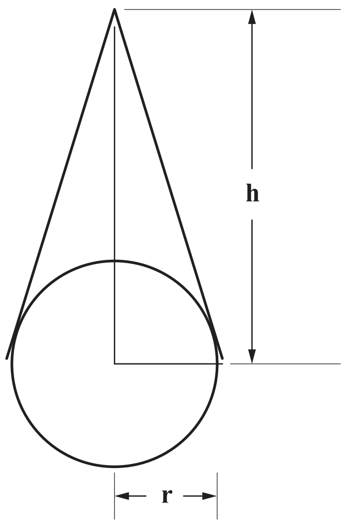
[2 marks]

b) the height

[1 mark]

(ii) Given that $r = 2.4m$ and $h = 4.8m$, determine the rate at which the volume, V, is changing.

Hint: $\frac{dV}{dt}=\frac{\partial V}{\partial r}.\frac{dr}{dt}+\frac{\partial V}{\partial h}.\frac{dh}{dt}$



[3 marks]

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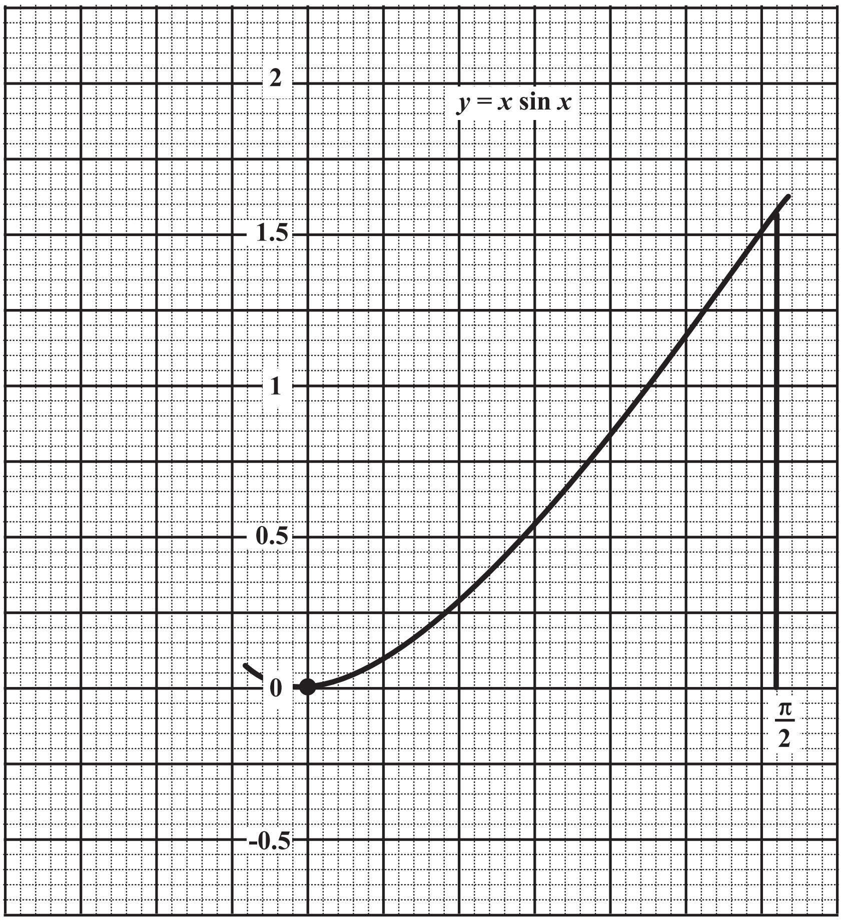
(c) The diagram below, **not drawn to scale**, shows the curve $y = x\sin x \ for \ 0 \le x \le \frac{\pi}{2}$.



(i) Determine the exact value of $\int_0^{\frac{\pi}{2}} x\sin x \, dx$.

[3 marks]

GO ON TO THE NEXT PAGE

(ii) Using the trapezium rule, with 2 intervals, estimate the value of $\int_0^{\frac{\pi}{2}} x \sin x \, dx$, giving your answer correct to 2 decimal places.

[2 marks]

(iii) With the aid of a geometrical representation, explain why the trapezium rule gives an overestimate of the value found in (c) (i).

[2 marks]

Total 20 marks

SECTION B

Module 2

2. (a) In a certain country, a construction company agrees to pay its construction workers a starting salary of \$2000 per month in the first year of employment. Workers will then be entitled to a yearly increase of 5% of the current years' monthly salary.

(i) How many years will it take Mr Jack to receive a salary of \$2555 per month?

[3 marks]

(ii) What will be the TOTAL salary paid to a person for the first three years of working with the company.

[4 marks]

(iii) No increases in salaries are given after 10 years working with the company. Calculate the monthly salary of a person who has been with the company for more than 10 years.

[2 marks]

(b) (i) A function is given by the formula $f(x) = 1 - 2x$ for all positive values of x. Generate the expansion of $(1 - 2x)^5$ giving each term in its simplest form.

[3 marks]

(ii) If x is small enough so that x^2 and higher powers of x can be ignored, show that $(1+x)(1-2x)^5 \approx 1-9x$.

[2 marks]

(c) (i) Show that the function $f(x) = x^3 + x - 3$ has a root between $x = 0$ and $x = 4$.

[2 marks]

(ii) Use the method of interval bisection to estimate the value of the root correct to 1 decimal place.

[4 marks]

Total 20 marks

SECTION C

Module 3

3. (a) At a fruit store, Mary paid \$30 for 2 kg of apples, 1 kg of cherries and 4 kg of plums. Gerry paid \$38 for 1 kg of apples, 2 kg of cherries and 1 kg of plums. Lester paid \$42 for 1 kg of apples, 1 kg of cherries and 2 kg of plums.

(i) Write this information as a matrix equation in the form $AX = B$.

[3 marks]

(ii) Using the row reduction method, calculate the cost per kilogram of each of the three fruits.

[5 marks]

GO ON TO THE NEXT PAGE

(b) A committee of 8 persons is to be seated at a round table.

(i) In how many ways can this be done

a) without restrictions

[1 mark]

b) if the chairman and the secretary must sit next to each other.

[2 marks]

(ii) Calculate the probability that the chairman and the secretary are NOT seated next to each other.

[2 marks]

(c) (i) The amount of salt, y kg, that dissolves in a tank of water after a time t minutes satisfies the first order differential equation $\frac{dy}{dt}+2y(t+10)=3$. Using a suitable integrating factor, show that the general solution of this differential equation is $y=t+10+\frac{c}{(t+10)^2}$ where c is an arbitrary constant.

[5 marks]

(ii) Initially the amount of salt put into the tank is 5kg. How much salt will be dissolved after 15 minutes?

[2 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATIONS®

PURE MATHEMATICS

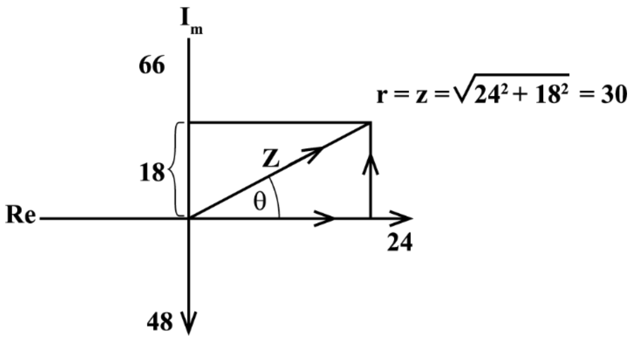
UNIT 2 — Paper 032

KEY AND MARK SCHEME

MAY/JUNE 2022

SPECIMEN PAPER

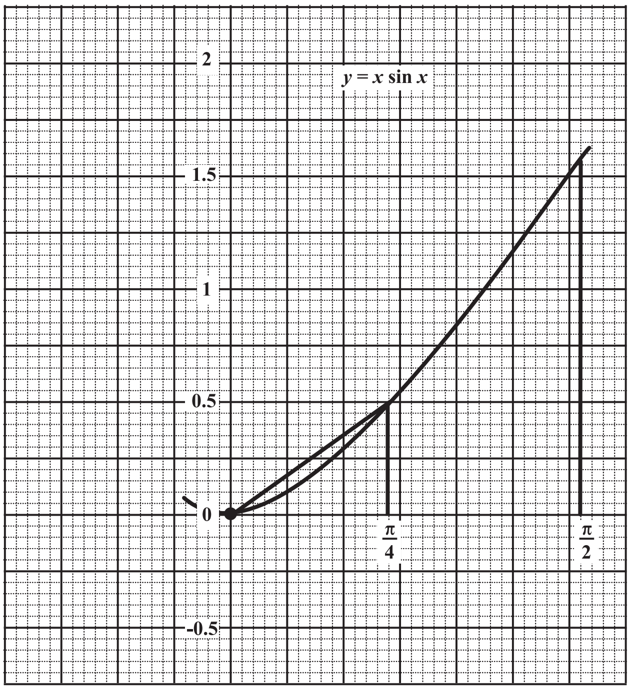
PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
1.	(a)	The impedance is: $Z = R + i\lvert X_L - X_C \rvert$ $\lvert z \rvert = \sqrt{R^2 + \lvert X_L - X_C \rvert^2}$ **[1 mark]** $= \sqrt{24^2 + \lvert 66-48 \rvert^2}$ **[1 mark]** $= \sqrt{576+324}$ **[1 mark]** $= 30$ **[1 mark]** $\arg Z = \tan^{-1}\left(\frac{18}{24}\right)$ **[1 mark]** $= 0.643\,radians$ The polar form of the impedance, $Z = 30(\cos 0.643 + i \sin 0.643)$ **[1 mark]**  **[1 mark]**	7
	(b) (i)	$\frac{\partial V}{\partial r} = \frac{2\pi rh}{3}$ **[2 marks]** $\frac{\partial V}{\partial r} = \frac{\pi r^2}{3}$ **[1 mark]**	3

PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
	(ii)	Given that $r = 2.4m,\ h = 4.8m,$ then, $\dfrac{dr}{dt} = 3ms^{-1}$ and $\dfrac{dh}{dt} = 1.2ms^{-1}$ **[1 mark]** $\dfrac{dV}{dt} = \dfrac{\partial V}{\partial r}.\dfrac{dr}{dt} + \dfrac{\partial V}{\partial h}.\dfrac{dh}{dt}$ $\dfrac{dV}{dt} = \dfrac{2\pi(2.4)(4.8)}{3} \times 3 + \dfrac{\pi(2.4)^2}{3} \times (-1.2)$ **[1 mark]** $= 23.04\pi - 23.04\pi$ $= 46.08\pi\ m^3s^{-1}$ **[1 mark]**	**3**
	(c) (i)	Let $v = x$ $\quad \dfrac{du}{dx} = \sin x$ **[1 mark]** $\dfrac{dv}{dx} = 1$ $\quad u = -\cos x$ $\int_0^{\frac{\pi}{2}} x \sin x\, dx = [-x\cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\, dx$ **[1 mark]** $= [\sin x]_0^{\frac{\pi}{2}}$ $= \sin\left(\dfrac{\pi}{2}\right) - \sin 0$ $= 1$ **[1 mark]**	**3**
	(ii)	x: 0, $\dfrac{\pi}{4}$, $\dfrac{\pi}{2}$ y: 0, 0.5554, 1.5708 $A = \left(\dfrac{1}{2}\right)\left(\dfrac{\pi}{4}\right)\left[(0 + 1.5708) + 2(0.5554)\right]$ **[1 mark]** $= 1.05306$ $= 1.05(2\ d.p)$ **[1 mark]**	**2**

PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
	(iii)	 **[1 mark]** Since the trapezia lies above the curve, the trapezium rule is an over-estimate of the true value. **[1 mark]**	2
		TOTAL	20
		Specific Objectives: Module 1, 1.6; 1.7; 1.8, 1.12, 2.8, 3.1, 3.2, 3.7, 3.11	

PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
2.	(a) (i)	$t_n = t_a r^{n-1}$ formula for the nth term of a sequence $t_n = 2550 \qquad t_a = 2000 \qquad r = 1.05$ **[1 mark]** $2555 = 2000(1.05)^{n-1}$ correct substitution into formula $\frac{2555}{2000} = 1.05^{n-1}$ $\frac{\log 1.2775}{\log 1.05} = n-1$ **[1 mark]** Correct application of logarithms n = 6.01 years **[1 mark]**	3
	(ii)	Since salaries are paid month, it will be necessary to calculate the sum per year, and then add. Year 1: 12 × 2000 = 24000 **[1 mark]** Year 2: 12 × 2100 = 25200 **[1 mark]** Year 3: 12 × 2205 = 26460 **[1 mark]** Total salary for 3 years = $75660 **[1 mark]** addition of their values	4
	(iii)	$t_{10} = 2000(1.05)^9$ substitution into formula = 3102.66 **[1 mark]** Monthly salary after 10 years will be $3102.66 **[1 mark]**	2
	(b) (i)	Generate the expansion of $(1-2x)^5$ giving each term in its simplest form. $1+5(-2x)+{}^5C_2(-2x)^2+{}^5C_3(-2x)^3+{}^5C_4(-2x)^4+{}^5C_5(-2x)^5$ Expansion with 4 to 6 terms correct **[2 marks]** **[3 terms correct, 1 mark]** $=1-10x+40x^2-80x^3+80x^4-32x^5$ **[1 mark]** simplifying terms	3

PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
	(ii)	$(1+x)(1-2x)^5$ $=1-10x+40x^2+...+x-10x^2+...$ **[1 mark]** Multiplying the brackets $=1-10x + x+...$ **[1 mark]** Choosing only correct terms $\approx 1-9x$	2
	(c) (i)	$f(0)=-3 \quad f(4)=65$ **[1 mark]** the two signs are different, therefore the functions cross the x-axis between 0 and **[1 mark]**	2
	(ii)	mid-point of [0,4] is $x=2$ **[1 mark]** $f(2) = 7$ root lies between $x = 0$ *and* $x = 2$ **[1 mark]** midpoint of [0,2] is $x=1$ $f(1) = -1$ roots lie between $x = 1$ *and* $x = 2$ midpoint of [1,2] is $x = 1.5$ $f(1.5) = 1.83$ root lies between $x = 1$ *and* $x = 1.5$ midpoint of [1, 1.5] is 1.25 $f(1.25) = 1.32$ root lies between $x = 1$ *and* $x = 1.25$ **[1 mark]** midpoint of [1, 1.25] is 1.125 $f(1.125) = 1.141$ root can be approximated $x = 1$ **[1 mark]**	4
		TOTAL	20
		Specific Objectives: Module 2, 2.2, 3.2, 4.1, 4.2	

PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
3.	(a) (i)	$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 30 \\ 38 \\ 42 \end{pmatrix}$ x is the cost of 1 kg apples **[1 mark]** y is the cost of 1 kg cherries z is the cost of 1 kg plums variables defined matrix A correct **[1 mark]** matrix B correct **[1 mark]**	3
	(ii)	Form an augmented matrix and use the method of row reduction to reduce the Augmented matrix to echelon form. $\left(\begin{array}{ccc\|c} 2 & 1 & 1 & 30 \\ 1 & 2 & 1 & 38 \\ 1 & 1 & 2 & 42 \end{array}\right)$ **Augmented matrix** **[1 mark]** $\left(\begin{array}{ccc\|c} 2 & 1 & 1 & 30 \\ 0 & 3 & 1 & 46 \\ 0 & 1 & 3 & 54 \end{array}\right)$ $\mathbf{2R_2 - R_1}$ $\mathbf{2R_3 - R_1}$ $\left(\begin{array}{ccc\|c} 2 & 1 & 1 & 30 \\ 0 & 3 & 1 & 46 \\ 0 & 0 & 8 & 116 \end{array}\right)$ $\mathbf{2R_3 - R_3}$	1

PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
		Row reduction to echelon form **[2 marks]** $8z = 116$ $z = \frac{116}{8} = 14.50$ $3y + z = 46$ $3y = 46 - z$ $y = \frac{46 - 14.5}{3}$ $= 10.50$ $2x + y + z = 30$ $2x = 30 - y - z$ $x = \frac{30 - 10.5 - 14.50}{2}$ $= 2.50$ $x = \$2.50;\ y = \$10.50;\ z = \$14.50$ **at least 2 variables correct** **[2 marks]**	**4**
	(b) (i)	In how many ways can the committee be seated a. With no restrictions 7! = 5040 **[1 mark]** b. The chairman and the secretary must sit together. 6! × 2 = 1440 **[2 marks]**	**3**

PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
	(ii)	Probability that the chairman and the secretary are NOT seated together $1-\frac{6!x2}{7!}$ OR $1-\frac{1440}{5040}=1-0.286$ $=1-\frac{2}{7}=\frac{5}{7}$ Probability **[1 mark]** = 0.714 Subtraction from 1 **[1 mark]**	**2**
	(c) (i)	Integrating Factor = $e^{\int \frac{2}{t+10}dt}$ **[1 mark]** $=e^{2\ln(t+10)}$ **[1 mark]** $=(t+10)^2 = 3\int (t+10)^2 dt$ $=3\frac{(t+10)^3}{3}+c$ **[2 marks]** $=(t+10)^3+c$ $y=\frac{(t+10)^3+c}{(t+10)^2}$ **[1 mark]** $=t+10+\frac{c}{(t+10)^2}$	**5**

PURE MATHEMATICS
UNIT 2 — Paper 032
KEY AND MARK SCHEME

	Question	Solutions	Total
	(ii)	At $t=0,\ y=5kg$, $5=10+\dfrac{c}{(0+10)^2}$ $-5=\dfrac{c}{100}$ $-500=c$ **[1 mark]** $y=25-\dfrac{500}{625}$ $y=25-0.8$ $=24.2kg$ **[1 mark]**	2
		TOTAL	**20**
		Specific Objectives: Module 3, 1.1, 1.2, 1.11, 2.5, 2.6, 2.9, 3.1	

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APPLIED MATHEMATICS
(Including Statistical Analysis)

INTEGRATED MATHEMATICS

Statistical Tables

and

List of Formulae

Revised April 2022

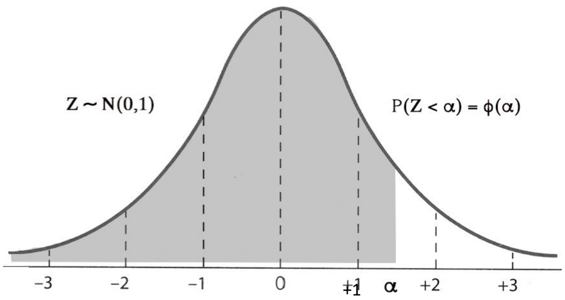
DO NOT REMOVE FROM THE EXAMINATION ROOM

Table 1: The Normal Distribution Function

If Z is a random variable, normally distributed with zero mean and unit variance, then ϕ (z) is the probability that $Z \le z$. That is, $\phi(z) = P(Z \le z)$.

The function tabulated below is ϕ(z), and is shown diagrammatically as

Standard Normal Distribution (area to the left of α)

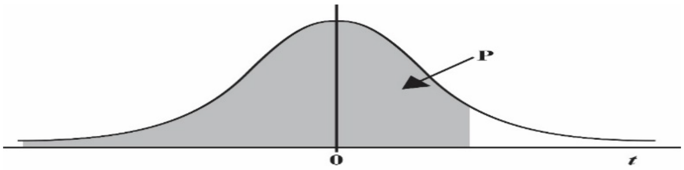


The Distribution Function, ϕ(z)

Z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6590	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0 7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9046	0.9148	0.9429	0.9441	1	2	4	1	2	4	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	1	2	3	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	1	2	3	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	1	1	2	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9760	0.9767	1	1	2	1	1	2	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	0	1	1	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	0	1	1	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	0	1	1	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	0	1	1	2	2	2
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	0	0	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Table 2: _t-Distribution_

If T has a t-distribution with v degrees of freedom then, for each pair of values of p and v, the table gives the value of t such that $P(T \leq t) = P$

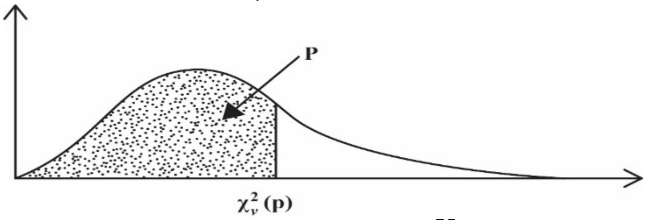


Critical Values for the *t*-distribution

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
V = 1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	4.317	5.208	5.959
12	0.695	1.356	1.782	2.179	2.681	3.055	4.029	4.785	5.408
13	0.694	1.350	1.771	2.160	2.650	3.012	3.833	4.501	5.041
14	0.692	1.345	1.761	2.145	2.624	2.977	3.690	4.297	4.781
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Table 3: Percentage Points of the x^2 Distribution

If X is a random variable, distributed as X^2 with v degrees of freedom then p is the probability that $X \leq \chi_v^2$ (p), where the values of the percentage points χ_v^2 (p), are tabulated in the table below. p is shown diagrammatically (when $v \geq 3$) as



Critical Values for the X^2- distribution

P	.01	.025	.050	.900	.950	.975	. 990	.995	.999
v = 1	0.0001571	0.0009821	0.003932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7 779	9.488	11.14	13.28	14 86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.52
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.646	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.72	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28 41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	104.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.76	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

Table 4: <u>Random Sampling Numbers</u>

18	11	36	26	88	81	11	33	64	08	23	32	00	73	04
57	33	88	37	26	10	79	91	36	03	07	52	55	84	61
72	02	11	44	25	45	92	12	82	94	35	35	91	65	78
89	83	98	71	74	22	05	29	17	37	45	65	35	54	44
44	88	03	81	30	61	00	63	42	46	22	89	41	54	47
68	60	92	99	60	97	53	55	34	01	43	40	77	90	19
87	63	49	22	47	21	76	13	39	25	89	91	38	25	19
44	33	11	36	72	21	40	90	76	95	10	14	86	03	17
60	30	10	46	44	34	19	56	00	83	20	53	53	65	29
03	47	55	23	26	90	02	12	02	62	51	52	70	68	13
09	24	34	42	00	68	72	10	71	37	30	72	97	57	56
09	29	82	76	50	97	95	53	50	18	40	89	40	83	29
52	23	08	25	21	22	53	26	15	87	93	73	25	95	70
43	78	19	88	85	56	67	56	67	16	68	26	95	99	64
45 0	69	72	62	11	12	18	25	00	92	26	82	64	3	
21	72	97	04	52	62	09	54	35	17	22	73	35	72	53
65	95	48	55	12	46	89	95	61	31	77	14	24	14	41
51	69	76	00	20	92	58	21	24	33	74	08	66	90	61
89	56	83	39	58	22	09	01	14	04	14	97	56	92	97
72	63	40	03	07	02	62	20	11	50	11	98	23	80	99

FORMULAE

PURE MATHEMATICS

For the quadratic equation: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d, \qquad S_n = \frac{n}{2}\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = a r^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1},\ r > 1, \qquad S_n = \frac{a(1 - r^n)}{1 - r},\ r < 1, \qquad S_\infty = \frac{a}{1 - r},\ |r| < 1$$

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 +, \ \dots\ + b^n,$$ where n is a positive integer.

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(r-1)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots r}x^r + \dots \text{ where } n \text{ is a real number and } |x| <|$$

Summations:

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1). \qquad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1). \qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Complex numbers:

$z^n = (\cos x + i \sin x)^n = \cos nx + i \sin nx$, where n is an integer and x is real

$e^{ix} = \cos x + i \sin x$ where x is real

$[r(\cos x + i \sin x)]^n = r^n(\cos nx + i \sin nx)$

Maclaurin's series:

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all real } x$$

$$\text{In}\,(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r!} + . \quad (-1 < x \le 1)$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots - \frac{x^r}{r!} - \ldots \qquad (-1 \le x < 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all real } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + . \quad \text{for all real } x$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^r(0) + \ldots$$

Taylor's series:

$$f(x) = f(a) + f'(a)\,\frac{(x-a)}{1!} + f''(a)\,\frac{(x-a)^2}{2!} + f'''(a)\frac{(x-a)^3}{3!} + + f^r(a)\frac{(x-a)^r}{r!} + \ldots$$

The trapezium rule $\int_a^b y\,\mathrm{d}x = \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + .. + y_n - 1)\}$,

$h = \frac{b-a}{n}$, where n is the number of intervals (strips)

The Newton-Raphson iteration $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

TRIGONOMETRY

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Arc length of a circle: $s = r\theta$, (θ measured in radians)

Area of a sector of a circle: $\quad \text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2}\ bc \sin A = \frac{1}{2}\ ca \sin B$

$$\text{If } \tan \frac{a}{2} = t \text{, then } \sin \alpha = \frac{2t}{1 + t^2} \text{ and } \cos \alpha = \frac{1 - t^2}{1 + t^2}$$

Trigonometric Identities:

$$cos^2\, \alpha + sin^2\, \alpha \equiv 1,\ 1 + tan^2\, a = sec^2 a, 1 + cot^2 a = cosec^2 a$$

$$\sin(\alpha \pm \beta) \equiv \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) \equiv \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) \equiv \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \qquad \alpha \pm \beta \neq (k + \frac{1}{2})\pi$$

$$cos\, 2\alpha = cos^2\, \alpha - sin^2\, \alpha = 2\, cos^2\, \alpha - 1 = 1 - 2\, sin^2\, \alpha$$

$$sin\, 2\alpha = 2\, sin\, \alpha\, cos\, \alpha$$

$$\sin \alpha + \sin \beta \equiv 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta \equiv 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta \equiv 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta \equiv 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2} \quad \text{or} \quad -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

STATISTICS

Frequency distributions

$$Mean\ \overline{x} = \frac{\Sigma fx}{\Sigma f}$$

$$Standard\ Deviation\ \sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\frac{\Sigma f(x - \overline{x})^2}{\Sigma f}} = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n}$$

Median $Q_2 = \left(\frac{n+1}{2}\right)^{th}$ *value*

Grouped data

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$ *where* x *=midpoint of each class, f is the frequency of each class.*

$$Median = l + \left(\frac{\frac{N}{2} - f_0}{f_1}\right) w \text{ where,}$$

l= lower limit of the median class

N = total frequency

f_0 *= frequency of class preceding the median class*

f_1 *= frequency of median class*

w = width of median class

Mode = $l + \left(\frac{f_1 - f_2}{2f_1 - f_0 - f_2}\right) w$

Where ,

l = lower limit of the modal class

f_1 *= frequency of the modal class*

f_0 *= frequency of the class preceding the modal class*

f_2 *= frequency of the class succeeding the modal class*

w = width of the modal class

Measures of spread or dispersion

Standard deviation for the population

s.d = $\sqrt{\frac{\sum fx^2 - \frac{(\sum fx)^2}{n}}{n}}$

where x = midpoint of each class

f = the frequency of each class

n = population size.

unbiased estimator of the variance of X is $\hat{\sigma}^2 = \frac{n}{n-1} \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

Product Moment Correlation Coefficient, *r*

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left[n\sum_{i=1}^{n} x_i{}^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i{}^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

Covariance Formula $= \frac{S_{x_i y_i}}{S_{x_i} S_{y_i}}$ where $S_{x_i y_i}$ is the co-variance of x and y,

$S_{x_i} S_{y_i}$ is the product of the standard deviation of x and y respectively

Regression line *y* on *x*

y = *a* + *bx* passing through $(\overline{x}, \overline{y})$ where

$$\overline{x} = \frac{\sum x}{n} \text{ and } \overline{y} = \frac{\sum y}{n}$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$b = \frac{S_{xy}}{S_{xx}}$, where S_{xx} is the variance of x.

$$a = \overline{y} - b\overline{x}$$

MECHANICS

Uniformly accelerated motion

$$v = u + at, \quad s = \frac{1}{2}(u + v)t, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

Motion of a projectile

Equation of trajectory is:

$$y = x \tan\theta - \frac{gx^2}{2V^2 \cos^2\theta}$$

$$= x \tan\theta - \frac{gx^2}{2V^2}(1 + \tan^2\theta)$$

Time of flight = $\frac{2V \sin\theta}{g}$

Greatest height = $\dfrac{V^2 \sin^2 \theta}{2g}$

Horizontal range = $\dfrac{V^2 \sin 2\theta}{g}$, maximum range = $\dfrac{V^2}{g}$ for $\theta = \dfrac{\pi}{4}$

Lami's Theorem

$\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$, where F_1, F_2, F_3 are forces acting on a particle

and α, β, γ are the angles vertically opposite F_1, F_2, F_3, respectively

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CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2005

PURE MATHEMATICS

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2005

INTRODUCTION

This is the seventh year that Mathematics Unit 1 was examined on open syllabus and the sixth year for Unit 2. The revised syllabus for each Unit was examined for the first time. Just over two thousand, five hundred candidates registered for Unit 1 and nine hundred registered for Unit 2.

Each Unit comprised three papers, Paper 01, Paper 02, and Paper 03. Papers 01 and Paper 02 were assessed externally and Paper 03 was assessed internally by the teachers and moderated by CXC.

Paper 01 in each Unit consisted of 15 compulsory, short-response questions. There were five questions in each of three sections, Section A, B and C corresponding to Modules 1, 2 and 3 respectively. The maximum number of marks for each question ranged from six to ten. In each Unit, candidates could earn a maximum of 120 marks for this paper representing 40 per cent of the assessment for the respective Unit.

Paper 02 in each Unit consisted of six compulsory extended response questions. There were two questions in each Section/Module. Each question was worth 20 marks. Candidates could obtain a maximum of 120 marks on Paper 02, representing 40 per cent of the assessment for the Unit. For each paper, marks were awarded for Reasoning, Method and Accuracy.

Paper 03 was compulsory. It was assessed internally by the teacher and moderated by CXC. For each Unit, candidates wrote three tests which assessed individually or collectively the three Sections/Modules. At least one test must exclusively assess mathematical modeling. This paper represented 20 per cent of the assessment for the Unit. This is the third year that Paper 03/2 was written by private candidates for Unit 1 and the second year for Unit 2.

GENERAL COMMENTS

UNIT 1

The performance of candidates continues to show improvement over pervious years. This is particularly encouraging this year since a revised syllabus was being examined for the first time. Questions on topics involving Curve-Sketching, Coordinate Geometry of the straight line and crcle, the Factor Theorem, Mensuration of Conic Sections, and Basic Differential and Integral Calculus were well done. Occasionally, amidst this broad coverage, there were pockets of weakness within these topics, this suggested that more practice is needed, at the level of preparation of candidates for the examination(s), to strengthen performance. As observed last year, general algebraic manipulation also requires further attention in order to eliminate faulty follow-through in problem solving.

A few topics continue to present challenges to candidates. These are Inequalities, Indices, Limits and Continuity/Discontinuity, and aspects of the Integral Calculus. Such topics should be targetted at the school level for special treatment. It was also observed that substitution in all forms was not treated very well.

The results showed some excellent performances in both papers and were generally very encouraging.

DETAILED COMMENTS

UNIT 1
PAPER 01
SECTION A
(Module 1: Basic Algebra and Functions)

<u>Question 1</u>

Specific Objective(s): (d) 4, 6; (c) 1, 3, 4.

The question sought to examine the candidates' ability to relate to its equation certain characteristics of the given graph of f(x) such as its intercepts with the axes. The Remainder/Factor theorem was also relevant to the complete solution.

(a) Many candidates substituted $x = 0$ and $x = 2$, obtaining expressions in terms of k and h. These candidates did not use the graph as required and failed to deduce the value of k as the y-intercept.

The mathematical use of words including STATE should be made clear to candidates in their tutorials. This would serve to make candidates aware of the requirements that such specific words may infer.

(b) Some candidates followed through with the correct value of k from Part (a) and easily found the value of h.

(c) Having obtained the correct value for k, and the correct value for h, candidates proceeded to use the remainder / factor theorem to factorize the expression completely. No instance was observed where candidates deduced that f(2) resulted in coincident roots, thus allowing for determining the third factor by simple inspection.

Answer(s):

(a) $f(0) = 12$, $f(2) = 0$

(b) $h = -1$, $k = 12$

(c) $f(x) = (x-2)$ $(x+3)$

Question 2

Specific Objective(s): (a) 1, 3, 4, 5, 6, 8; (b) 1, 2

This question tested basic properties of two real numbers, both of which are negative, or one negative and one positive, and of the modulus function. Inequalities with quadratic expressions were also involved.

(a) A significant number of candidates failed to apply correctly the concept of a modulus function. Some of them separated the inequality to read

x￼ - 3 < 2￼

and proceeded to square both sides. Having obtained a quartic equation, they could not make a suitable substitution for xỗ and hence find the correct solution set. Some candidates used the positive value for x only and gave the incorrect range of values of x. Other candidates, who used both positive and negative values for x, gave the solution set over both ranges of values of x, and did not use the fact that $x < 0$ was given condition.

(b) Many candidates used suitable real numbers to represent k, x and y. Very few used a purely algebraic approach.

Answer(s): (a) $-3 < x < 0$

Question 3

Specific Objective(s): (c) 1; (e) 1, 2.

Surds and small powers of the variable x, its reciprocal ￼ and (x + ￼) were the focus of this question.

(a) It was most surprising to see a significant number of candidates who found it difficult to rationalize denominators in surd form. This question should not be beyond the mathematical skills of candidates at this level. Few obtained the maximum mark.

(b) (i) The expansion of ￼ proved difficult for some candidates. Errors were made in obtaining the correct expansion $x^2 + 2 +$ ￼.

(ii) Several attempts at simplification of ￼ some of which involved the term containing 3 ￼ failed. Few candidates were able to obtain the maximum for this question.

Answer(s): (b) (ii) $x^3 +$ ￼ $= -2$

Question 4

Specific Objectives: (f) 1, 7(i), 8, 10

The theme of this question was simultaneous equations in two unknowns, one equation linear and one quadratic.

Except for minor errors in simplification the overall performance was very good. It was not uncommon, however, to see candidates stating that $(2y - 3)^2 = 4y^2 \pm 9$.

Answer(s): $x = 1, y = 2$ and $x = -5/2, y = ¼$

Question 5

Specific Objectives: (d) 1, 6; (f) 1

This question tested the basic properties of injectiveness, substitution and solution of equations in a single unknown.

(a) Candidates showed a satisfactory understanding of the concept of a one-to-one function.

(b) In spite of this topic being covered at CSEC level, many candidates found it difficult to obtain the correct expression for f [f (x)].

Answer(s): (b) $x = 2$

SECTION B
(Module 2: Plane Geometry)

Question 6

Specific Objective(s): (a) 1, 2, 3, 4, 7(ii), 8

This question tested some of the basic properties of perpendicular lines, as well as the coordinates of a point which divides a line segment in a given ratio.

(a) This part was generally well done. Some candidates seemed unaware of the perpendicularity relationship between gradients. Some did not use the mid-point correctly.

(b) This part was generally not well done. Some candidates used an incorrect formula ($\underline{p}$ = instead of . Others used an alternative solution $\underline{p} = \underline{b}$ + ¼ or $\underline{p} = \underline{a}$ + ¾ .

A few candidates observed that p was the mid-point of MP and used the formula (p = ½ ($\underline{m} + \underline{b}$).

Common error:

Answer(s): (a) (i) M ≡ (1, −1) (ii) gradient = - 4/5 (iii) equation: 4y = 5x – 9

(b) Cord. of P: (￼)

Question 7

Specific Objective(s): (b) 18, 20, 21

This question examined the minimum value of a trigonometric function by converting it to the form R cos ￼ Most candidates attempted this question, but very few obtained the maximum marks.

A common source of error was the incorrect expansion of R cos (θ + ∝), leading to ∝ = -35.3° (0.615^c) instead of ∝ = 0.615^c. A few candidates wrote down the minimum value of f (θ) as ￼, instead of ￼.

Another common error was solving ￼ cos (θ + 0.615) = −1 instead of solving θ + 0.615 = π. The use of degrees instead of radians was a common feature.

Answer(s): (a) f (θ) = ￼ as (θ + ∝), î = 0.615^c

(b) minimum f (θ) = ￼

(c) θ = 2.53^c

Question 8

Specific Objective(s): (c) 1, 4, 5

This question dealt with the condition for the existence of complex roots of a quadratic equation as well as the expression of a complex number in the form $x + iy$, $i^2 = -1$.

Most candidates attempted this question.

(a) Most candidates were aware of the discriminant "$b^2 – 4ac$" but some tried to solve $b^2 – 4ac > 0$. Some solved the inequality $k^2 < 9$ by writing down the solution -3 < k < 3, but failed to observe that k^2 = ￼, for all k, hence ￼< 9 implies ￼ < 3.

A few candidates solved the inequality *kỗ* < 9 by using the graphical method. Some candidates wrote down the solution of the quadratic equation, using the quadratic formula, and then proceeded to solve the inequality. Again, a few candidates presumed that there were two equal complex roots and wrote down *(x+iy)ô = x￼2kx+9.*

Most candidates attempted this part and obtained full marks. Some used the alternative method ￼ = *x+yi* and cross multiplied to obtained real and imaginary components. A few candidates did not separate the real imaginary components in their response.

Answer(s) : (a) -3<*k*<*3*

(b) $\tilde{\hat{o}}$ + OBJ

Question 9

Specific Objective(s) : (d) 1, 2, 3, 4, 5, 7, 8

This question dealt with the expression of a vector in the form xi+yj and of a point as a position vector.

(a) Most candidates attempted this part and scored full marks. Some candidates used an incorrect $\underline{i}$, $\underline{j}$ notation example OBJ = OBJ = OBJ

They very rarely used the notation OBJ, OBJ OBJ inserted of A,B,C.

(b) This part was generally poorly done.

Common Errors: OBJOBJ = A + B
= Ax B
OBJ = C + D = C OBJ D

The use of the correct vector notation was generally poor. Many careless mistakes such as $-\underline{i} - 2\underline{j} + 2\underline{i} + 5\underline{j} = -\underline{i}$ and $3\underline{j}$ and $-\underline{i} - 2\underline{j} + 5\underline{j} = i - 3\underline{j}$ were made.

The following were made alternative method was also used.

OBJ = OBJ $\underline{b} - \underline{a} = \underline{d} - \underline{c}$

$\underline{d} = \underline{b} - \underline{a} + \underline{c}$ = (2,5) – (1,2) + (0, -4)

= (1, -1) = $\underline{i} - \underline{j}$

Answer(s) : (a) $\underline{i} - 2\underline{j}$, $2\underline{i} + 5\underline{j}$, $-4\underline{j}$

(b) $\underline{i} - \underline{j}$

Question 10

Specific Objective(s): (d) 9, 10; (b) 5, 13, 19

This question dealt with parallel vectors that are expressed in terms of a position in *i, j* form.

The very large number of no responses and zeros, attested to the unfarmiliarity of candidates with this form of question. Many candidates did not know how to respond. Several candidates formed the trigonometric equation correctly but thereafter were unable to solve it correctly.

Several candidates were familiar with the dot product.

A significant number of candidates claimed that the vectors were either equation or perpendicular, that is, $\underline{a} = \underline{b}$ or that OBJ - OBJ= 0.

Answer(s) : $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{7\pi}{6}$, $\frac{4\pi}{3}$

SECTION C
(Module 3: Calculus 1)

Question 11

Specific Objective(s): (a) 1, 2, 3, 4; (b) 5

This question provided a means of differentiating $y = \tilde{\hat{o}}$, with respect to *x,* from principles, by expressing

OBJ −OBJ as OBJ from the given result

OBJ+OBJOBJ-OBJ = h.

(a) Many candidates did not rearrange the terms in the given result to express OBJ as OBJ and did not benefit from the lead provided. Those candidates who did, obtain the majority of the credit for this part of the question.

(b) The majority of candidates did not see the connection of this part with part (a).

Question 12

Specific Objective(s): (a) 8, 9, 10.

This question tested discontinuity of a rational function over the real numbers and the location of roots in a closed interval. Many candidates showed familiarity with the topics.

(a) Most candidates knew the condition for discontinuity of the function but a few did not factorize the quadratic expression $x2$- $2x$-8 correctly.

(b) Very many candidates used the values $f(2)$ and $f(3)$ correctly to set up the change of sign of $f(x)$ in the interval but did not use the Intermediate Value Theorem and the continuity of $f(x)$ to the task.

Answer(s) (a) $x = 4$, $x = -2$

Question 13

Specific Objective(s): (b) 2, 7, 8, 18, 19, 25

The question tested the candidates' ability to find the first and second derivatives of a polynomial function and to relate these derivatives to using the gradient of the curve and to finding the equation of the normal at a point P on the curve.

(a) Several candidates were successful in answering this part correctly but a few had difficulty in finding the value of the constant k.

(b) Most candidates who attempted this question obtained the correct answer.

(c) There were several good answers to this part; however, source candidates lost marks by using the wrong gradient, others, through faulty algebraic manipulation.

Answer(s)

(a) $k = -8$
(b) ￼ at $x = 1$
(c) Equation : $2y = x - 23$

Question 14

Specific Objective(s): (b) 14, 15, 16, 17, 18, 19, 20, 21.

The question tested knowledge about stationary points and the nature of such points of a polynomial function f.

(a) This part was very well done by many candidates. Some difficulties were experienced by a few candidates due to faulty algebraic manipulation.

(b) As for part (a) this part was well done but errors were made by a few candidates in obtaining the second derivative, and some confusion in distinguishing the maximum and minimum points was evident.

Answer(s)

(a) Stationary points: (3, -21) , (-1, 11)

(b) Minimum at x = 3, maximum at x = -1.

<u>Question 15</u>

Specific Objective(s): (b) 23; (c) 3, 4, 7(i), 8, 10(i).

The question tested the candidates knowledge on areas between a curve and the x-axis for a
specific range of values of x.

(a) The majority of candidates were successful in obtaining the coordinates of the points P, Q and R.

(b) There were several very good attempts at this part of the question. The main stumbling block in obtaining correct answers resulted from candidates not taking the absolute value of the area O Q R.

Answer(s)

(a) P ￼ (-1, 3), Q ￼ (1, -1), R ￼ (2, 0)

(b) Area = ￼ square units.

UNIT 1
PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)

<u>Question 1</u>

Specific Objective(s): (c) 1; (d) 4, 6, 9; (e); (*f*) 6, 7.

The question tested the candidates' knowledge on graphs of the modulus function, indicies and surds.

(a) There were many good solutions to this part of the question. Several candidates obtained full marks for this part.

(b) There were some very good solutions but equally so many attempts failed because of faulty manipulation of indices.

(c) This part was not well done, but a few candidates did present good solutions gaining full credit.

Answer(s)

(a) Table Values (x, *1 f(x)* 1) : (-1, 3), (0, 0), (1, 1), (3, 3)

(b) $k = 3$ or ꞉

(c) (i) $x = 2$ (repeated),

(ii) ￼

<u>Question 2</u>

Specific Objective(s): (a) 10; (f) 3; (g)

This question tested the principle of mathematical induction, solutions of simultaneous equations and solutions of inequalities involving rational functions.

(a) This part of the question was not well done. The initial step for n = 1 was as far as many candidates were able to reach in setting out the induction process. More practice in proceeding from n = $k\tilde{\hat{o}}$ to n = $k+1$ appears to be needed.

(b) There were a few very good efforts at this part but in general it was not well done.

(c) This part of the question proved to be the most manageable for the candidates although errors in the algebra marred the attainment of correct answers in some cases.

Answer(s)

(b) (i) $= p$-2

(ii) solution set = ￼

(c) $2 < x <$ ￼

SECTION B
(Module 2: Plane Geometry)

Question 3

Specific Objective(s) : (a) 1, 2, 3, 7, 11, 14, 8.

This question dealt with the geometric of the circle and tested knowledge of the centre, radius and tangent.

(a) Very well done. The vast majority of the candidates found this part easy.

(b) As for part (a), in view of the connection between the two parts.

(c) Well done, but for a small number candidates who made a few arithmetic blunders.

(d) Very many good answers were presented. That the tangent at A was perpendicular to OA was not appreciated by a few candidates.

(e) This part was found to be easy when parts (a) to (d) above were well established.

Answer(s)

(a) *a* ￼ 1, *b* = -1 (b) (i) *O* ￼ *(1, -1),* (ii) radius = 5 units

(d) equation: $4y + 3x = 24$

(e) *B* : *(-2, -5).*

Question 4

Specific Objective(s): (b) 1, 3, 12, 13, 14, 16, 17; (c) 4, 5, 6; (d) 4, 9, 10, 11 9(i).

The question tested elementary properties of the sector and cone, as well as trigonometric functions, vectors and basic manipulation of complex numbers.

(a) Candidates found this part easy and obtained full marks for their efforts.

(b) (i) There were many good derivations of this identity.

(ii) The majority of candidates were not able to obtain the condition for the perpendicularity of a̲ and b̲ in terms of ￼ and so did not see the relevance of part (b) (i) above. At that point, their efforts as solution fell apart.

(c) There were several very good solution but there were too many faulty algebra produced wrong solutions (e.g. $i\tilde{\hat{o}} = -1$.

Answer(s)

(a) (i) arc ABC = cm
(b) =

(c) $z = 17 + 6i$ 18.0

SECTION C
(Module 3: Calculus 1)

Question 5

Specific Objective(s): (a) 3, 4, 6 (ii); (b), 4, 7, 8, 9, 10, 18 19; (c) 4, 7, 10(ii)

This question tested the topics of limits involving the trigonometric function sin kx, differential equations and volume generated by rotating the area between curves about the x-axis.

(a) (i) Many candidates showed a lack of knowledge of the concepts of limits. Instances of
or were seen as responses.

(ii) The substitution $x =$ was used to obtain but candidates were unable to obtain this limit, having failed at (i).

(iii) This part did not prove any easier than (i) and (ii). The concepts of products and quotients of limits escaped many candidates.

(b) This question was well done.

(i) There was no difficulty of note in the responses to this question.

(ii) Interestingly many candidates quoted the formula for the volume of solid generated by rotating a function of x-axis as $2x$
Some candidates calculated the volume as π and failed to subtract the volume of the cone which may have been either by simple mensuration or by integration.

Answer(s):

(i)

(ii)

(c) (i) P

(ii) V = cubic units.

Question 6

Specific Objective(s): (b) 8, 10, 12, 13, 14, 17; (c) 4, 8, 9

This question tested the chain rule for differentiating a function of a function, stationary points of functions and values for which functions are increasing/ decreasing, and substitution as a means of relating one integral to another.

(a) Some candidates experienced difficulties in parts of this question. Differentiation of a composite function of x and the multiple angle of trigonometric functions were instances of such difficulties. The simplification of the final answer was also badly done in some cases.

(b) (i) This question was well done.

(i) Many candidates stated [OBJ]>0 and gave the solution as $x > 5$ and $x > 1$ for y increasing, and [OBJ] with $x < 5$ and $x < 1$ for y decreasing. A few instances of a graphical approach were observed.

(c) (i) This question was poorly done. Not surprisingly the weakness in algebra were evident.

(ii) Candidates found the deduction [OBJ] beyond their understanding. Performances were generally poor.

Answer(s)

(a) $10x$[OBJ]

(b) (i) $x = 1$, $x = 5$

(i) $x > 5$ or $x < 1$

(c) [OBJ]

UNIT 1
PAPER 03/B
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

(a) Specific Objective(s): (c) 1, 6; (f) 5

This part of the question covered completion of the square involving the determination of given constants.

A few candidates obtained full marks on this part. Others who attempted it lost marks through faulty algebra.

Answer(s)

(a) $h = 2$, $k = 3$

(b) Specific Objective(s): (a) 1, 3, 5, 6;

Aspects of inequalities were tested in this part which was well-done. In (ii) some candidates did not seem to know how to use the condition p+q = 1.

(c) Specific Objective(s): (d) 3, 4, 6.

This part of the question used a quadratic functions to represent a mathematics model. Most of the small number of candidates sketched the graph of the function correctly but were unable to determine correct answers to part (ii).

Answer(s)

(i) a) 11 (b) \$25 (c) 5

SECTION B
(Module 2: Plane Geometry)

Question 1

(a) Specific Objective(s): (a) 1, 2, 4, 5, 7 (ii), 9

This part of the question covered the coordinate geometry of the straight line, perpendicular lines and the intersection of straight lines. This part of the question was well done.

Answer(s)

(a) (i) N (ii) PN = units

(b) Specific Objective(s): (b) 5, 13, 17, 19

A mathematical model was portrayed by a trigonometric function of the parameter t. Few candidates used the complete information in the table to obtain the four equations necessary for the answer and as a consequence there were few correct solutions.

Answer(s)

P =5 , q = 30

SECTION C
(Module 3: Calculus 1)

<u>Question 3</u>

(a) Specific Objective(s): (a) 3, 5, 7

Limits and continuity were examined in this part of the question.

Candidates showed familiarity with both concepts but poor factorization of the functions (numerator and/or denominator) led to incorrect solutions. There were a few good solutions, nevertheless.

Answer(s)

(i) OBJ

(ii) *f(x)* continuous

(b) Specific Objective(s): (b) 5(i)

Differentiating from first principles was examined in this part.

There were good attempts at solution of this part, but several candidates did not use the limit procedure correctly.

Answer(s)

$2x + 2$

(c) Specific Objective(s): (b) 9; (c) 5, 6 (ii), 8, 11

The topic of mathematical modelling was tested in this part by means of the differential and internal calculus.

Few candidates obtained the correct differential equation for the model, and of those, only a small percentage was able to solve the problem completely.

Answer(s)

(i)

(ii) 4 minutes

GENERAL COMMENTS
UNIT 2

The general caliber of candidates in Unit 2 was of a very high standard with a small number of candidates recording outstanding performances in this first year of a revised syllabus. Indeed, there were very encouraging responses to the additional topics which were examined.

Notwithstanding the semblance of an improved performance, there were a few candidates who were inadequately prepared for the occasion. The lack of preparation showed up in the inability to carry out processes previously covered in Unit 1 particularly, but also in CSEC, and if executed successfully, would have led to greater accomplishment at this level.

Topics such as integration and differentiation in Calculus, simple probability and counting problems, and approximation to roots of equations, evoked favourable responses but general weakness continue to manifest themselves in areas such as mathematical modeling, indices and logarithms, and series. Indeed, substitution also reared its head as a new area of weakness. Most of these deficiencies can be rectified by extended practice on the respective themes. Teachers should also ensure that the preparation of candidates for the examinations is not excessively formula-driven or too heavily dependent on the use of calculators, but that basic principles and processes are emphasized.

On the whole the performance of the candidates was very satisfactory.

DETAILED COMMENTS

UNIT 2
PAPER 01
SECTION A
(Module 1: Calculus II)

<u>Question 1</u>

(a) Specific Objective(s): (a) 1

The question tested the ability of candidates to differentiate a function $f\tilde{\hat{o}}$ and its derivative f'.

(i) Almost all candidates were able to obtain f' successfully.

(ii) Most candidates were able to $\lambda f(x)$ and equate it to $f'(x)$. However many of them did not equate the coefficients of the constant term, x_1,, x_2 to obtain a_1, a_2 and a_3 in terms of and a_0 . Pool substitution was the cause for incorrect results.

Answer(s)

$a_1 =$ a_0 $a_2 =$ $a_3 =$

(b) Specific Objective(s): (b) 1, 5

The question tested differentiation of functions of a function.

This question was done well by most candidates who attempted it.

In (i), many candidates obtained the correct solution and in (ii) most candidates used the product rule to obtain the correct result.

Answer(s)

(i)

(ii)

<u>Question 2</u>

(a) Specific Objective(s): (b) 5; (c) 4, 6

This question, involved the use of logarithms in the topic of change of base.

(i) This part was surprisingly very poorly done.

(ii) Several candidates had difficulty converting from base e to base 10. Many others did not use the fact that 9 = 3 to obtain a connection between log 9 and log 3.

Answer(s)

(i) log_{10}

(ii) $log_e\ 9 =$

(b) Specific Objective(s): (a) 10

This question examined the use of logarithms in solving equations.
Very many candidates performed well on this question. It was well done.

Answer(s)

$X =$

Question 3

Specific Objective(s): (b) 5; (c) 4, 6

This question tested the use of substitution in differentiating and integrating trigonometric functions.

(a) Some candidates did not use the identity tan￼ = ￼ and so failed to apply the quotient rule effectively in obtaining the desired result. In some cases, the substitution was not done properly.

(b) Many candidates did not observe the connection between parts (a) and (b) and hence did not obtain the required result. A very few cleverly used integration by parts for this part of the question.

In many cases, the constant of integration was omitted.

Answer(s)

(b) ￼ (constant of integration).

Question 4

Specific Objective(s): (b) 5, 6

The question covered the topic of the differentiation of products of trigonometric functions and the formation of differential equations involving such functions.

(a) This part of the equation was very well done by many candidates although some did have difficulties applying the product rule. A few other had problems expressing ￼ in the required form.

(b) Many candidates used alternative forms of ￼ to obtain ￼. The question was well done.

Question 5

Specific Objective(s): (c) 5, 6, 7

The question tested aspects of the integral calculus including and involving the use of substitution.

(a) There were many good solutions to this part of the question once the

substitution
u = x￼+ 1 or some such was used. There were some problems with the constants in the solution. Some candidates observed that the question was of the form ￼ and dealt with it accordingly.

(b) Too many candidates did not use the given substitution properly but attempted integration by parts instead. In many such cases the integrand in the variable x was not replaced completely by a new integrand involving u only. Otherwise, there were some good solutions to this part.

Answer(s)

(a) *3 ln*￼ (Constant Of Integration)

(b) -￼ 9 (c. o. i).

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 6

Specific Objective(s): (a) 2

This question examined sequences.

Many candidates attempted this question with approximately half of them scoring full marks. Some candidates experienced great difficulty with substitution in the given recurrence relation and this reduced considerably their chances of success.

(a) Candidates who did not observe the condition *nỗ* had many problems obtaining definitive answers. Some others made mistakes in the algebraic manipulation of the equations involved.

(b) Many candidates who got past part (a) were able to complete this part successfully.

Answer(s)

(a) $u_1 = 4$ or $u_1 = -1$

(b) $u_3 =$ ￼ or $u_3 =$ ￼

Question 7

Specific Objective(s): (b) 4, 7

This question examined the topic on series with particular reference to the A.P.

Many candidates attempted this question but several of them did not know how to find the *nth* term a_n. As a consequence there were not many complete solutions.

(a) This part was not well done. Too many candidates found it difficult to obtain a_n.

(b) For those who were able to do part (a), this part was well done.

(c) This part was done very well by those candidates who successfully answered parts (a) and (b).

Answer(s)

(a) $a_n = 4n-6$

(c) (i) -2

(ii) 4

Question 8

Specific Objective(s): (c) 1

This question tested simple properties of equality and sums of binomial coefficients ^nC_k, $k \geq 1$.

Not many candidates were able to express ^nC_k correctly in terms of factorials. This suggests that several were only exposed to ^nC_k in the context of a calculator key. As a consequence, this question its entirety was not done as well as was anticipated.

(a) These basic building blocks for questions on the binomial coefficients seemed not to be familiar to many candidates.

(b) Unfamiliarity with the coefficients in (a) expressed in general terms made it difficult for some candidates to do this part successfully.

(c) There were not many good solutions to this part since it depended on the earlier parts (a) and (b).

Answer(s)

(a) $^nC_k =$ [OBJ] $^nC_{k-1} =$ [OBJ]

Question 9

Specific Objective(s): (c) 1, 3

This question tested the binomial expansion of the expression with a view to finding the value of the constant a based on a given relationship between specific terms in the expansion.

There were not many good solutions to such a basic question on the binomial expansion of this sort. Several candidates did not write down the coefficients of x and x in the expansion correctly and this spoiled their chances of obtaining correct answers to the constant a. Some others ignored the value -2 for a as the fourth root of 16.

Answer(s)

$A = \pm 2$

Question 10

Specific Objective(s): (d) 1, 2

This question examined the topic of errors.

There were many good solutions to this question. On the whole it was quite well done.

(a) Most candidates were able to find the upper and lower boundaries correctly. A few made minor errors in calculation but were able to follow through quite successfully.

(b) (i) Several candidates were able to find the maximum and minimum areas correctly, and this led naturally to the correct minimum absolute error of 24.75.

(ii) In finding the maximum percentage error, many candidates got the area of 625 but had some difficulty in finding the error of 25.25.

Answer(s)

(a) 24.5 length < 25.5

(b) (i) Min. abs. error = 24.75

(ii) Max % error = = 4.04

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 11

Specific Objective(s): (a) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

This question explored basic counting principles and simple probability.

(a) Not many candidates used basic counting principles to solve this problem. Many tried to used ${}^{n}c_{r}$ or ${}^{n}p_{r}$ to find a solution.

(b) (i) There were several attempts at this part but few applied straightforward techniques to arrive at the correct solution.

(ii) This part depended on (a) above, so without the correct solution to (a) and possibly (b)(i), not many correct answers were achieved.

Answer(s)

(a) 625

(b) (i) 250

(ii) 0.6

Question 12

Specific Objective(s): (a) 4, 5, 6, 7, 10

This question examined the topics of sample spaces and probability.

This question was quite well done. Many candidates were able to gain close to full marks on this question.

(a) Very well done by many candidates but a few did not write out the full space and lost some credit as a consequence.

(b) (i) This part was very well done.

(ii) Many candidates obtained full marks for this part.

Answer(s)

(a) .

(b) (i) (ii)

Question 13

Specific Objective(s): (b) 1

This question examined the basic properties of product and transpose of matrices.

Candidates found both parts of question easy and many scored full marks. The question was very well done.

Answer(s)

(a) (i) AB =

(ii) B^TA^T =

(b) (AB) = = B^TA^T

Question 14

Specific Objective(s): (b) 1, 3, 4, 5, 6

This question covered the topic of augmented matrix, and echelon form as they apply to the solution of systems of linear equations.

There were several attempts by candidates but many candidates failed to obtain complete solutions because of minor errors in arithmetic and algebra. Despite these blemishes, many good solutions were achieved.

Answer(s)

(a) =

(b)

(c) $x = 1$, $y = 0$, $z = 2$.

Question 15

Specific Objective(s): (c)

The topic examined in this question is mathematical modeling, posed here in the context of the rate of change of the volume V of a sphere of radius r.

The quality of responses to this question was very poor. Many candidates who attempted it found it difficult to obtain the differential equation in (a) without which a solution for r in (b) could not be reached.

Answer(s)

(a) ￼ (k is constant of proportionality)

(b) (included on question paper)

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

<u>Question 1</u>

Specific Objective(s): (a) 2, 3, 4, 5, 6, 9, 10

This question tested the relationship from the logarithmic and exponential functions, including their graphical relationship.

(a) This part was generally well done. Common Errors: reflection about the x or y-axis or the origin. Some who tried to reflect about the line y = x, failed to draw the asymptote to the y-axis. Some candidates did not understand reflection about y = x.

Most candidates calculated the value of v but not of p.

(b) This part was very well done. A few candidates used the relationship $a^{\log_a b} = b$ to solve the problem.

(c) This part was generally well done.

(i) Misinterpreting e^{-x} as $-e^{x}$

(ii) Expressing e^{2x} *as* e^{x2}

(iii) Expressing $3e^{-x}$ as ￼

(iv) Using the same letter x in the substitution of e^{x} i.e. $x = e^{x}$.

(v) Failure to state $ln\ 1=0$

Many students obtained full marks.

Answer(s)

(a) (i) Sketch graph

(ii) Reflection of each other about $y = x$, and inverse function

(iii) $v = 33.1$ $p = -1.00$

(b)

(c) $x = 0$ or $x = \ln 3$

Question 2

Specific Objective(s): (b) 3, 4; (c) 1, 3

(a) This part tested the use of the chain rule to find the gradient of and normal to, a curve given by its parametric equations.

Most candidates attempted this question and were comfortable using the chain rule
OBJOBJOBJ = OBJ

Common Errors:

(i) OBJ

(ii) Gradient of normal OBJ gradient of tangent = 1.

(b) This part tested the expression and integration of a proper rational function that was decomposed as partial functions.

This part was generally well done. Many candidates found it very difficult to calculateOBJ *dx*.. Some evaluated it variously as *ln ỗ* , OBJ *ln xỗ* , 2*x kn* OBJ. Evaluation of a definite integral was understood by most candidates. Many mistakes with the algebra were seen.

Answer(s)

(a) (i) OBJ (ii) OBJ

(b) (i) OBJ

(ii) OBJOBJ

SECTION B
(Module 2: Sequences, Series and Approximations)

<u>Question 3</u>

Specific Objective(s): (b) 1, 10; (c) 1, 2, 3

This question examined the summation of series by the method of differences, convergences of geometric series and properties of the binomial coefficients.

(a) (i) This part of the question was not well done. Some candidates used Mathematical Induction to solve this problem. Many did not realize that by simply writing out the terms for each and summing the desired result would follow.

(ii) Most candidates did not appreciate that as *nỗ* , .

(b) Only a few correct solutions to this part were seen. Again, this part was very poorly done. The main weakness was the inability to solve inequalities.

(c) Most candidates made the two correct substitutions in this part to obtain the required results.

Answer(s):

(b) $x > 4$ *or* $-1 < x < 1$ *or* $x < -4$

(c) (i) Substitution is $x = 1$

(ii) Substitution is $x = -1$

<u>Question 4</u>

Specific Objective(s): (e) 1, 3

This question examined the behaviour of a given polynomial function in a specific interval and the Newton–Raphson method for finding approximations of a root of the function in the interval.
Several candidates attempted this question with varying degrees of success. Many of them obtained at least half marks for their efforts.

(a) (i) Most candidates were able to correctly differentiate the given function but did not recognize a range of values for x was required, in this case, They also did not pay attention to the word 'strictly' when describing the behaviour of the function.

(ii) This part was well done but the majority of the candidates lost credit for neglecting to mention the I.V.T. and the continuous nature of the function over the interval.

(iii) Again the continuous nature of the function was ignored.

(b) Candidates were able to improve their scores with this Newton-Raphson question. The formula and manipulation of variables were well done.

SECTION C
(Module 3: Counting, Matrices and Modelling)

<u>Question 5</u>

Specific Objective(s): (a) 1-7

The question tested some basic tenets of counting principles as well as fundamental concepts of probability.

The vast majority of candidates attempted this question and many obtained at least 40 per cent of the marks.

(a) (i) This part was well done.

(ii) This part was not so well done. Many candidates had difficulty in obtaining the correct denominator in calculating the probability in each case.

(b) (i) This part was well done.

(ii) This part presented problems to several candidates.

(iii) Several partial answers were seen. Many candidates obtained correct answers to parts of this question.

Answer(s)

(a) (i)

(ii) (a) (b) (c)

(b) (i) 2024 (ii) (iii)

Question 6

Specific Objective(s): (b) 2; (c)

This question covered determinants and mathematical modelling.

Several candidates attempted this question but there were few completely correct answers.

(a) Many candidates had difficulty in expanding the determinant to obtain the cubic equation $x^3 - 7x - 6 = 0$. Some who reached that stage did not recognize that the Factor Theorem would be of assistance in solving for x. Not many candidates used any form of the Gauss-Jordan method of solution.

(b) Not many candidates relished this modelling question. Several interpreted the information in a manner which led to an A.P and not a G.P. Among those candidates who recognized the G.P model some incorrectly used the common ratio r as 0.04 and not 0.96.

Answer(s)

(a) $x = -3, 1 \text{ or } 2$

(b) (i) 1200

(ii) 30000

(iii) 30000

(iv) Maximum number = 30,000

UNIT 2
PAPER 03
(Module 1: Calculus)

Question 1

Specific Objective(s): (a) 9, 10

This question examined the use of logarithms to investigate a mathematical model in the form of a straight line of fit data derived from a biological experiment.

There were very few candidates registered for this paper. At least one candidate performed creditably.

Answer(s)

(a) (ii) (a) ￼ (b) $x = log_{10}\, x$ (c) $d = log_{10}\, b$

(b) Table 2:￼ , (1.48, 3.21) , (1.60, 3.40) , (1.70, 3.57)

(c) (i) (a) gradient = ￼

(b) $b = 10$,

(c) $n = 1.5$, $d = 1$

(ii) $x = 31.6$ for $y = 1800$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

Specific Objective(s): (b) 4, 8

This question geometric series in the context of mathematical modelling.

The few candidates who registered for this paper all attempted this question with some success.

Answer(s)

(a) (i) Beginning of year 4–A+AR+AR￼+AR^3
Beginning of year 5 –A+AR+AR^2+AR^3+AR^4

(ii) End of year 4 –AR+AR^2+AR^3+AR^4

End of year 5 – $AR+AR^2+AR^3+AR^4+AR^5$

(b) Beginning of nth year – $A+AR+AR^2+\text{----}+AR^{n-1}$

(c) Payout at end of n^{th} year

$AR+AR^2+AR^3+\text{----}AR^n$

$= AR\ (1+R+R^2+\text{----}R^{n-1})$

$= \$$, $R > 1$

(d) $n = 20$, $r = 5$ Amount of payout = \$8975

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 3

Specific Objective(s): (b) 1, 2, 5, 6, 7, 8; (c)

This question covers the topic of matrices in the context of mathematical modelling of a testing process in a chemical plant.

Attempts at this question were partially successful.

Answer(s)

(a) , so A is non-singular

(c) $A^{-1} =$

(d) $X = A^{-1}\ Y =$ $=$

PAPER 03
INTERNAL ASSESSMENT
MODULE TESTS

In general the exercises used in the internal assessment tests were relevant to and appropriate for the objectives stated in both Units 1 and 2 of the CAPE Mathematical syllabus. Like in the previous years, most of the questions used in the tests were taken from past CAPE Mathematics examination papers. Unfortunately, there were instances where one examination was set to test all three modules. The range of difficulty also varied significantly. In a number of cases, the tests were very detailed in content and the time allocated for completion was either too long (some were in excess of two hours) or too short. There were a few cases where the tests did not reflect adequately a sufficient coverage of the syllabus and the topic of mathematical modelling was not formally included.

Most tests were submitted with question papers, solutions and detailed marking schemes indicating clearly the distribution of the marks; however, there were far too many samples submitted which did not contain all the components necessary for the conduct of a complete analysis. In some extreme cases, question papers were not submitted in accordance with the guidelines in the syllabus and this made the moderation process far too cumbersome. Strict adherence to the guidelines for module tests is encouraged in order to enhance the efficiency and accuracy of the process.

Assessment was generally of a good standard except in a few cases where the allocations of final marks was difficult to follow. There appeared to be no major problems with the new topics introduced into the syllabus and tested for the first time in 2005.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2006

PURE MATHEMATICS

PURE MATHEMATICS

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2006

INTRODUCTION

This is the second year that the current syllabus has been examined. There has been a significant increase in the number of candidates writing the examinations, approximately 4430 for Unit 1compared to 2405 in 2005 and 1500 compared to 885 for Unit 2. Performances varied across the spectrum of candidates with an encouraging number obtaining excellent grades, but there continues to be a large cadre of candidates who seem unprepared for the examination.

GENERAL COMMENTS

UNIT 1

Overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as the Factor/Remainder Theorem, Coordinate Geometry as contained in the Unit, Basic Differential and Integral Calculus, and Curve-sketching. However, many candidates continue to find Indices, Limits, Continuity/Discontinuity and Inequalities challenging. These topics should be given special attention by teachers if improvements in performance are to be achieved. General algebraic manipulation of simple terms, expressions and equations also require attention.

DETAILED COMMENTS

UNIT 1
PAPER 01
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

This question sought to examine, in Part (a), knowledge relating to substitution and the factor/ remainder theorems, and in Part (b), to the use of summation via the Σ notation.

(a) (i), (ii) Candidates continue to demonstrate weaknesses in making substitutions for values in algebraic expressions. Substituting $x = 1$ resulted in the expression $1^4 - (p+1)^2 + p$. However, beyond this result some candidates failed to conclude that is a factor of $f(x) = x^4 - (p+1)x^2 + p$, for .

Many candidates attempted the division

$$x-1\overline{\big)\,x^4-(p+1)x^2+p}$$

but could not proceed beyond the point of obtaining

$$\begin{array}{r l} & \quad x^3 \\ x-1 \big) & \overline{x^4-(p+1)x^2+p} \\ & \underline{x^4-x^3} \end{array}$$

A few candidates earned full marks in this question, demonstrating that they fully understood the concepts of substitution and the remainder/factor theorems.

(b) The majority of candidates who could not show the required result failed to use the fact that

$$\sum_{r=1}^{n} 1 = n.$$

Substitution for $\sum_{r=1}^{n} 3r = 3\frac{n}{2}(n+1)$ posed little difficulties.

Answer: (a) (ii) $p = 4$

Question 2

This question tested the modulus function, sets and simple identities on real numbers:

(a) Generally, this question was poorly done. Many candidates solved the equation $|x-4| = h$, using the values of $x = 2$ and $x = 7$, to obtain values of $h = 2$ and $h = 3$. However, they could not proceed to the largest value of h.

Some candidates attempted to solve

$$(x-4)^2 = h^2$$

and had difficulties in answering the question. Very few candidates gained full marks on this question.

(b) Expansion of algebraic expressions continues to be an area of weakness, particularly those involving mixed terms such as x, y and $\frac{1}{2}y$. Those candidates who expanded the left-hand side correctly and simplified the result were able to find the correct value of k.

Answers: (a) Largest $h = 2$

(b) $k = \frac{3}{4}$

Question 3

The topics examined in this question were indices, surds and inequalities.

(a) (i), (ii) Rational inequalities that involve solving $\frac{f(x)}{g(x)} \le (\ge) c$, generally result in candidates solving $f(x) \le (\ge) c[g(x)]$.
Partial solutions do not give the full range of values of x for which the inequality is true. A few candidates showed a fair understanding of the methods required for finding the correct solutions. Many candidates used the technique requiring a table with change of signs for $ax + b$ and $x + 1$ to obtain the range of values of x for the solution. No candidate used the technique of the number line and the region of like signs for greater than or equal to zero (≥ 0) or the region of unlike signs for less than or equal to zero (≤ 0).

(b) It is surprising that many candidates are still finding it difficult to use simple manipulation of indices to evaluate results. Evidence of failing to express $\sqrt{2}$ as $2^{1/2}$ and to simplify $8^{-1/3}$ were observed. Many candidates failed to obtain full marks for showing the final expression $2^4(\sqrt{2})$.

Answer(s): (a) (i) a = 1, b = -2; (ii) $x > 2$ or $x < -1$.

Question 4

This question focused on functions and their properties.

(a) (i) Several candidates were able to get the correct values for p and q.
Many candidates were able to simply read the values from the graph by using the boundary values of the domain and the expression for $f(x)$.

(ii) The concept of range was not fully understood by many candidates.

(b) (i),(ii),(iii) Candidates continued to show poor understanding of surjective, injective and bijective functions. Some candidates used the horizontal line test for the injective function. Others used the vertical line test for the surjective function.

Many of the responses were in fact essays explaining when a function is surjective and/or injective. More mathematical examples should be practised by candidates to enhance understanding of these concepts as they relate to functions.

Answer(s): (a) (i) $p = 2,\ q = 1$; (ii) $1 \le f(x) \le 2$

(b) (i) surjective since $1 \le f(x) \le 2$ for each $-1 \le x \le 1$

(ii) not injective since $f(-1) = f(1) = 2$

(iii) no inverse since since f is not injective

Question 5

This question examined the solution of a system of two simultaneous linear equations with two unknowns.

(a) Not many candidates used the method of the non-singular matrix to obtain the condition for a unique solution of a system of equations. Candidates used the idea of the coefficients of x and y not being equal in order to find the condition for a unique solution. However, candidates failed to reason that $n \in$ R hence lost marks for this question.

(b), (c) It was not difficult for several candidates to deduce the values of m for inconsistent and infinitely many solutions, after finding the correct solution to Part (a). Some incorrect values were given for n.

Answer(s): (a) $m \neq 4$ for any $n \in R$

(b) $m = 4,\ n \neq 2$

(c) $n = 2,\ m = 4$

SECTION B
(Module 2: Plane Geometry)

Question 6

This question tested some of the salient properties of the intersection of perpendicular lines and the perpendicular distance of a given point from a given line.

(a) Most candidates recognized that the gradient of PQ was 2 and correctly found the equation of the line PQ. Few candidates found it difficult to determine the required gradient but using the gradient obtained, they were able to find an equation.

(b) Several candidates obtained full marks for this part. Those who followed through from Part (a) also recognized that by solving the pair of equations simultaneously, they would find the coordinates of point Q.

(c) Most candidates used the correct formula in finding the exact length of the line segment PQ. However, the majority, after having found $\sqrt{5}$ proceeded to approximate to 3 significant figures.

In general, the question was well answered.

Answer(s): (a) $y = 2x + 3$; (b) Q = (1, 5); (c) $PQ = \sqrt{5}$ units

Question 7

The question tested knowledge of the cosine formula.

Many candidates did not know how to find $\cos\frac{2\pi}{3}$. Some seemed unfamiliar with the cosine rule while others did not know the meaning of 'exact length'. For a topic which has been examined so often , too many candidates found the question difficult; howver, despite the short-comings of some candidates, there were a few excellent answers.

Answer(s): (a) AC = 13; (b) AB = $13\sqrt{2}$

Question 8

The focus of this question was quadratic equations in trigonometric functions, and trigonometric identities.

In general, this question was reasonably well-done; however, poor algebraic manipulation hinderered several candidates' progress in both parts. Due care is required in writing signs when a substitution is made. Some candidates did not give the answers in Part (a) in radians while a few had difficulties in solving a quadratic equation involving a trigonometric function.

Answer(s): (a) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Question 9

The question examined complex numbers and the roots of quadratic equations as they relate to the coefficients of the equations.

The overall performance on the question was poor. Again poor algebraic manipulation was evident and in some instances the roots of the equation were not multiplied to obtain a value for k. In Part (b), multiplying numerator and denominator of the left hand side by 3+4i did not occur to many candidates while to some, the rationalisation and equating of real and imaginary parts presented insurmountable challenges. A few candidates did well on this question.

Answer(s): (a) $k = 13$ (b) $u = 11$, $v = 2$

Question 10

This question tested properties of vectors including perpendicularity.

The response to this question was satisfactory, nevertheless, several candidates lost credit for faulty algebraic manipulation. While the basic concepts seem to be understood, errors due to carelessness spoiled the correctness of answers for many.

Answer(s): $x = {}^{-}3$, $y = 1$

SECTION C

Question 11

The question tested basic knowledge of limits and continuity/discontinuity.

Factorization and substitution were the two main areas of weakness in the efforts of the candidates. Most candidates showed familiarity with the concepts but errors in factorization of either numerator or denominator were the main obstacles towards achieving complete and correct solutions.

Answer(s): (a) -3 (b) $x = \pm\sqrt{3}$

Question 12

The focus of this question was differentiation and the critical values of a function.

Most candidates attempted this question but not many obtained maximum marks. Several did not seem to be familiar with the term 'critical' as it applies to curves. Many errors occurred due to faulty algebra and weakness in simplifications.

Answer(s): (a) critical value at $(4, {}^{-1}/_{8})$, a minimum

(b) $f'(x) = 4x \sin x^2 \cos x^2$

Question 13

This question tested knowledge of stationary points of a cubic curve and of methods used to obtain the equation of a normal to the curve at a given point.

(a) Many candidates obtained full marks on this part of the question but others had difficulties in separating out the values of x for the stationary points.

(b) There were many good answers to this question but again faulty algebraic manipulation was problematic in a few cases.

Answer(s): (a) $A \equiv (-2, 16)$, $B \equiv (2, -16)$

(b) Equation of the normal is $y = \frac{1}{12}x$ or $12y = x$

Question 14

This question focused on area under the curve and the integration of simple functions to obtain such an area.

Several candidates obtained full marks for this question. A few had difficulty expressing the shaded region in Part (a) as a difference of two integrals while others ignored $\int_2^3 dx$ in the calculations in Part (c). A few candidates chose to find the area of the triangle as a means of calculating the required area in Part (c).

Answer(s): (a) $\int_2^3 \frac{16}{x^2}\, dx - \int_2^3 (\frac{1}{2}x - 1)\, dx$

(b) $A = 2.42\ units^2$

Question 15

This question focused on a fundamental principle of definite integrals and the use of substitution towards the evaluation of such integrals.

Not many candidates attempted this question. However, among those who attempted it, a few earned full marks and most of the others at least 4 marks. The main difficulty seemed to be a misinterpretation of the question which required direct use of

(i) the given result in Part (a) applied to $f(x) = x \sin x$
(ii) the identity $\sin(\pi - x) = \sin x$

The only integration involved required candidates to find $\int_0^{\pi} \cos x \, dx$ in Part (c).

UNIT 1
PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)

<u>Question 1</u>

This question tested the candidates' ability to solve two simultaneous equations in two unknowns, one being quadratic and one being linear, as well as to demonstrate the relationship between the sum and product of roots and coefficients of $ax^2 + bx + c = 0$.

The question was generally well answered with many candidates scoring the maximum 20 marks.

(a) In cases where candidates attempted to make y the subject of either formula there were problems in expanding the brackets after the substitution was made.

For example, $x - 3\left(\dfrac{6 - x^2}{x}\right) + 1$ was **<u>incorrectly</u>** expanded as

$$x - \frac{18 - 3x^2}{3x} + 1$$

(b) Many candidates

(i) equated $\alpha + \beta$ incorrectly

(ii) represented $\alpha^2 + \beta^2$ incorrectly

(iii) failed to put the expression $x^2 + 2x - 2$ equal to zero as required (that is, a quadratic equation).

Answer(s): (a) $x = -\frac{9}{4},\ y = -\frac{5}{12}$ and $x = 2,\ y = 1$

(b) (i) $\alpha + \beta = -4,\ \alpha\beta = 1$

(ii) $\alpha^2 + \beta^2 = 14$

(iii) Equation:

$x^2 + 2x - 2 = 0$

<u>Question 2</u>

This question tested the principle of mathematical induction and the use of the sigma (Σ) notation.

Many candidates performed below average in this question, especially in (b) and (c). Only a very small percentage of candidates earned the maximum 20 marks.

(a) From the responses it was evident that candidates showed some improvement over previous years. However, few candidates earned full marks.

Some weaknesses observed were:

- The use of the right hand (RHS) only (instead of both LHS and RHS) to establish that P(1) is true.

- The inductive step was incorrectly obtained by some candidates who replaced k with $k+1$, thus obtaining $P(k+1) = \frac{1}{2}(k+1)(k+2)$ instead of $P(k+1) = \frac{1}{2}k(k+1) + k + 1$.

- Incorrect conclusions involving $\forall n \in Z$, $\forall n \in R$ instead of $\forall n \in Z^+, \forall n \in N$ or equivalent.

(b) (i) There were poor responses to this part. Candidates demonstrated a lack of understanding of the concept tested in this part of the question. Some candidates multiplied the expression by 2 thereby **incorrectly** obtaining

$2\left(\frac{1}{2}n(n+1)\right)$ for $\sum_{r=1}^{2n} r$.

(ii) This part of the question was poorly done by the majority of candidates who failed to recognize that $\sum_{r=n+1}^{2n} r = \sum_{r=1}^{2n} r - \sum_{r=1}^{n} r$.

(c) Many candidates failed to see that this part was a continuation from (b) (i) and (ii), so that very few correct answers were obtained.

For example, $\sum_{r=n+1}^{2n} r = 100$ was incorrectly interpreted by the weaker candidates as $n+1=100$, that is, $n=99$.

Answer(s): (b) (i) $\sum_{r=1}^{2n} r = n(2n+1)$

(ii) $\sum_{r=n+1}^{2n} r = \frac{1}{2}n(3n+1)$

(c) $n=8$

SECTION B
(Module 2: Plane Geometry)

Question 3

This question dealt with the geometry of the circle and tested the candidates' ability to find the centre and radius, given the equation of a circle in the Cartesian form, to obtain parametric equations from the Cartesian form and to find the points of intersection of a curve with a straight line.

(i) This part was generally well done, however, some candidates encountered problems converting the given equation into the form $(x-a)^2 + (x-b)^2 = r^2$. Problems arose when candidates had to complete the square. Candidates who expanded to find 'f' and 'g' usually forgot to change signs for the coordinates of the centre. Others factorised **incorrectly** to find the centre usually by grouping like terms, for example, $x(x+2)+y(y-4)=4$ to obtain **incorrectly** that radius = 4 and centre = (2,-4).

(ii) This part was poorly done. Candidates generally did not show any understanding of the concepts involved. A preferred method was substitution of the parametric equations using $\sin^2\theta + \cos^2\theta = 1$ to obtain the original equation given in (i).

(iii) This part of the question was generally well done; however, substitution of $y=1-x$ into $x^2+2x+y^2-4y=4$ was the preferred method leading to the quadratic equation $2x^2+4x-7=0$ from which was obtained the required solution.

(b) This question was well done. Most candidates, however, did not exhibit a full understanding of 'General Solution' and stopped after finding the principal values of θ.

Answer(s): (a) (i) Centre (-1, 2); radius = 3 units

(b) $\theta = 2n\pi \pm \frac{\pi}{3}$, $2n\pi \pm \pi$

Question 4

This question covered topics related to trigonometric functions of the form $a\cos x + b\sin x$ and complex numbers.

Most candidates showed familiarity with the concepts involved in both parts of this question, however, in Part (a) the notion of stationary point confused some candidates.
In Part (b), many candidates did not see the connection between (iii), (i) a) and b), and the roots of quadratic equations.

Some good answers were received:

Answer(s): (a) (i) R= 4.1, $\alpha = 14°$;
(ii) $x = 104°$

(b) (i) a) 5+i, b) 18-i, c) $-\frac{6}{25} - \frac{17}{25}i$

(ii) Equation: $z^2 - (5+i)z + (18-i) = 0$

SECTION C
(Module 3: Calculus 1)

Question 5

This question covered the topics of differentiation from first principles of the function $y = \sin 2x$ and the application of differentiation in obtaining the gradient and equation of a tangent to a given curve.

This question was generally well done by several candidates. Part (a)(iii) proved to be challenging for some candidates, particularly those who experienced difficulties in obtaining A and B correctly in (a) (ii).

In Part (b), some candidates had minor difficulties in differentiating $y = hx^2 + \frac{k}{x}$ but apart from those, many candidates found this part easy.

Answer(s): (a) (i) $\lim_{\delta x \to 0} \frac{\sin \delta x}{\delta x} = 1$; (ii) $A = 2x + \delta x, B = \delta x$

(b) (i) $h = 2, k = -1$; (iii) Equation: $2y = 12x - 9$

Question 6

The topics tested in this question involved integration by means of the rectangular rule, and differentiation and integration of rational functions.

(a) (i) Most candidates used the trapezium rule instead of the required rectangular rule.

(ii) Many candidates were unable to show the required equation for the approximate area S using the given sum $\sum_{r=1}^{n-1} r = \frac{1}{2}n(n-1)$. The factorization of the expression for S in (a) (i) was clearly not recognized by the candidates.

(b) (i) This part was well done. Students correctly identified that the quotient rule was needed. There were some instances where candidates rearranged the expression for $f(x)$ and used the product rule as an alternative.

(ii) Candidates' performance on this part of the question was satisfactory. Most students took notice of the "Hence" part of the question and realized that the integral of the expression given involved using a scalar multiple of $f(x)$.

(c) There were some good results for this question. Some candidates, however, did not notice that the integration process involved a negative index and proceeded to treat the index as a positive number. For most candidates, the 'solving process' was well done.

Answer(s): (b) (ii) $\frac{6}{5} = 1.2$; (c) $u = 2$.

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

This question focused on the modulus function, indices, quadratic equations and properties of functions in general.

(a) (i) There were a few good responses to this part among the small number of candidates, nevertheless, some candidates obtained only one value of x because one or other of the two possible equations $x+4=2x-1$, $x+4=-(2x-1)$ was ignored.

(ii) Indices continue to present difficulties to many candidates. Too often candidates **incorrectly** obtained $\frac{x^2}{4}$ from the expression $\frac{3^{x^2}}{81}$.

(b) Candidates seemed not to understand the basic definition of a function and so had difficulty in doing Parts (i) and (ii), and in recognising a function as a set of ordered pairs.

Answer(s): (a) (i) $x=5$ or $x=-1$ (ii) $x=4$ or $x=-2$

(b) (i) v maps to both 1, 3 or w is not mapped to any $b \in B$.

(ii) Delete one of the arrows from v to 1 or 3 and map w to any $b \in B$.

(iii) $g = \{(u,1), (v,1), (x,2), (y,4), (w,b), \textit{any } b \in B\}$ or

$g = \{(u,1), (v,3), (x,2), (y,4), (w,b), \textit{any } b \in B\}$

SECTION B
(Module 2: Plane Geometry)

Question 2

This question tested a linear function model of an experiment, trigonometrical identities and some basic properties of complex numbers.

(a) The initial value $d=0$ corresponding to $w=500$ in the table in (i) was routinely missed by the candidates and this adversely affected the outcomes to the entire Part (a). Few correct answers were received.

(b) Both identities presented difficulties due mainly to faulty algebra, however, there were one or two correct derivations in both cases.

(c) Candidates showed some familiarity with complex numbers but a few seemed not to know how to find $\arg(z)$ in Part (ii). None used the fact that $z\,\bar{z} = |z|^2$ which connected Part (i) with Part (iii).

Answer(s): (a) (i)

d (day)	0	25
w (gm)	500	1500

(ii) (a) $w = f(d) = 40d + 500$

(b) $w = 900$

(iii) $d = 42$

(c) (i) $|z| = 1$

(ii) $\arg(z) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

(iii) $z\bar{z} = 1$

SECTION C
(Module 3: Calculus 1)

Question 3

The topics covered in this question were limits, integration and volume of rotation.

(a) There were some good answers to the limits posed in this part.

(b) This part was not well done. The separation of $\int_2^3 [f(x) + 4]\, dx$ did not come readily to all but a very few of the small number of candidates.

(c) That rotation was around the y-axis was ignored by almost all of the candidates.

Answer(s):

(a) (i) $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x \to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}$

(ii) $\lim_{x \to 4} \frac{\sqrt{x}-2}{x^2-5x+4} = \lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} \lim_{x \to 4} \frac{1}{x-1}$

$= \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$

(b)

$$\int_2^3 [f(x)+4]\, dx + \int_3^5 f(x)\, dx = \int_2^5 f(x)\, dx + \int_2^3 4\, dx$$

(c) (i) $\frac{\pi}{2}$ (ii) π

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 was of a high standard with a small number of candidates reaching an outstanding level of proficiency. However, there were some candidates who were inadequately prepared for the examination.

Topics in Calculus, simple probability, approximation to roots of equations and series seemed well covered but weaknesses continue to manifest themselves at the level of algebraic manipulation, including substitution, which frustrate the processes required to complete the problem-solving exercises posed in the questions. As recommended in previous years, extended practice on respective themes needs to be undertaken in order to eradicate such deficiences and raise the level of performance in the identified areas of weakness.

The results on the whole were very encouraging.

DETAILED COMMENTS

UNIT 2
PAPER 01
SECTION A
(Module 1: Calculus II)

Question 1

This question examined the use of logarithms in solving equations. The majority of the candidates performed well on this question.

(a) Candidates performed better in this part of the question than in Part (b). Despite this, some errors were evident. For instance, $(\log x)^2$ was incorrectly interpreted as $\log x^2$ or $2 \log x$.

(b) Many candidates were aware of the principle/procedure involved, but premature rounding off of values affected the accuracy of the answer. In some cases, log 5-log 3 was incorrectly represented as $\frac{\log 5}{\log 3}$ instead of log (5/3)

Answer(s): (a) $x = 4$ or $x = 2$

(b) $x = 3.15$

Question 2

This question tested the candidates' ability to differentiate (using the chain rule or otherwise), combinations of trigonometric and logarithmic functions as well as to find the derivative of $e^{f(x)}$ and ln $f(x)$, where $f(x)$ is a differentiable function of x.

The question was generally well done by the many candidates who attempted it, with approximately half of them scoring the maximum mark.

(a) Many candidates omitted the brackets and wrote

$e^{2x+\sin x}2+\cos x$ or $2+\cos x\, e^{2x+\sin x}$

instead of $(2+\cos x)\, e^{2x+\sin x}$

(b) The most common errors were:

(i) $\frac{d}{dx}(\tan 3x) = \sec^2 x$ or $\sec^2 3x$

(ii) $\frac{d}{dx}\left(\ln(x^2+4)\right) = \ln\left(\frac{2x}{x^2+4}\right)$ or $\frac{1}{x^2+4}$

Answer(s): (a) $\frac{dy}{dx} = e^{2x+\sin x}(2+\cos x)$

(b) $\frac{dy}{dx} = 3\sec^2 3x + \frac{2x}{x^2+4}$

Question 3

This question tested the ability of candidates to use the concept of implicit differentiation to obtain the gradient of the curve at a point P and to use it to find the equation of the normal at a point P on the curve.

(a) There were many good solutions to this part of the question. However, some candidates experienced problems with the implicit differentiation and the product rule. The transposing of terms in the equation posed a challenge in a few cases as well.

(b) This part of the question was generally very well done. About 95% of the candidates were able to obtain the correct gradient of the normal from the gradient of the curve in Part (a), and the subsequent equation.

Answer(s): (a) $\frac{dy}{dx} = \frac{-1}{2}$

(b) Equation: $y = 2x + 5$ or $y - 2x = 5$

Question 4

This question examined the candidates' knowledge about applying the chain rule to find the first and second derivatives of trigonometric functions involving sin 2*A* and cos 2*A*.

The response to this question was very good with many candidates earning the maximum mark.

(a) Many candidates differentiated $\sin 2A + \cos 2A$ as composite functions. This was efficient and full marks were obtained. However, few candidates transformed $\sin 2A + \cos 2A$ using the trigonometric identities $\sin 2A = 2\sin A \cos A$, and $\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$. This was longwinded and often included errors where candidates were unable to complete the solution successfully. Some candidates used the chain rule effectively in this regard.

(b) Candidates had a good level of success in obtaining $\frac{d^2y}{dx^2}$ using composite functions. Some candidates used other methods which were complex/complicated, without success. The proof of $\frac{d^2y}{dx^2} + 4y = 0$ was well done using the correct substitution for $\frac{d^2y}{dx^2}$ and y.

Answer(s): (a) $\frac{dy}{dx} = 2\cos 2x - 2\sin 2x$

(b) Since $\frac{d^2y}{dx} = -4\sin 2x - 4\cos 2x$

$= -4y$

$\therefore \frac{d^2y}{dx^2} + 4y = 0$

Question 5

Candidates were required to use given substitutions to integrate functions.

(a) This part of the question was generally well answered with the majority of candidates earning the maximum of 4 marks for this part. There were some attempts at integration by parts although the question specifically stated "use the substitution given".

(b) This part proved to be a bit more problematic, as it involved three substitutions. There were difficulties in obtaining $dx = u\ du$, with some candidates incorrectly obtaining $dx = \frac{du}{u}$ instead.

The omission of the constant of integration was not frequently seen. The manipulation of indices, and transposition continue to be challenging to some candidates.

Answer(s): (a) $\frac{1}{9}\sin^9 x + k$ (constant of integration)

(b) $\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + k$ (constant of integration).

or $\frac{1}{10}\sqrt{(2x+1)^5} - \frac{1}{6}\sqrt{(2x+1)^3} + k$ (constant of integration).

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 6

(a) This part of the question tested candidates' knowledge and ability to manipulate recurrence relations. Most candidates attempted this question, however, there was a high percentage of candidates who responded incorrectly. The main source of error encountered was that candidates substituted values for n and showed by example that $u_{n+2} = -u_n$ and $u_{n+4} = u_n$. Most candidates showed $u_{n+4} = u_n$ by first finding u_{n+3} and substituting (a long method). They generally did not note the relation

$$u_{n+4} = -u_{n+2} \rightarrow u_{n+4} = -(-u_n).$$

(b) This part of the question required candidates to write specific terms from the recurrence relations. Most candidates were able to identify correctly the required terms. However, a common error encountered was the use of u_1 as -3 (disregarding the given data; $u_1 = 1$).

Answer(s): (b) $u_1 = 1$ (given), $u_2 = -3$, $u_3 = -1$, $u_4 = 3$

Question 7

This question tested knowledge about summing a geometric series to n terms as well as finding x and d given the sum and product of three consecutive terms, $x-d$, x and $x+d$ of an arithmetic series.

Most candidates attempted this question with about ninety percent gaining at least six marks.

(a) Most candidates identified the common ratio and were able to use the sum formula for a GP.

(b) Many candidates who attempted this part obtained the maximum marks. The majority summed the terms and found the value of x. They then substituted this value in the product equation and solved correctly for d. However, a few candidates ignored the fact that $d > 0$ and left their answer as $d = \pm 2$.

Answer(s): (b) (i) $x = 7$, $d = 2$

Question 8

The question tested the use of the binomial term ${}^{n}C_{r}$, quadratic equations and inequalities.

Approximately 80% of the candidates attempted this question in which 60% were able to earn full marks. However, there were some candidates who experienced difficulties in parts of this question, for example, with the binomial expansion of ${}^{x-2}C_{2}$.

Such candidates lacked the basic building block on binomial coefficients which seemed not to be known by some of the candidates. Some candidates also failed to answer the questions asked in Part (a) and went straight ahead to solve for x, for example, ${}^{x-2}C_{2} = {}^{5}C_{2} \Rightarrow x-2=5 \Rightarrow x=7$.

Answer(s): (b) $x=7$

Question 9

This question focused on the expansion of the expression $(1+ux)(2-x)^3$ in powers of x up to the term in x^2. Candidates were required to find the value of the constant $'u'$, based on a given condition of a specific coefficient in the expansion.

(a) Many candidates were able to expand the expression properly but a significant number of them did not stop at x^2; instead they expanded the expression completely.

(b) Several candidates had elementary problems with signs and as a result, they were unable to obtain the correct coefficient of x^2 and lost marks. Thus, although they identified the terms properly, they failed to give the correct answer for u.

Answer(s): (a) $8+(8u-12)x+(6-12u)x^2+...$

(b) $u=\dfrac{1}{2}$

Question 10

The question tested the candidates' knowledge of intersecting graphs and the algebraic equation represented at the point of intersection. It also covered roots in a specific interval of the real number line.

Most candidates were able to write down the equation required. The majority of candidates were able to determine that the function (equation) had values with different signs at the end points of the interval.

However, a large number of candidates did not point out that the function should be continuous. Some were insightful enough to say that the function was both continuous and differentiable.

While candidates established the existence of the root within the interval through a difference in signs of the value of the functions, a large number of them did not use the words 'intermediate value theorem' seemingly unaware that this was the theorem in use.

Generally, this question was well done.

Answer(s): (a) $e^x = -x \text{ or } e^x + x = 0$

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 11

This question tested the candidates' skills in using the binomial term ${}^nC_r$ in counting problems.

This question was generally well done by the majority of candidates. A few candidates who were unable to answer the question, applied the concept of 'permutation' rather than 'combination' that was required.

Answer(s): (a) 70 (b) 224 (c) 425

Question 12

The question tested arrangements of objects.

(a) This question was generally well answered. However, some candidates had difficulty in distinguishing between permutations and combinations.

(b) This part proved to be more difficult than Part (a) as candidates could not determine the denominator as (7!), in the calculation of the required probability.

Answer(s): (a) (i) 3600 (ii) 2400 (b) $\frac{3600}{7!} = \frac{5}{7} = 0.714$

Question 13

The topics covered related to determinants and methods of their evaluation.

This question was successfully answered by a large majority of the candidates. Two methods were used by the candidates to answer the question. Of these, the more popular method related to the use of 'minors'.

Question 14

The topics covered in this question were systems of equations, cofactors of a matrix, transpose of a matrix, matrix multiplication and determinants.

(a) Almost all candidates earned full marks on this part.

(b) (i) Most candidates confused the matrix of cofactors of the matrix A with the determinant of A.

(ii) Almost all candidates were able to write the transpose of B but too many were unable to form the matrix product $B^T A$.

(iii) Few candidates, even among those who calculated $B^T A$ correctly, were able to deduce the det A.

Answer(s): (a) $\begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 3 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

(b) (i) $B = \begin{pmatrix} -2 & -1 & 3 \\ -2 & 5 & 3 \\ 0 & -3 & -3 \end{pmatrix}$ (ii) $B^T A = \begin{pmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{pmatrix}$

(iii) $|A| = -6$

Question 15

The question tested the candidates' knowledge of the differential calculus in determining rates of change. Few candidates attempted this question. The majority of those who attempted the question earned very low marks.

(a) For Part (a) most candidates earned 1 mark.

(b) For Part (b) most candidates failed to differentiate the function correctly. Most candidates substituted the values of h and r directly into $A = 2\pi r^2 + 2\pi rh$. Candidates attempted to use various methods to answer this question which had no relationship whatsoever to the question.

Answer(s): (a) $\frac{dr}{dt} = 1.5$

(b) $\frac{dA}{dt} = 54\pi\, cm^2 / \sec$ when $r = 4$, $h = 10$

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

This question tested differentiation of a function of a function by means of the product rule as well as mathematical modelling related to an exponential function.

(a) (i) Basically, most candidates were able to apply the Product Rule for differentiation. However, many cases were observed where the differential of

$\ln^2 x$ was **incorrectly** given as $\frac{2}{x}$ and 2 ln x.

In addition, many candidates lost marks for inability to simplify the differential to show the required result.

(ii) Those candidates who attempted to apply the Product Rule for the result at Part (a) found it difficult to differentiate the expression $\ln x\,(3\ \ln x + 2)$. Many candidates used the result at Part (a) as $3x^2 \ln^2 x + 2x^2\ \ln x$ and proceeded to apply the Product Rule. Failure of these candidates to differentiate $\ln^2 x$ correctly resulted in their inability to show the required result. Many weaknesses in algebra were evident in the candidates' work.

(b) (i) A number of candidates gave the correct answer to this part of the question. Many of them substituted the values of $N = 50$ when $t = 0$ but failed to determine $e^{-r(0)} = 1$. Instead they carried forward the expression $1 + ke$. This resulted in the value of k given in terms of e. Failure to give the limit of ke^{-rt} for large t complicated the answers.

(ii) A common error seen in the responses to this question was NOT calculating the EXACT value of r. The majority of students got the equation $e^{-r} = 0.2$ but used the calculator to find r.

(iii) Having obtained the values for k and r there were no difficulties getting the correct answer to this question.

A few candidates gained full marks for the entire question.

Answer(s): (b) (i) $N = 800$ (ii) $k = 15$, $r = \ln 5$ (iii) $N = 714$

Question 2

The topics tested in this question related to partial fractions, integration of rational functions and reduction formulae.

(a) (i) A few candidates **erroneously** used the result

$$\frac{1+x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x^2+1}$$

to find the partial fractions.

Other errors included wrong calculations for the values of x which were substituted.

(ii) A few candidates incorrectly evaluated $\int \frac{-x}{x^2+1}\,dx$, particularly with respect to the minus sign. Many candidates failed to include the constant of integration.

(b) (i) Some candidates found this part of the question difficult. It was clear that integration by parts was not fully understood by these candidates. Other candidates had difficulty evaluating.

$$\left(xe^x - e^x\right)_{0.}^{1}$$

(ii) Generally most candidates demonstrated that they knew how to proceed with this question. However, writing the integral as

$$I_n = x^n e^x - \int_0^1 e^x\, nx^{n-1}dx$$

without including the limits of integration in the first integral, resulted in

$$I_n = x^n e^x - nI_{n-1}.$$

Candidates simply quoted the given result for I_n thus obtaining partial credit.

(iii) Many candidates attempted to find $\int_0^1 x^3 e^x\ dx$. The attendant difficulties were expected. Some candidates found it very difficult to use the formula for I_n and the link to I_1.

A few candidates gained full marks for the question.

Answer(s): (a) (i) $\frac{1+x}{(x-1)\,(x^2+1)} = \frac{1}{x-1} - \frac{x}{x^2+1}$

(ii) $\int \frac{1+x}{(x-1)\,(x^2+1)}\,dx = \ln|x-1| - \tfrac{1}{2}\ln(x^2+1) + k\ (const)$

(b) (i) $I_1 = 1$ (iii) $I_3 = 6 - 2e$

SECTION B

(Module 2: Sequence, Series and Approximations)

Question 3

This question tested arithmetic progressions, mathematical induction and sequences.

(a) (i) This part required that candidates show that $\sum_{r=1}^{m} \ln 3^r$ is an arithmetic progression.

Many candidates listed the terms in the progression with a comma between them, for example, ln 3, $\ln 3^2$, $\ln 3^3$, and not ln 3 + $\ln 3^2 + \ln 3^3 + \ldots\ldots \ln 3^m$ as was expected. Many answers did not indicate the m^{th} term of the progression. Approximately 80 per cent of the candidates showed working to indicate the first term 'a' and the common difference, 'd'. Approximately 60 per cent of these candidates calculated the numerical value of ln 3, rounding it off to the 3 s.f for the most part, but some also rounded off to 1 s.f.

It must be noted that it is **unnecessary** to use 10 significant figures in the value for ln 3. Almost fifty per cent of the candidates made a final statement to indicate that they understood that the progression was arithmetic in nature.

(ii) Few candidates added all 20 terms.
Approximately eighty per cent of candidates earned full marks for this question.

(iii) Many candidates gained full marks for this part.

(b) (i) Approximately five per cent of the candidates performed exceptionally well on this item.
The remaining ninety per cent had a vague idea as to the steps involved in proof by mathematical induction.
Steps such as:
Prove true for n = 1, assume true for n = k, were missing for the most part.

The final statement was also missing.

(ii) Approximately ninety-five per cent of the candidates did not do this question correctly. Most of them did not understand that they should apply the technique of completing the square.

Answer(s): (a) (i) A.P. with first term ln 3 and common difference ln 3.
(ii) Sum to 20 terms is 210 ln 3.

(b) (ii) $x_{n+1} - x_n = (x_n - \frac{1}{2})^2 > 0$

$\Rightarrow x_n < x_{n+1}$

Question 4

This question tested the ability of the candidate to sketch curves and use the Newton-Raphson method to find the non-zero root of $\sin x - x^2$.

Part (a) was poorly done as all but a few students had problems sketching $y = \sin x$ and $y = x^2$. Most drew $y = x^2$ as either the straight line y = x or as a V-shaped curve. Candidates also had some problems sketching y = sin x. Candidates need to be reminded of the need to use an appropriate scale on each axis as many did not show the point of $(\frac{\pi}{2}, 1)$ for sin x nor the (1, 1) for x^2. In too many cases, the points of intersection were way off. Candidates should also have stated the domain for the sketches.

Part (b) was attempted by several candidates although they just mentioned that the 2 curves 'intersected at 2 points, hence there were 2 real roots'. Hardly anyone wrote that x = 0 and x = α (the non-zero root).

Part (c) was also attempted by many candidates. Most of them knew that they were supposed to use the Intermediate Value Theoren, but many used values of x < 0 and values other than $\frac{\pi}{4}, \frac{\pi}{2}$.
Some used 0 and then said that 0 was positive. Many did not mention that the function was continuous.

Part (d) was attempted by almost all candidates. This was handled well by many, but the common mistake was the use of 0.7 as degrees instead of radians, hence, an incorrect answer was obtained. A few quoted the Newton-Raphson formula incorrectly. Many candidates did not notice that only one iteration was required and went on to find $x_2, x_3 \ldots$ and up to x_8 (in a few cases).

Answer(s): (c) Interval: $[0, \frac{\pi}{2})$, (d) $x_2 = 0.943$

SECTION C

(Module 3: Counting, Matrices and Modelling)

Question 5

This question tested the candidates' knowledge of simple permutations and probability in Part (a) while Part (b) examined their knowledge based on a probability model.

(a) (i) The majority of candidates who attempted this question realised that they were dealing with a 4-digit number, although a few considered a 6-digit number. Many candidates realised that the number must start with the digits 3, 4 or 5, but a few included the digit 6.

(ii) In general this question was poorly answered, however, responses of candidates indicated that they were familiar with the concept of probability.

(b) (i) It was clear that many candidates were not prepared to apply an unfamiliar formula to calculate the required probability, for example, binominal model. Some candidates exhibited serious misconceptions of probability as values were given outside of the interval $[0, 1]$.

(ii) Some candidates gave no consideration to possibilities that would have exhausted the sample space and chose to perform longer, rigorous calculations in order to arrive at their solution.

Answer(s): (a) (i) 180 (ii) Prob $= \dfrac{96}{180} = 0.533$

(b) (i) 0.543 (ii) 0.457

<u>Question 6</u>

The question tested basic knowledge about the product of conformable matrices and of finding the inverses of invertible matrices. Modelling is also included.

Almost all candidates attempted this question with many good responses. Part (a) focused on standard routine matrix operations while Part (b) focused on mathematical modelling incorporating matrices.

In Part (a), candidates frequently made arithmetic errors in calculating AB and this made it difficult to deduce A^{-1} from (a)(i). Others resorted to alternative methods of finding A^{-1}.

In Part (b), the weaker candidates seemed to have been challenged by the wording of the problem. The majority of them interchanged c, b, z, with p, q, r, and generated meaningless equations.

Another common error made by candidates was attempting to find M^{-1} although it was given in Part (b)(v).

Answer(s): (a) (i) AB = 4I (ii) $A^{-1} = \frac{1}{4}, \; B = \frac{1}{4}\begin{pmatrix} 2 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

(b) (i) z - grass = 2p + 2q + 6r

(ii) 2p + 4q = c

2p + 2q + 6r = z

6p + 4q + 4r = b

(iii) $\begin{pmatrix} 2 & 4 & 0 \\ 2 & 2 & 6 \\ 6 & 4 & 4 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} c \\ z \\ b \end{pmatrix}$

(iv) $x = M^{-1}D$

(v) p = 3, q = 6, r = 2.

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)

SECTION A
(Module 1: Calculus II, Algebra and Fractions)

Question 1

The qestion tested the candidates' knowledge of functions, natural logarithms and differential equations based on a simple biomathematical model.

There were some good responses to this question from among the few candidates who wrote the examination. Some experienced difficulty in writing down the differential equation in Part (a), but for those candidates who got past Part (a) the results were encouraging. Part (b)(i) also posed some minor challenges.

Answer(s):

(a) $f'(t) = k\,f(t)$

(b)(i) $f\ (0) = 10^6 = 1\,000\ 000$; $f\ (2) = 2\ x\,10^6 = 2\,000\,000$.

(iii) $f\ (7) = 10^6(2^{7/2}) \approx 11\ 313\ 709$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

This question examined sequences by means of mathematical modelling.

There were some excellent solutions to this problem from the small number of candidates. The only aspects of real difficulty for one or two candidates were Parts (a)(ii) and (iii).

Answer(s): (a)(i) row 1 - entries: Year $4 - P(1-\frac{1}{q})^3$; Year $5 - P(1-\frac{1}{q})^4$

row 2 - entries: Year $3 - P(1-\frac{1}{q})^3$; Year $4 - P(1-\frac{1}{q})^4$

Year $5 - P(1-\frac{1}{q})^5$

(ii) G.P. with common ratio $1-\frac{1}{q}$

(iii) $P\ (1-\frac{1}{q})^n$.

(b)(i) q = 5

(ii) $6553.60

(iii) n = 17

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 3

The question covered the topic of random selections and row reduction of the augmented matrix of a given system of equations.

The candidates found this question manageable. Only one or two had difficulty concluding the inconsistency of the system in Part (b).

Answer(s): (a) (*i*) $\frac{56}{2002} = 0.028$ (*ii*) $\frac{560}{2002} = 0.280$ (*iii*) 0.972

PAPER 03

INTERNAL ASSESSMENT

Module Tests

Approximately 138 centres (161 Teachers) in Unit 1 and 90 centres (98 Teachers) in Unit 2 were moderated.

In general, there was a marked improvement in the quality, consistency of marks awarded and the presentation of the Internal Assessment (Module Tests) by the teachers throughout the participating territories. Unit 2 was of a very good standard.

Although most of the questions used were taken from past CAPE Pure Mathematics Examination Papers, it was evident that few teachers made a conscientious effort to be original and creative in the tests designed.

This year, the majority of the samples of tests were submitted with question papers, solutions and detailed marking schemes with the marks allocated to the cognitive levels as specified in the syllabus. There were eight of 161 teachers in Unit 1 and two out of 98 teachers of Unit 2 who submitted samples without the required documents, therefore, making the moderation process more difficult.

It should be noted that the majority of teachers satisfied the objectives outlined by CXC CAPE Pure Mathematics Syllabus, Unit 1 and Unit 2. However, some common mistakes observed throughout the moderation process included:

(1) Test items examined were not consistent with the allotted time for the examination as some were either too long or too short. In one instance, there were 15 items with several parts to be completed by the candidates in 1 hour.

Teachers are reminded that the module test should be of 1 1/2 hours' duration.

(2) A few teachers continued to award fractional marks.

(3) On the question papers, teachers should indicate the time allotted, the total score for the examination, as well as instructions for the test.

(4) In a few cases, teachers tested topics in Unit 1 that were not in the Unit 1 Syllabus, for example, Logarithms, Partial Fractions and Implicit Differentiation. It must be noted that ‘3-dimensional vectors’ is not on the CAPE syllabus.

(5) A maximum of 5 samples is required for moderation. Please note that additional samples are not needed, unless there is a specific request from the Council. (Refer to FORM PMATH - 2)

(6) A few teachers are using the incorrect PMATH form to record marks. The scores for the three Modules (1, 2 and 3) scores must be recorded on the same form. (Form PMATH 2-3 Unit 2).

(7) Marks for the candidates should be clearly identified for each question at the side of the student’s solution; and the total at the top.

(8) The maximum mark that is allocated to each question on the question paper should be reflected in the allocation of marks on the solution and the mark scheme.

Overall, for the efficiciency of the moderation process, teachers should make every effort to adhere to the guidelines provided in the CAPE Pure Mathematics Syllabus.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2007

PURE MATHEMATICS

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2007

INTRODUCTION

This is the third year and the final time that the current syllabus has been examined. **A revised syllabus will be examined in 2008**. There has been a significant increase in the number of candidates writing the examinations with approximately 5021 writing Unit 1 compared to 4 430 in 2006 and 2 521 compared to 1 500 for Unit 2. Performances varied across the entire spectrum of candidates with an encouraging number obtaining excellent grades, but there continues to be a large number of candidates who seem unprepared to write the examinations particularly for Unit 1; a more effective screening process needs to be instituted to reduce the number of ill-prepared candidates.

GENERAL COMMENTS

UNIT 1

The overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as the Factor/Remainder Theorem, Coordinate Geometry, Basic Differential and Integral Calculus and Surds. However, many candidates continue to find Indices, Limits, Continuity/ Discontinuity and Inequalities challenging. These topics should be given special attention if improvements in performance are to be achieved. Other areas needing consolidation are general algebraic manipulation of simple terms; expressions and equations; substitution, either as a substantive topic in the syllabus or as a convenient tool for problem solving.

DETAILED COMMENTS

UNIT 1
PAPER 01
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (c) 1, 2, 3, 4, 6.

This question tested the use of the factor theorem to find factors and to evaluate an unknown coefficient in a polynomial given one of the factors. This question was attempted by almost all candidates. The majority of the candidates performed well on this question.

(a) The correct answer was obtained by the majority of candidates, however, a few of the candidates substituted incorrectly using $x = -1$ instead of $x = 1$ for the factor $x - 1$.

(b) This part of the question was generally well done by candidates, with various methods being used to determine the remaining factors such as long division, factor theorem, comparing coefficients and synthetic division. However, many candidates solved (for roots) after factorising and expressed the factors as $x = -1, -2$. Also, a small percentage of candidates expressed the remaining factors in quadratic form, failing to factorise completely.

Some candidates did not recognize that a non-zero remainder after division implied that no factors existed.

Answer(s):

(a) $p = 2$

(b) $(x + 1), (x + 2)$

Question 2

Specific Objective(s): (e) 1, 2; (f) 2.

This question examined the candidates' ability to solve an equation as well as to perform operations involving surds.

The question was generally well done by the many candidates (approximately 90 per cent) who attempted it, with approximately half of them scoring the maximum mark.

(a) Most candidates were able to express 27 as 3^3. However, approximately 25 per cent of the candidates were unable to express $(3^x)^2$ as 3^{2x}, expressing it instead as 3^{x+2} or, as 3^{x^2}, in a few cases. Despite the errors made with the indices, most candidates showed competence in their ability to equate the indices and to solve for x.

(b) Most candidates (approximately 75%) recognized the need to rationalize the denominator of the fraction by using its conjugate. However, some careless errors were made in the expansion, grouping and simplification of the surds, hence preventing some candidates from obtaining full marks in this part of the question. There were errors with fundamental operations such as $\sqrt{3}\sqrt{3} = \sqrt{6}$ and $\sqrt{3}\sqrt{3} = 9$.

Answer(s):

(a) $x = 6$ (b) $x = 13,\ y = -7$

Question 3

Specific Objective(s): (d) 1, 3, 9.

This question tested the ability of candidates to obtain coordinates of a given point after translations of the graph $y = f(x)$ and applying the principles of one-to-one functions in solving an equation involving composite functions.

This question was popular among the majority of candidates.

(a) Candidates were aware that a translation was involved. However, many failed to effect the correct translation; applying it to the wrong coordinate in some cases and to the wrong direction in other cases.

(b) The proof that $f(x)$ is a one-to-one function was poorly done by the majority of candidates. Much practice is needed with these proofs from first principles, that is, the general proof (instead of the proof for specific/particular values of x which was popular among candidates). Some candidates used proof by counter example for NOT one-to-one.

In the second part, the more confident candidates opted for the “hence” approach, while the majority used the “otherwise” approach in solving for x. The composite functions posed a challenge for the weaker candidates, with many simplifying and substituting incorrectly, for example, replacing x with $(3x – 2)$ to work out the composite as in $f(x-3) = 3x - 2 - 3$ instead of the **correct approach $f(x - 3) = 3(x - 3) - 2$**.

Answer(s):

(a) (i) $A'(1, 1)$

(ii) $A''(-2, 3)$

(b) (ii) $x = -5$

Question 4

Specific Objective(s): (a) 6; (f) 5.

This question examined the candidates’ ability to obtain the solution sets of inequalities involving the modulus function and to express a quadratic function in the form of a completed square.

The response to this question was good with approximately 80 per cent of the candidates scoring at least 6 marks.

(a) Some candidates encountered problems in performing the operations necessary for the removal of the modulus sign. A common error observed was $(x-4)^2 - 6 > 0$ instead of the correct inequality $(x-4)^2 > 6^2$ (or equivalent) which is obtained by squaring both sides.

Final answers were given as equations (roots) in some instances and not as a solution set in inequality form. A number of the candidates who used a graphical method to obtain their answer had difficulty expressing their final answer as an inequality.

A number of **errors** occurred with transpositions and sign changes, for example:-
$(x-4) - 6 > 0 \quad \Rightarrow \quad x - 4 > 6 \quad \Rightarrow \quad x > 2$ instead of $x > 10$.

(b) The question on completing the square was better manipulated than Part (a), however, a few errors were made with simple fraction addition such as $w = 2 + \frac{1}{12} = \frac{1}{6}$. Some candidates equated the coefficients and **incorrectly** obtained $w = 2$.

Answer(s): (a) $x > 10$ or $x < -2$ (b) $u = -3,\ v = \frac{1}{6}, w = \frac{25}{12}$

Question 5

Specific Objective(s): (f) 1, 7(i), 8(i), 8(ii), 10.

This question tested the candidates' ability to solve simultaneous equations in two unknowns, one being quadratic and one being linear.

The majority of the candidates attempted this question and a large percentage of them earned at least 5 out of 7 marks. The candidates who used the substitution $y = 5 - 3x$ were more successful than the ones who used $x = \frac{5 - y}{3}$ which presented a challenge to weaker candidates in terms of simplifying to obtain the correct quadratic equation.

A few candidates tried the elimination method without much success and in most cases, could not obtain the correct quadratic equation. Some candidates who obtained the quadratic equation in the form $-2x^2 + 5x - 2 = 0$ had difficulty factorizing it. Some candidates incorrectly solved $-2x + 1 = 0$ as $x = -\frac{1}{2}$. Also, a significant number of candidates solved for x and forgot to solve for y to complete the solutions.

Simple multiplication errors such as $3 \times \frac{1}{2} = 3\frac{1}{2}$ are not expected at the CAPE level.

Several errors in factorizing quadratic equations were noted as well, the most popular being:-

$$-2x^2 + 5x = 2 \quad \Rightarrow \quad x(-2x + 5) = 2 \quad \Rightarrow \quad x = 2 \text{ or } x = \frac{3}{2}$$

Answer(s): $x = \frac{1}{2}, y = 3\frac{1}{2}$ and $x = 2, y = -1$

SECTION B
(Module 2: Plane Geometry)

Question 6

Specific Objective(s): (a) 1, 2, 7 (i), 8, 9

The question tested the candidates' ability to solve problems in coordinate geometry dealing with equations of lines, points of intersection and perpendicularity of lines.

Approximately 85 per cent of the candidates attempted the question with the majority earning 4 or 5 marks out of the maximum 9 marks. A common mistake made by candidates related to treating M as the mid-point of one or both of the lines AC or BD. A few candidates also used BD perpendicular to AC and obtained incorrect solutions.

Answer(s): (a) (i) Equation of AC is $3y = x + 2$

(ii) Equation of BD is $y = 2x - 6$

(b) $M \equiv (4, 2)$

Question 7

Specific Objective(s): (b) 1, 5, 18, 20

This question was a test of the trigonometric form $R\cos(\theta + \alpha)$.

Many candidates attempted the question but there was limited success in most cases.

(a) Some candidates ignored the form $R\cos(\theta + \alpha)$ in attempting to manipulate $\cos\theta - \sin\theta$ within the parameters specified. A few performed well, nevertheless.

(b) Candidates who were familiar with the form in Part (a) performed well in Part (b). However, some were unable to obtain the general solution which suggests that this form of the solution needs further practice.

Answer(s): (a) $\cos\theta - \sin\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)$

(b) $\theta = 2n\pi, \quad 2n\pi - \frac{\pi}{2}, \; n \in Z$

Question 8

Specific Objective(s): (b) 3, 6

Knowledge of the area of a sector and of the area of a triangle was required to solve this problem.

In general, the question was popular with the candidates. However, many seemed unwilling to use radian measure, thus reducing the maximum mark they could achieve on the question. Several candidates experienced difficulties in using the formulae for the respective areas, in terms of π, of the sector and the triangle.

Answer(s): (a) π cm (b) $\left(3\pi - \frac{9}{4}\right)\text{cm}^2$.

Question 9

Specific Objective(s): (c) 4, 5, 7

The question tested the candidates' ability to obtain the conjugate of a complex number, to manipulate a complex number and its conjugate and to find the modulus of a complex number.

Approximately 90 per cent of the candidates responded to this question. Many of them were unable to earn the maximum seven marks, mainly because they seemed unfamiliar with the conjugate of a complex number. Several **incorrect** representations of the conjugate of $4 + 3i$ such as $-4 - 3i$, $3 - 4i$; $3i + 4$, $-4 + 3i$ and $\frac{1}{5}(4 + 3i)$ were seen.

A significant number of candidates did not express $\frac{\bar{z}}{z}$ in the form $a + bi$ as required; they left the answer as $\frac{7-24i}{25}$. Others who correctly obtained $\frac{7}{25} - \frac{24}{25}i$ disregarded the fractions for and b, and found the modulus in Part (b) as $\sqrt{7^2 + 24^2}$.

Answer(s): (a) $\frac{\bar{z}}{z} = \frac{7}{25} - \frac{24}{25}i$ (b) $\left|\frac{\bar{z}}{z}\right| = 1$

Question 10

Specific Objective(s): (d) 1, 3, 5, 8, 9, 10

This question tested basic elements of vector algebra in terms of magnitude, position vectors and perpendicularity.

In manipulating the vectors in this question, candidates made several errors, many of which were numerical in nature. As a consequence, several candidates did not earn maximum marks for their efforts.

Answer(s): (a) (i) $\overrightarrow{AB} = -\boldsymbol{i} - 6\boldsymbol{j}$ (ii) $|\overrightarrow{AB}| = \sqrt{37}$ (iii) $\overrightarrow{OM} = \frac{8}{3}\boldsymbol{i}$

(b) $\overrightarrow{OA}$ is not perpendicular to $\overrightarrow{OB}$ since $\overrightarrow{OA} \cdot \overrightarrow{OB} \neq 0$.

SECTION C
(Module 3: Calculus I)

Question 11

Specific Objective(s): (a) 3, 4, 5, 6, 7

This question tested knowledge of limits and continuity or discontinuity of functions.

(a) The majority of candidates recognised the indeterminate form when $x = -2$ was substituted in the original expression. However, a large number of them experienced difficulty in factorizing the cubic expression in the numerator. Some candidates factorized and cancelled correctly but then encountered difficulties with substituting $x = -2$ in the rational function which remained after cancellation.

Some candidates used L'Hopital's rule with a few doing so satisfactorily.

(b) Most candidates associated "continuity" with putting the denominator equal to zero but some of these had difficulty in drawing the correct conclusion when the results $x \neq 6$ and $x \neq -3$ were obtained.

Answer(s): (a) $\frac{3}{2}$ (b) Continuous for all $x \neq 6$, $x \neq -3$

Question 12

Specific Objective(s): 4, 7, 8, 9, 10, 11

This question examined differentiation of rational functions and integration of a trigonometric function.

(a) About 95 per cent of the candidates attempted this part of the question and were able to apply the quotient rule for differentiation, however, many errors were made in simplifying the resulting algebraic expression to arrive at the correct answer.

Some candidates wrote $f(x)$ as $(x^2 - 4)(x^3 + 1)^{-1}$ and successfully applied the product rule to obtain the answer.

(b) Only about 60 per cent of the candidates attempted this part of the question which is based on Specific Objectives (c) 7 and 8. Many candidates did not seem to know how to differentiate $u = \sin 2x$ to obtain $du = 2\cos 2x\, dx$, while others among those who did obtain the expression for du, failed to substitute correctly to transform the original integral to $\int \frac{1}{2} u\, du$; the point seemed to have been missed that in substituting $u = \sin 2x$ all occurrences of x in the integrand $\sin 2x \cos 2x$ and dx should be replaced with some function of u so that a new integrand appears under the integral sign expressed entirely in u and du.

Not many candidates used the "or otherwise" approach, but a few observed that $\sin 2x \cos 2x = \frac{1}{2}\sin 4x$ and proceeded to obtain the correct solution of $\frac{1}{4}$.

Answer(s): (a) $f'(x) = \dfrac{-x^4 + 12x^2 + 2x}{\left(x^3 + 1\right)^2}$

(b) $\frac{1}{4}$

Question 13

Specific Objective(s): (b) 2, 7, 8, 9, 25

This question required candidates to obtain constant coefficients in a cubic equation, given points on the curve and the gradient at one of the points.

(a) This part of the question was attempted by almost all of the candidates, not all of whom earned maximum marks because of weaknesses in the algebraic manipulation. Several candidates did not show that $r = 0$ since the curve passed through the origin and so complicated the calculations which depended on this fact. Other candidates failed to differentiate and could not make use of the gradient at P being equal to 8 in order to obtain a second equation.

(b) Many candidates did not use the fact that the gradient was given as 8 at P but derived it. Others substituted $x = 2$, $y = 1$ at P (1, 2) instead of $x = 1$ and $y = 2$ to find the equation of the normal.

Answer(s): (a) $p = 3$, $q = -1$, $r = 0$ (b) $8y + x = 17$

Question 14

Specific Objective(s): (b) 7, 8, 9, 16, 17, 19, 20, 21

The question tested candidates' ability to find coordinates of stationary points of a curve and to determine the nature of the stationary points.

(a) The majority of candidates who attempted this question knew that the function had to be differentiated, however, after finding the first derivative and equating the result to zero only 60 per cent of them were able to arrive at the correct solution to the quadratic equation obtained. Of those who got the x-values correct, a considerable number could not correctly find the y-values.

(b) Most candidates knew that the nature of the stationary points revolved around "something" being "positive" or "negative", but some were uncertain what that "something" was.

Approximately 20 per cent of the candidates gained full marks on this question.

Answer(s): (a) (2, 16) and (–2, –16)

(b) (2, 16), a maximum; (–2, –16), a minimum

Question 15

Specific Objective(s): (c) 3, 6 (ii), 7(ii), 10 (i)

The question tested the candidates' ability to use the integral calculus to find the area between two curves.

(a) Most candidates knew that the points of intersection of the straight line and the parabola had to be found but a few made mistakes in factorisation. Several candidates earned the maximum marks.

(b) There were many good answers to this part, however, a few candidates made mistakes in using the incorrect form of the integrand representing the difference between the equations of the straight line and the parabola. Some others used wrong limits and obtained incorrect results.

Answer(s): $P \equiv (-1, 1)$, $Q \equiv (3, 9)$ (b) $\frac{32}{3}$ units^2

UNIT 1

PAPER 02

SECTION A

(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 1, (c) 1, 5, (e) 2, (f) 8

The question tested knowledge of real roots and real factors. Overall, the question was attempted by more than 90 per cent of the candidates. However, only a few managed to earn maximum marks.

(a) (i) The majority of the candidates recognised that the expression $x^4 - 9$ is the difference of two squares and hence were successful in factorising to give $(x^2 - 3)(x^2 + 3)$. Many candidates failed to state the factors as $\left(x-\sqrt{3}\right), \left(x+\sqrt{3}\right)$ and $(x^2 + 3)$.

(ii) It was evident that some candidates had difficulty with the terms 'real factors' and 'real roots'. Most of the candidates gave $\sqrt{3}$ as the only root.

(b) (i) Although all of the candidates attempted this question, some of them experienced difficulty squaring an equation involving a fraction. Instead of getting,

$u^2 = x^2 + 8 + \frac{16}{x^2}$, most candidates wrote $u^2 = x^2 + \frac{16}{x^2}$ or $u^2 = x^4 + 8x^2 + 16$.

A few candidates failed to simplify u^2, rather leaving it as $x^2 + \frac{4x}{x} + \frac{4x}{x} + \frac{16}{x^2}$.

(ii) Many candidates misinterpreted this question. They used the method of substitution to obtain $u^2 - 9u + 20 = 0$. In other words they used an 'or otherwise' approach rather than a deductive approach as suggested. Most of the candidates failed to recognise that since $x \neq 0$, then $x^2 \neq 0$, but rather ignored the presence of x^2 in $f(x)$.

(iii) A poor interpretation of Part (ii) led to much difficulty in solving Part (iii). Nevertheless, most candidates managed to utilize the factor theorem in order to solve for x when $f(x) = 0$. A number of candidates gave 0 as one of the possible values.

Answer(s): (a) (i) $\left(x+\sqrt{3}\right)$, $\left(x-\sqrt{3}\right)$ and $\left(x^2+3\right)$ (ii) $x = \sqrt{3}$, $x = -\sqrt{3}$

(b) (i) $u^2 = x^2 + 8 + \dfrac{16}{x^2}$ (iii) $x = 1, 2, 4$

Question 2

Specific Objective(s): (a) 1, 4, 10, (c) 2

This question tested the use of the summation notation, sums and products of quadratic equations and the principle of Mathematical Induction.

(a) Many candidates attempted to solve this question using the principle of mathematical induction. This misinterpretation led to incorrect solutions. Also, most candidates failed to equate correctly $3\,S_{2n}$ and $11\,S_n$ as a result of some careless mistakes such as:

(i) Incorrectly substituting n instead of $2n$ in finding S_{2n}

(ii) Inaccurately expanding brackets for instance, $3(2n^2 + n) = 6n^2 + 6n$ and $2\left({}^{11}\!/_{2}\left(n^2+n\right)\right) = 11n^2 + n$.

Several candidates had problems solving the quadratic equation $n^2 - 5n = 0$.

Many of the candidates who solved the quadratic equation $n^2 - 5n = 0$ correctly to obtain $n = 0$ and $n = 5$, failed to state that only $n = 5$ satisfies $n \in N$.

(b) Most of the candidates attempted this question and were able to earn maximum marks. However, a few failed to identify correctly the sums and roots of both equations. For example, instead of $(2\alpha + \beta) + (2\alpha - \beta) = 8$, they wrote $\alpha + \beta = 8$.

(c) Most of the candidates attempted this question. However, the majority performed extremely poorly. A number of candidates could not indicate all the steps clearly. The ability to use critical thinking and make logical deduction continues to be a major weakness.

Candidates preparing to write the examinations should find the "Pure Mathematics Resource Material" document recently published by CXC helpful in studying Mathematical Induction, and other topics as well.

Answer(s): (a) $n = 5$

(b) (i) $p = \alpha + \beta$, $q = 4\alpha^2 - \beta^2$ (ii) $\alpha = 2, \beta = 12$
(iii) $p = 14$, $q = -128$

SECTION B
(Module 2: Plane Geometry)

Question 3

Specific Objective(s): 1, 2, 5, 7 (ii), 8, 9, 10, 13, 14

This question dealt with the geometry of the circle and tested the candidates' ability to find the lengths of the radius and of the tangents, the equation of a circle and points of intersection of the circle with the x-axis from given data about the circle.

Approximately 80 per cent of the candidates attempted this question. There were several good solutions. Varying methods were used to obtain results, showing that candidates had a good grasp of the relevant material.

Answer(s): (a) (i) radius = 5 units (ii) $(x-5)^2 + (y+4)^2 = 5^2$
(iii) $A \equiv (2, 0)$, $B \equiv (8,0)$ (iv) $3x + 4y = 24$ (v) $P \equiv (0, 6)$

Question 4

Specific Objective(s): (b) 5, 7, 8, 12, 13, 14, 15, 16

This question challenged the candidates' ability to apply basic trigonometric ratios in establishing identities, and in evaluating angles in the context of a triangle.

Overall performance was below expectation. The main difficulty seems to stem from candidates' inability to recall information from CSEC level on basic trigonometric ratios. While many candidates readily used the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta$ in Part (a) (i), many did not make the connection with Part (a) (ii). In Part (b), the use of the ratios for the triples (3, 4, 5) and (5, 12, 13) in right-angled triangles, the fact that the exterior angle r is equal to the sum of the interior opposite angles p and q, and the sum of the angles in the large triangle being 180 giving $p + t = 120 - q$, all proved difficult for the majority of candidates.

Answer(s): (b) (i) $^4/_5$ (ii) $^{12}/_{13}$ (iii) $^{63}/_{65}$ (iv) $\frac{3\sqrt{3} - 4}{10}$

SECTION C
(Module 3: Calculus 1)

Question 5

Specific Objective(s): (b) 8, 9, 10, 11, 12, 13

This question examined differentiation and relationships between first and second derivatives of a given function with a hint of mathematical modelling.

(a) (i) This question was generally well done by most candidates, however a few candidates demonstrated weaknesses in differentiation, especially in the concept of the chain rule.

(ii) Most candidates who got the right answer for Part (i) were able to follow through to show this part.

(iii) The overall performance on this part was poor. Candidates did not recognize **the use of the product rule**. Most candidates attempted to differentiate the expression for dy/dx to obtain d^2y/dx^2, but failed to obtain the correct expression as a result of the algebraic manipulation involved.

(b) Generally, this question was poorly done. A few candidates were able to attain full marks. Most candidates did not seem to understand the maximum and minimum concept of cosine.

Many weaknesses were seen in the differentiation of the expression $h = 2(1 + \cos(\pi t/450))$. A common error was $dh/dt = -2\sin(\pi t/450)$.

Answer(s): (a) (i) $\dfrac{dy}{dx} = \dfrac{5x}{\sqrt{5x^2 + 3}}$

(b) (i) 4 metres (ii) $t = 450$ min. (iii) $\dfrac{\pi}{450}$ m/min

Question 6

Specific Objective(s): (b) 23, (c) 7 (ii), (iii), 8, 9, 10 (ii)

This question focused on a fundamental principle of definite integrals and the use of substitution towards the evaluation of such integrals. Curve sketching and the calculation of volume is also included.

(a) As in the previous year, the result $\int_o^a f(x)dx = \int_o^a f(a - x)dx$ was not fully understood by the candidates.

(i) This question was poorly done by most candidates. Very few candidates recognized the relationship between the constants a and $\pi/2$.

(ii) A few candidates obtained the correct answer, but most candidates had difficulty obtaining the answer $\pi/4$. Not many candidates used the formula for cos $2x$ in terms of $\sin^2 x$ or $\cos^2 x$ as a means of calculating I.

(b) (i) This question was well done by most candidates, however, a few did not recognize that the graph was a parabola which concaved upwards.

(ii) This question was generally well done by the majority of candidates. Some candidates used $V = \pi \int y^2 \, dx$ instead of $\pi \int x^2 \, dy$. A few candidates calculated incorrect values for the limits.

Answer(s): (b) (ii) $\frac{\pi}{2}$ units^3

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 6, (c) 1, (d) 1, 3, 5, (g)

This question focused on properties of functions and the solution of inequalities.

(a) Most candidates were unable to substitute competently into the given function.

(i) Most candidates made simple errors such as: $h\left(\frac{1}{t}\right) = \frac{1}{\frac{1}{t} + t}$ and $h\left(t^2\right) = \left(t + \frac{1}{t}\right)^2$

(ii) A number of candidates failed to simplify $h(t)h\left(\frac{1}{t}\right) - h(t^2)$

(b) The response to this question showed that most candidates had difficulty in solving inequalities. A number of candidates multiplied both sides of the inequality by $(x + 2)$ instead of the more direct approach of multiplying both sides by $(x + 2)^2$. A few candidates accurately ensured that zero was on one side of the equality. However, a few demonstrated poor algebraic manipulations in attempting to simplify the inequality.

(c) Most of the candidates demonstrated a poor understanding of key terms used in functions.

Answer(s): (a) (i) $\left(t + \frac{1}{t}\right)^2$ (ii) 2

(b) $-3 < x < -2$

(c) (i) range of $h = \{x \in \text{R}: x \geq 2\}$ (iii) domain of $h^{-1} = \{x \in \text{R}: x \geq 2\}$

SECTION B
(Module 2: Plane Geometry)

Question 2

Specific Objective(s): (a) 5, 6, 8, 9, 14, 16

The question focused on the ellipse represented in terms of parametric equations and by means of its Cartesian equation. The equation of a tangent to the ellipse was also examined.

(a) Several candidates attempted this question, many of whom recognised that the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ was the means to obtaining the desired equation. Some candidates who were not informed of the correction ($y = 2\sin\theta$) obtained instead the form $\frac{x^2}{9} + \frac{y^2}{16} = 1$; they were awarded full credit.

(b) Only a few candidates were able to solve correctly this equation. The majority of the candidates were unable to derive the gradient of the tangent which they could have obtained by using the chain rule $\left(\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}\right)$ or with knowledge of implicit functions. Candidates' inability to derive the equation of the tangent led to some problems in Part (c).

(c) (i) A number of candidates assumed a certain equation for the tangent. Consequently, a few of them earned a reasonable number of marks due to their knowledge of some basic geometric concepts (x and y intercepts, the area of a triangle, the length of a given line and perpendicular lines).

(ii) Those candidates who obtained the coordinates of Q and R were able to find the area of ΔQOR quite easily.

(iii) Candidates were able to calculate the length of QR by means of the formula for the distance between two points with known coordinates.

(iv) Candidates successfully utilized the formula for the length of perpendicular from O to QR as $\left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|$.

No candidate utilized the result of (c) (ii) (that is, the area of triangle $A = \frac{1}{2}bh$) to find the length of the perpendicular.

Answer(s): (b) $\frac{\sqrt{3}}{6}x + \frac{y}{4} = 1$

(c) (i) $Q \equiv \left(2\sqrt{3}, 0\right)$, $R \equiv (0, 2)$ (ii) Area of $\Delta QOR = 2\sqrt{3}$ units^2

(iii) $QR = 4$ units (iv) Perp. = $\sqrt{3}$ units in length

SECTION C
(Module 3: Calculus 1)

Question 3

Specific Objective(s): (a) 3, 4, 5, 6 (b) 9, 11, 15 - 17, 19 - 21

This question deals with limits, critical values of a given cubic function and differentiation leading to a maxima and minima.

(a) (i) The majority of the candidates who attempted this question were unable to factorise the expression $(x - 1)$ in terms of $x^{1/3}$.

(ii) A number of candidates attempted to solve this question using the "hence" approach, although their results to Part (i) were incorrect. As a result, only a few candidates were able to obtain maximum marks.

(b) (i) Most candidates attempted this question and were successful in obtaining full marks. Many candidates were confused with critical values of y and critical point; in most cases candidates gave the latter, that is, $\left(\frac{-1}{3}, \frac{32}{27}\right)$, (1, 0).

(c) (i) Most of the candidates were successful in showing that $S = 2\pi r^2 + \frac{20}{r}$. However, a few candidates were unable to show that S has a minimum value when $r^3 = 5/\pi$.

At least 90 per cent of the candidates attempted this question. With the exception of Part (a), the question appeared manageable to several candidates.

Answer(s): (a) (ii) $\frac{1}{3}$

(b) (i) $\left(-\frac{1}{3}, \frac{32}{27}\right)$ and (1, 0) (ii) $\left(-\frac{1}{3}, \frac{32}{27}\right)$ – a max.; (1, 0) – a min.

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 was of a high standard with a small number of candidates reaching an outstanding level of proficiency. However, there were some candidates who were inadequately prepared for the examination.

Topics in Calculus, Simple Probability, Approximation to Roots of Equations, and Series seemed well covered. Weaknesses continue to manifest themselves at the level of algebraic manipulation, including substitution, which frustrate the processes required to complete the problem-solving exercises posed in the questions. As indicated in previous years, extended practice on respective themes needs to be undertaken in order to eradicate such deficiencies and raise the level of performance in the areas of weakness identified.

However, the results on the whole were very encouraging.

DETAILED COMMENTS

UNIT 2
PAPER 01
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 3, 10

This question examined the exponential function and the use of logarithms in solving equations.

(a) Almost all candidates attempted this question with approximately 30 per cent obtaining full marks, however, some candidates experienced difficulties in factorising the expressions $e^{2p} - 2e^{p}$ and $e^{-p} - 2e^{-2p}$.

(b) Candidates were asked to solve an equation of the form $a^x = b$.

Most candidates attempted this part of the question with a satisfactory degree of success.

Overall, about 35 per cent of the candidates scored between 6 and 8 marks on this question.

Answer(s): (a) $p = \ln 2,\ q = \frac{1}{2}$ (b) $x = \frac{\log 3}{2\log 2} = 0.792$

Question 2

Specific Objective(s): (b) 2, 3, 5

This question focused on parametric equations of a curve, gradients and differentiation of functions.

(a) There were several good responses from the candidates on this part of the question, nevertheless, a few candidates had difficulties differentiating $\frac{4}{t}$ while others seemed unaware of the chain rule and were unable to finish the question.

(b) The quality of responses to this question was mixed. Some candidates had difficulty differentiating one or both of $\tan^2(3x)$ and $\ln(x^3)$. A few candidates obtained full marks for Part (b).

Answer(s): (a) gradient = –1 (b) $\frac{dy}{dx} = 6 \tan 3x \sec^2 3x + \frac{3}{x}$

Question 3

Specific Objective(s): (c) 1, 3

This question involved the use of partial fractions in finding integrals.

(a) This question was well done with a success rate of approximately 96 per cent.

(b) Most candidates were aware that $\int \frac{1}{b+x}\, dx = \ln|b + x|$ but too many seemed unaware that

$$\int \frac{1}{b-x}\, dx = -\ln|b - x|.$$

Some candidates did not include a 'constant of integration' in each case.

Answer(s): (a) $P = 1,\ Q = 1$ (b) $\ln|3 + x| - \ln|2 - x| + \text{constant}$

Question 4

Specific Objective(s): (c) 4, 5, 6, 7

This calculus question covered the topics of integration by substitution and integration by parts.

(a) This part of the question was generally well done, however, many candidates who did not use the substitution completely were confronted with the task of manipulating an integrand which was a function of both u and x.

(b) Several candidates attempted this part but 80 per cent of them could only reach as far as $x \tan x - \int \tan x \, dx$, not being able to evaluate $\int \tan x \, dx$.

In both parts, the constant of integration was omitted.

Answer(s): (a) $\left(\frac{(2x - 5)}{24}\right)^6 + \frac{(2x - 5)^5}{4}$ + const. (b) $x \tan x + \ln(\cos x)$ + const.

Question 5

Specific Objective(s): (c) 7, 9

This question models a manufacturing process by means of a first order differential equation. The use of integrating factors was tested.

Approximately 85 per cent of the candidates attempted this question with about 9 per cent obtaining full marks.

The major difficulty encountered by candidates was the failure in obtaining the correct integrating factor. Some candidates used 'c' as the constant of integration which confused their otherwise correct solution.

Answer: $c = 5x + \frac{205}{2} e^{-2x} - \frac{5}{2}$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 6

Specific Objective(s): (a) 1, 2

This question tested the candidates' knowledge of, and ability to manipulate, recurrence relations as they apply to sequences.

About 98 per cent of candidates, attempted this question and performed extremely well. Most scored full marks which indicated that they were well prepared in this area.

Candidates provided a variety of approaches to obtain the solution.

Answer(s): (b) $u_3 = 2^2$, $u_5 = 2^3$

Question 7

Specific Objective(s): (b) 4, 8

Geometric progressions were covered in this question.

Approximately 95 per cent of the candidates attempted this question. Many were able to generate the equations $a + ar^2 = 50$, $ar + ar^3 = 150$ connecting the first term a and common ratio r, but some encountered difficulties in solving these two equations to find a and r. Those candidates who obtained the correct values of a and r were able to complete the question and earn maximum marks.

Answer(s): (a) $r = 3$ (b) $a = 5$ (c) sum = 605

Question 8

Specific Objective(s): (c) 2, 3

This question examined the candidates' ability to extract the independent term in a binomial expansion.

Candidates found this question easy with about 95 per cent submitting good answers and approximately 90 per cent of those obtaining maximum marks.

Answer(s): ${}^{10}C_6\, 2^6(-5)^4$

Question 9

Specific Objective(s): (c) 1, 2, 3

This question on the binomial theorem tested the candidates' knowledge about binomial coefficients in terms of factorials.

Most candidates who attempted the question were able to do both parts of (a) but had considerable difficulty in obtaining the result in (c) which suggests that more practice is needed in the area of the manipulation of expressions involving factorials.

Answer(s): (a) (i) $\dfrac{(2n)!}{n!\,n!}$ (ii) $\dfrac{(2n-1)!}{n!\,(n-1)!}$

Question 10

Specific Objective(s): (b) 10

The topic covered the method of differences in the summation of series.

Approximately 80 per cent of the candidates attempted this question and of these 85 per cent obtained less than 5 marks while 15 per cent obtained between 5 and 8 marks.

Candidates were comfortable with the method of finding the solution to Part (a) but had some difficulties with substitution. A few candidates were unable to cope with the summation notation Σ.

Answer(s): (b) $\sum_{r=3}^{n} \frac{r+3}{r(r-1)(r-2)} = \frac{1}{2}\left[\frac{7}{2} - \frac{2n+3}{n(n-1)}\right]$

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 11

Specific Objective(s): (a) 2, 3, 4

This question tested arrangements of objects.

(a) Many candidates failed to deduce the number of combinations of the vowels. Some candidates confused combination with permutation.

(b) The question was generally well done by most candidates. Some candidates confused addition with multiplication. Candidates were unable to differentiate between AND and OR.

Answer(s): (a) $\frac{8!}{3!\ 2!}$ (b) 186

Question 12

Specific Objective(s): (a) 5, 6, 7, 8, 9

This question tested some basic properties of counting and probability.

(a) Many candidates wrote the number 36 instead of representing the answer as ordered pairs.

(b) (i) Many candidates did not subtract 1/36 from 12/36.

(ii) This part was well done by many candidates.

(iii) Many candidates did not subtract 1/36 from (11/36) + (6/36) and obtained the correct answer.

Answer(s): (a) $\{ (a, b) : a, b \in \mathrm{N}, 1 \le a, b \le 6\}$

(b) (i) $\frac{11}{36}$ (ii) $\frac{6}{36}$ (iii) $\frac{4}{9}$

Question 13

Specific Objective(s): (b) 1, 2

This question focused on the products, transposes and determinants of square matrices.

This question was reasonably well done by most candidates. A few candidates showed weaknesses in calculating the determinant.

Answer(s): (a) $|X| = -365$ (b) $Y^T X = \begin{pmatrix} 7 & 68 & 29 \\ 56 & 78 & 4 \end{pmatrix}$

Question 14

Specific Objective(s): (b) 7, 8

This question covers the calculation of the inverse of non-singular matrices and the solution of matrix equations.

(a) Most candidates who used row reduction were unable to attain the final answer for A^{-1}.

(b) Many candidates wrote $X = YA^{-1}$ instead of $X = A^{-1}Y$. This was a common error.

A few candidates attempted to derive the answer using simultaneous equations but experienced difficulties.

Answer(s): (a) $A^{-1} = \frac{1}{2}\begin{pmatrix} 5 & 6 & -15 \\ -7 & -8 & 21 \\ -1 & -2 & 5 \end{pmatrix}$ (b) $X = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

Question 15

Specific Objective(s): (c) 1, 2

This question examined rate of increase in the context of mathematical modelling of properties of a sphere.

This question was generally well done by most candidates. A few students showed weaknesses in algebraic operations.

Incorrect differentiation was seen by a few candidates.

Answer(s): (a) $\frac{dr}{dt} = \frac{11}{16\pi}$ cm s^{-1} (b) $\frac{ds}{dt} = 55$ cm^2 s^{-1}

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 6, 8, 10, 11

This question related to logarithmic and exponential functions, and their graphs.

All candidates attempted this question with varying degrees of success.

(a) About 90 per cent of the candidates attempted this part. Change of base and the value of x satisfying the equation $\log_8 x = -2$ presented some candidates with major challenges.

(b) The table was generally well done in (i). In (ii), most candidates changed the scale to accommodate the range of values for the graph while others did not plot the point at $x = 3$; in both instances, candidates were credited with maximum marks.

About 96 per cent of the candidates who attempted this part of the question obtained at least 4 of the 5 marks that were allocated.

In Part (iii) b), about 50 per cent of the candidates obtained the end points for the range and far less got the inequality completely correct.

Answer(s): (a) $x = 2, \frac{1}{64}$ (b) (iii) a) $x = 0$, b) $-1 \leq x < 0$

Question 2

Specific Objective(s): (c) 7, 8

This question tested the candidates' ability to use reduction formulae.

(a) About 50 per cent of the candidates who attempted this question earned maximum marks. Many were familiar with the identity $\sec^2 x = 1 + \tan^2 x$ and applied it correctly.

(b) There were several good responses to this part, with many candidates obtaining full marks.

(c) (i) About 25 per cent of the many candidates who attempted this part of the question obtained full marks. The majority of candidates earned at least 4 of the 7 marks.

(ii) About 15 per cent of the candidates who attempted this part obtained full marks. Some candidates tried to use I_n for $n = 0$ even though it was stated earlier that $n \geq 2$.

Answer(s): (b) $n \tan^{n-1} x \sec^2 x$ (c) (ii) $I_4 = \frac{\pi}{4} - \frac{2}{3}$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objective(s): (a) 2, 5; (b) 4, 8, 9; (c) 1

This question tested the candidates' ability to apply the principle of mathematical induction to factorials; identifying and obtaining the general term of a geometric series as well as its sum to infinity.

The majority of the candidates attempted this question. However, a minority was able to gain near maximum marks.

(a) The Mathematical Induction was very poorly done. It was very clear that most students lacked the understanding of the **process** of induction; not even being able to recognize what was to be proved.

The basic step of proving that $n! = 1! = u_1 = 1$(given) was hardly seen. Most candidates wrote a very clear and concise memorized conclusion even though a proper inductive step was absent. Many candidates added the $(k + 1)^{th}$ term to the k^{th} term although the question dealt with $n!$ – a clear lack of understanding.

Some other weaknesses observed were:

– The inductive step was attempted by some students who replaced k with $k + 1$ thus obtaining $u_{k+1} = (k + 1)!$ instead of $u_{k+1} = (k + 1)u_k = (k + 1)k! = (k + 1)!$

– Incorrect conclusions involving for all $n \in Z$, for all $n \in$ R, were frequently seen, instead of for all $n \in Z^+$, for all $n \in N$ or equivalent.

(b) In this part, many candidates failed to recognize that the question asked for the n^{th} term which should be simplified into a single fraction in this case. This prevented some candidates from obtaining full marks. Careless simplification errors were made, especially with the indices because of the lack of brackets, for example, $2 - (n-1)$ was incorrectly written as $2 - n - 1 = 1 - n$.

In Part (ii), many candidates used a particular solution approach, when a general solution approach was necessary to show that the series S is a geometric progression.

Answer(s): (b) (i) $a_n = \frac{18}{3^n}$ **OR** $6\left(\frac{1}{3}\right)^{n-1}$ **OR** $2 \text{ x } 3^{2-n}$

(ii) $\frac{a_n}{a_{n-1}} = \frac{1}{3}$, a constant

(iii) $a = 6, \quad r = \frac{1}{3}$

(iv) $S_\infty = 9$

Question 4

Specific Objective(s): (b) 4, 7; (c) 1; (e) 1, 2, 4

This question examined the use of the Intermediate Value Theorem in testing for the existence of a root in an equation; the Newton-Raphson Method in finding successive approximations to a root in an equation and mathematical modelling involving an arithmetic series.

For this question, Part (b) proved more challenging to the candidates than Part (a).

(a) (i) The main difficulties candidates encountered with this part are as follows:–

- Not stating that the function is “continuous”
- Attempting to use the given formula to get the root
- Using the derivatives $f'(0)$ and $f'(1)$ in an attempt to show that $f(x)$ has a root in the interval (0,1).

However, many candidates correctly used $f(0)$. $f(1) < 0$ or the sign change criterion between $f(0)$ and $f(1)$, as well as mentioning the Intermediate Value Theorem, in this part of the question.

(ii) The main areas of concern regarding the candidates’ approach were as follows:

- Writing the Newton-Raphson Formula incorrectly as

$x_2 = \frac{f(x_1)}{f'(x_1)}$ or $x_2 = -\frac{f(x_1)}{f'(x_1)}$ instead of the correct form $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$.

- Having difficulty in simplifying the expression $x_n - \frac{f(x_n)}{f'(x_n)}$ due mainly to the omission of the necessary brackets, that is, $\frac{x_1(4x_1^3 - 4) - x_1^4 - 4x + 1}{4x_1^3 - 4}$ instead of $\frac{x_1(4x_1^3 - 4) - \left(x_1^4 - 4x + 1\right)}{4x_1^3 - 4}$
- Substitution of particular values in the Newton-Raphson Formula in order to prove the given expression.

(b) (i)
- Not recognising the loan repayment as an A.P., hence writing **<u>all 12</u>** instalments, then summing.
- Using P = \$570 to work out the 12 payments then summing to prove that A = \$10 800 rather than using A = \$10 800 to prove that P = \$570.

(ii) It was noted that the students had more difficulty with (b) (ii) than with (b) (i).

- Writing the loan balance as (10 800 – nth instalment) instead of (10 800 – S_n) where S_n is the sum of the first n instalments.
- Using the sum of a G.P. instead of the sum of an A.P.
- Incorrect expansion and simplification of the brackets when forming an expression for the sum of the instalments.
- Using 10 800 = S_n instead of (10 800 – S_n) as an attempt to find an expression for the remaining debt.

Answer(s): (b) (ii) $D = 10\,800 - 540n - 30n^2$

SECTION C
(Module 3: Counting, Matrices and Modelling)

<u>Question 5</u>

Specific Objective(s): (a) 4, 7, 9, 10

This question tested the candidates' knowledge of simple counting principles and probability.

There were several good responses to this question with many candidates scoring between 12 and 20 marks. A significant number earned maximum marks.

(a) Some candidates experienced confusion between permutation and combination. More practice is recommended to clarify the distinction.

(b) A few candidates included the column totals in their calculation of the number of males and the number of females.

In Part (ii), some candidates did not recognise the problem as inclusive and so many found P (males) + P (news) rather than P (males) + P (news) – P (males and news).

Generally, attention should also be paid to computational correctness in problems of this kind.

Answer(s):

(a) (i) 210 (ii) $\frac{55}{210} = 0.262$

(b) (i) $\frac{48}{100} = 0.48$ (ii) $\frac{70}{100} = 0.70$ (iii) $\frac{20}{100} = 0.20$

(iv) $1 - \frac{30}{100} = \frac{70}{100} = 0.70$

(c) (i) $p = 0.20$ (ii) 0.45

<u>Question 6</u>

Specific Objective(s): (b) 1, 2, 3, 4, 5, 6

The question examined solutions to systems of linear equations by matrix methods, as well as properties of matrices and determinants.

Although almost all the candidates attempted the question, generally, it was not well done. Very few candidates were able to obtain full marks in this question.

(a) (i) In a few cases, candidates were not able to express the system of equations in complete matrix form, that is, $AX = Y$.

(ii) Although candidates were able to identify the augmented matrix, in many instances this was not written using valid notation. Some candidates confused the augmented matrix with the adjugate matrix.

(iii) Most candidates were able to perform elementary row operations. There were, however, many arithmetic errors. There also seemed to be a problem in identifying when the matrix was in echelon form with candidates getting a simpler matrix to work with but not a matrix in the desired echelon form.

(iv) Most candidates were able to find the value of α from their reduced matrix.

(v) The majority of candidates were able to solve the system of equations up to a point. Many got as far as stating $y = -1$, $x + z = 11$. Only a few candidates were able to go further to choose x or z arbitrarily and state the final required answer.

(b) (i) This was generally well done, however, many candidates seemed not to be able to identify the 3 x 3 identity matrix, I. Some students misinterpreted $KI - A$ as $K(I - A)$.

(ii) Most candidates who correctly found KI-A were able to at least make a valid attempt at finding the determinant. In many instances, there were errors in the simplification of the determinant and this led to the wrong cubic equation. Most candidates were able to use the factor theorem correctly to factorise the cubic. There were cases of candidates correctly factorising the valid cubic equations but losing marks by making errors such as $k^2 - 3 = 0 \Rightarrow k = \pm 3$ or by simply stating that $k^2 - 3 = 0 \Rightarrow k^2 = \sqrt{3}$.

Answer(s): (a) (i) $\begin{pmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 35 \\ \alpha \end{pmatrix}$ (ii) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 3 & -2 & 3 & 35 \\ 2 & 1 & 2 & \alpha \end{array}\right)$

(iii) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & \alpha - 21 \end{array}\right)$ (iv) $\alpha = 21$

(v) $x = 11 - z,\ y = -1, z$ arbitrary

(b) (i) $kI - A = \begin{pmatrix} k & 1 & -1 \\ 1 & k & -1 \\ -1 & -1 & k-1 \end{pmatrix}$ (ii) $|kI - A| = 0 \Rightarrow k = 1, k = \pm\sqrt{3}$

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)

SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (c) 1, 2, 3

The question posed a mathematical modelling problem based on proportionality, exponentials and the solution of a differential equation.

Most of the candidates who took the paper were unable to form the differential equation required in Part (a).

While 10% of the candidates gained maximum marks, the majority earned less than 10% of the 20 marks.

Several candidates were unable to manipulate the exponential and logarithmic functions.

Answer(s): (a) $\frac{dA}{dt} = kA, \quad k < 0$ or $\frac{dA}{dt} = -kA\ , \quad k > 0$

SECTION B
(Module 2: Sequences, Series and Approximation)

Question 2

Specific Objective(s): (b) 4, 6, 8, 9

This question examined geometric series and mathematical modelling.

The general performance on this question was poor with the majority of candidates earning between 0 and 5 marks.

(a) Several of the candidates were unable to recall that the common ratio r of a convergent geometric series must satisfy the condition $|r| < 1$.

(b) (i) Some candidates were unable to express the answers to 2 significant figures.

(ii) Most candidates did not recognise that maximum output meant sum to infinity.

Answer(s): (a) $x < 6, \quad -1 < x < 1, \quad x > 6$

(b) (i) a) 1 300 000 to 2 sig. fig. b) 920 000 to 2 sig. fig.

(ii) 29 to 2 sig. fig.

SECTION C
(Module 3: Counting, Matrices and Modelling)

<u>Question 3</u>

Specific Objective(s): (a) 1, 2, 6, 7, 9, 10

The question covered arrangements of objects, probability, and solutions of system of equations. Mathematical modelling is also evident.

(a) Candidates were able to find the probability with no restrictions in (i), but had severe difficulties in obtaining the correct answer to Part (ii).

(b) Generally, this part of the question was well done with 95 per cent of the candidates obtaining the correct answers.

Answer(s): (a) (i) 10! (ii) 0.8

(b) (i)

$$2x + 2y + z = 5950$$
$$4x + y + z = 11\,450$$
$$5x + 3y + 2z = 14\,600$$

(ii)

$$\begin{pmatrix} 2 & 2 & 1 \\ 4 & 1 & 1 \\ 5 & 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5\,950 \\ 11\,450 \\ 14\,600 \end{pmatrix}$$

(iii)

$$x = 2800$$
$$y = 100$$
$$z = 150$$

PAPER 03

INTERNAL ASSESSMENT

Module Tests

The main features assessed are:

- The mapping of the items tested to the specific objectives in the syllabus;
- Coverage of content of each Module test;
- The appropriateness of the items tested for the CAPE level;
- The presentation of the sample (Module Tests and students' scripts);
- The quality of the teachers' solutions and mark schemes;
- The quality of the teachers' assessment – consistency of marking using the mark scheme.

In general, the question papers, solutions and detailed mark schemes met the CXC CAPE Internal Assessment requirements for both Unit 1 and Unit 2.

Too many of the Module tests comprised items from CAPE past examination papers. Untidy 'cut and paste' presentations with varying font size were common place. Teachers are encouraged to use the past CAPE examination papers ONLY as a guide and to include original and creative items in the Module tests.

The stipulated time for Module Tests ($1 - 1\frac{1}{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module Tests are too extensive or insufficient.

The specific objectives tested in a particular module test must be from that Module.

In most cases, the Internal Assessments that were based on collective testing of Modules were inadequate in terms of the coverage of the Unit.

Improvement is needed in the presentation of samples for moderation. (See Recommendations below).

The moderation process relies on the validity of teacher assessment. In a few instances, the students' scripts reflected evidence of rewriting of some solutions after scripts were formally assessed by the teacher. Also, there were a few cases where students' solutions were replicas of the teacher's solutions – some contained identical errors and full marks were awarded for incorrect solutions.

To enhance the quality of the design of Module tests, the validity and accuracy of teacher assessment and the validity of the moderation process, the following Internal Assessment guidelines are recommended.

Recommendations (Module Tests):

(i) Design a separate test for each Module.

(ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and the same marking scheme used. ONE sample of FIVE students will form the sample for the centre.

(iii) **In 2008, the format of the Internal Assessment remains unchanged. [Multiple Choice Examinations will not be accepted].**

Please note Recommendations for Module Tests and Presentation of Sample.

Recommendations for Module Tests and Presentation of Samples

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required at cover for each Module test.

- Name of School and Territory; Name of Teacher;
- Unit Number and Module Number;
- Date and duration ($1 - 1\frac{1}{2}$ hours) of Module Test;
- Clear Instructions to candidates;
- Total Marks allotted for Module Test;
- Sub-marks and total marks for each question must be clearly indicated.

2. COVERAGE OF SYLLABUS CONTENT

- The number of questions in each Module Test must be appropriate for the stipulated time of 1 - 1 $\frac{1}{2}$ hours;
- CAPE Past Examination papers should be used as a guide ONLY;
- Duplication of specific objectives and questions must be avoided;
- Specific objectives tested must be within the syllabus.

3. MARK SCHEME

- Detailed mark schemes must be submitted; holistic scoring is not recommended;
- FRACTIONAL MARKS MUST NOT BE AWARDED;
- The total mark for Module tests must be clearly stated on the teacher's solution sheet;
- The student's mark must be entered on the FRONT page of the student's script.

4. PRESENTATION OF SAMPLE

- Students' responses must be written on normal size paper, preferably 8 $\frac{1}{2}$ " x 11";
- Question numbers are to be clearly written in the left margin;
- The total score for each question marked on students' scripts must be clearly written in the right margin;
- ONLY original students' scripts must be sent for moderation.
  Photocopied scripts will not be accepted;
- Module Tests must be typed using a legible font size, (or if handwritten must be neat and legible)
- The following are required for EACH Module test:

  - A question paper
  - Detailed solutions with detailed mark schemes
  - The scripts of the candidates comprising the sample
    (Students' scripts in the sample are to be organized by Modules)

- Marks recorded on PMaths 1 - 3 and PMath 2 - 3 must be rounded off to the NEAREST WHOLE NUMBER;
- In cases where there are five or more registered candidates, FIVE samples must be sent.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2008

PURE MATHEMATICS
(Trinidad & Tobago)

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2008

INTRODUCTION

This is the first year that the revised syllabus for Pure Mathematics is examined. The new format of Paper 01 is multiple choice (MC) and Papers 02 and 03 have retained the format with extended-response questions. Circumstances dictated that examination papers for the Trinidad and Tobago (T&T) candidates were not the same as those for the Rest of the Region (ROR), nevertheless, the Internal Assessment (IA) of candidates from Trinidad and Tobago was included in the overall IA for the entire region and a common report has been written for that aspect of the examination process. A copy of that report is appended under the heading PAPER 03.

Generally, the performance of candidates was very satisfactory with a number of excellent to very good grades. There still remained, however, too large a number of weak candidates who seemed unprepared for the examination. Approximately 3 500 scripts were marked.

GENERAL COMMENTS

The new topics in the revised Unit I syllabus are Cubic Equations, Indices and Logarithms, and L'Hopital's rule, with Complex Numbers moved to Unit 2. Of these new topics, candidates showed reasonable competence in Cubic Equations and Logarithms, but some seemed not to have been exposed to L'Hopital's rule. Among the old topics comprising Unit 1, candidates continue to experience difficulties with Indices, Mathematical Induction and Summation Notation ($\sum$). General skills at algebraic manipulation including substitution at all levels continue to pose challenges. A new area of difficulty has emerged, the topic of Trigonometric Identities. Strong performances were recorded in Differentiation, the Plotting of Graphs, Vectors, and Coordinate Geometry. This was encouraging. Some effort should be made in providing students with practice in connecting parts of the same question in order to facilitate efficient solutions.

DETAILED COMMENTS
UNIT 1
PAPER 01

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily. The mean score was 61.0 per cent and standard deviation was 8.6.

PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (d) 3, 8, 10; (b) 4; (f) 3, 5; g (2)

This question tested properties of the roots of quadratic equations, cubic equations, the modulus function and graphs.

(a) (i) There were several attempts at this part of the question. Many candidates wrote the condition for real roots without the 'greater than' sign in $b^2 - 4ac \geq 0$ and so did not obtain full marks. Others made errors in simplifying $(-2h)^2$.

(ii) This was one of the new topics and presented some challenges, but there were many encouraging attempts. Very few candidates used the fact that $5(5 - k)(5 + k) = 105$ to obtain the values of k. More practice is recommended.

(b) (i) (iii) This part of the question was very well done. Most candidates did not use the graph to find the values of x in Part (iii), but simply read the values from the table.

Answer(s): (a) (i) $h \geq 4$ **or** $h \leq -8$

(ii) $p = 71$, $k = \pm 2$

(b) (iii) $f(x) = g(x)$ when $x = 0$ **or** $x = 4$

Question 2

Specific Objective(s): (a) 5, 6, 8; (c) 1, 2, 3, 5

This question tested knowledge about indices, logarithms and the principle of mathematical induction.

(a) Several candidates attempted this part of the question, many of whom realized that each term could be expressed in the form 3^x, $x \in N$. Due to errors in the algebraic manipulation, only about 50 per cent of those attempting the question obtained the correct answer.

(b) (i) Not many candidates knew how to derive this result although several of them knew how to use it as they demonstrated in Part (ii).

(ii) This was very well done by the many candidates who attempted it, although they found the underlying principle at (i) hard to derive.

(c) There were many good attempts at this question with several candidates obtaining at least 90 per cent of the marks. The step from $n = k$ to $n = k + 1$, which is the main task in the principle of mathematical induction, still eludes many. More practice is recommended.

Answer(s): (a) $3^4 = 81$

(b) (ii) $y = 5$

SECTION B
(Module 2: Trigonometry and Plane Geometry)

Question 3

Specific Objective(s): (a) 9, 10, 12, 13; (c) 8, 9, 10

This question tested properties of vectors, trigonometric identities and solutions of trigonometric equations.

(a) This part of the question was quite well done although Part (iii) did pose a challenge to a few candidates.

(b) Several successful attempts were made in solving this part, however, some candidates had difficulty expanding cos 2A.

(c) The manipulation of the trigonometric identity troubled some candidates in this part of the question. It was also noted that not many candidates used the 'otherwise' route in Part (iii) to solve $\sin 3\theta = \sin \theta$.
More practice of this type of question and better use of the formula sheet are recommended.

Answer (s): (a) (i) $\lambda = -2$

(ii) $\lambda = 2$

(iii) $\lambda = 4 \pm 2\sqrt{3}$

(c) (iii) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

Question 4

Specific Objective(s): (b) 5, 7, 9

The question dealt with tangents to circles and basic properties of circles in the context of coordinate geometry.

(a) (i) Most candidates were able to find the coordinates of P, A and B. Approximately 90 per cent of attempts were successful.

(ii) Several candidates found it difficult to obtain the value of λ. Those who were successful substituted the coordinates of P but made simple errors in extracting the correct value of λ. Many obtained $\lambda = \frac{10}{3}$ instead of $\frac{-10}{3}$.

For those candidates who had a value of λ to carry forward, further marks were obtained from Parts (a) to (d).

(b) (i) Many candidates did not relate the result required to the trigonometric relationship $\sin^2 t + \cos^2 t = 1$ and hence missed out on the simplicity of the process in obtaining the required equation.

(ii) This part of the question was more of a challenge for the candidates. Few obtained full marks.

Answer(s): (a) (i) $P \equiv (1, 10)$, $A \equiv (2, 3)$, $B \equiv (6, 5)$
(The coordinates of A, B may be interchanged)

(ii) a) $\lambda = -\frac{10}{3}$

b) $3x^2 + 3y^2 - 16x - 40y + 113 = 0$

c) $|PQ| = \frac{5\sqrt{5}}{3}$

d) $|PM| = 3\sqrt{5}$

SECTION C
(Module 3: Calculus I)

Question 5

Specific Objective(s): (a) 4, 7; (b) 8, 9(i), 10, 16; (c) 13, 14, 15

The question examined knowledge about limits, L'Hopital's rule for limits, differentiation of rational functions and maxima in mensuration.

(a) The topic of L'Hopital's rule for finding limits is a new topic in the revised syllabus and some candidates did not seem to be familiar with it. As a consequence, candidates were not penalized for using other methods to solve the particular problem posed.

(b) (i) This part of the question was generally well done. Several candidates obtained full marks for both a) and b).

(ii) Most candidates who attempted this part of the question obtained full marks. Several of them found $\frac{d^2y}{dx^2}$ by differentiating $\frac{dy}{dx}$ as a quotient. A small number of candidates used implicit differentiation to find $\frac{d^2y}{dx^2}$.

(c) (i) This part was very well done with the majority of candidates who attempted it gaining full marks.

(ii) Several candidates succeeded in doing this part correctly although some had difficulty in substituting h in the expression for V. A few did not find the second derivative in order to obtain the value of h for V a maximum.

Answer(s): (a) $\frac{4}{5}$

(b) (i) a) $\frac{dy}{dx} = \frac{1}{(1-4x)^2}$

(c) (ii) $h = 4$

Question 6

Specific Objective(s): (c) 1, 3, 4, 5, 6, 7, 8 (i), 9

The question tested knowledge and skill in differentiation and integration. Generally the question was well done, with approximately 70 per cent of candidates obtaining at least 16 of the 25 marks allocated.

(a) This part of the question was the most challenging for the candidates. Many seemed not to be familiar with integrating functions using 'substitution', which should be a regular procedure for problems whenever the integrand is not straightforward. Many candidates found difficulty manipulating $x\mathrm{d}x$ to change from the variable x to the variable u. More practice is recommended.

(b) Too many candidates were unable to relate the gradient of the curve with the need for integrating to obtain the equation for the curve. Several candidates treated $x^2 - 4x + 3$ as the function $f(x)$ rather than $f'(x)$ and proceeded in the wrong direction.

(c) (i) Candidates found this part of the question easy, however, some struggled with the algebra involved.

(ii) There were some excellent responses to this part of the question. Common errors included:

a) Incorrect choice of limits
b) Attempting to combine the equations of the line and curve into a single function to integrate
c) Attempting to use approximation to find the area despite the stipulation to obtain the exact value

Answer(s): (a) $\frac{1}{3}\sqrt{3x^2 + 1}$ + a constant

(b) Equation of C is $\frac{x^3}{3} - 2x^2 + 3x - 1$

(c) (i) A ≡ (1, 3), B ≡ (0, 5), C ≡ (4, 0)

(ii) Exact value of area = 13 units^2

UNIT 1
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 7; (c) 2, 3; (d) 7, 9; (f) 5(ii)

This question tested candidates' abilities in solving logarithmic equations, and their knowledge of the factor theorem, concept of a decreasing function, and the sigma notation relating to an arithmetic progression.

(a) (i) This part of the question was satisfactorily done.

(ii) This part of the question was not well done. Candidates found it difficult to express the given logarithmic equation in index form thus allowing for the solution of a simple linear equation.

(b) (i) A significant number of candidates were unable to use the intercepts of the curve to determine the constants required. Preparation for specific topics seem to be stereotype. Apparently, the use of the factor theorem is studied without any reference to the relationship of a curve and its intercepts.

(ii) Most of the candidates stated the range as seen on the graph but included the point where $x = -1$. Instances were seen where candidates attempted differentiation of f(x) to find the required range. More practice on graphs and how to use graphs to determine some features of a function should be done.

(iii) This part of the question was satisfactorily done.

Answer(s): (a) (i) p

(ii) $x = 9$

(b) (i) $h = 4, \; k = -1, \; m = -2$

(ii) $-1 < x \leq 0$

(c) 15 350

SECTION B
(Module 2: Trigonometry and Plane Geometry)

Question 2

Specific Objective(s): (b) 1, 2, 6, 7, 9.

This question tested candidates' abilities to determine a Cartesian curve from given parametric equations, finding a tangent and a normal to a Cartesian curve in linear form and in terms of its parameter, intersection of a line and a curve, and the distance between two points on a curve.

The majority of candidates performed poorly with approximately 10 per cent of them giving no responses.

Symbolic representation and application of the given data as required was a big challenge to most candidates. The algebraic skills demonstrated were very weak.

Such candidates require more preparation and practice to perform satisfactorily at these examinations.

Answer(s): (b) (ii) $y + t_1x - at_1^3 - 2at_1 = 0$

(iv) $2a(1 + t_1^2)$

SECTION C
(Module 2: Calculus1)

Question 3

Specific Objective(s): (a) 4, 5; (c) 2, 4, 5, 6; (b) 11.

This question tested the concept of limits and of definite integration, as well as the use of a simple model involving rate of change.

(a) (i) This part of the question was well done since candidates were given a useful hint which simplified the rational function.

(ii) Most candidates were able to follow through with the result from (i) to perform well on this part of the question.

(b) (i) This part of the question was satisfactorily done by most of the candidates.

(ii) Some candidates had difficulty separating the integral and using the result given for $\int_{1}^{4} f(x)\,dx = 7$. In addition candidates could not use the fact that for a continuous function $\int_{1}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx = \int_{1}^{4} f(x)\,dx$. Candidates therefore could not obtain the correct answer. A number of candidates attempted integration of the problem in the form given with obvious difficulties.

(c) (i) The majority of candidates merely found $\frac{dV}{dh}$, apparently not aware that finding $\frac{dV}{dt}$ required multiplication of $\frac{dV}{dh}$ by $\frac{dh}{dt}$.

(ii) The candidates were required to find $\frac{dh}{dt}$ but many of them failed to do so since their result at (i) was incorrect. There were no correct responses to this part of the question.

Answer(s): (a) (i) $\frac{1}{6}$

(ii) $\frac{1}{48}$

(b) (i) $u = 2$

(ii) 4

(c) (i) $\frac{dV}{dt} = \frac{1}{3}\pi\left(48h - 3h^2\right) \times \frac{dh}{dt}$

(ii) $\frac{25}{7\pi}$ cms^{-1}

$= \pi h(16 - h) \times \frac{dh}{dt}$

GENERAL COMMENTS

UNIT 2

Topics satisfactorily covered were those relating to solution of Exponential Equations, Calculus of Composite Functions, (including Inverse Trigonometric Functions), First-order Differential Equations, Solution of Second-order Differential Equations, Series, Mathematical Induction, Permutations and Simple Probability, Approximations to Roots of Equations, Series, Complex Numbers (including De Moivre's theorem), and Matrix Algebra.

This examination tested the new topics which included Calculus of Inverse Trigonometrical Functions and the Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series, Reduction to Row-Echelon Form, and Row Reduction of an Augmented Matrix, Complex Numbers with application of Demoivre's Theorem for integral n.

The majority of candidates continue to display weaknesses in tasks requiring algebraic manipulation or involving substitution. It is imperative that more emphases be placed on these areas of weaknesses. Extensive practice in the use of substitution and algebraic manipulation is demanded if candidates are to be well prepared to show improved performances in these areas. Candidates continue to demonstrate a lack of appreciation for questions which allow for "hence or otherwise". They fail to see existing links from previous parts of the questions and never seem disposed to using "otherwise" thus employing any other suitable method for solving the particular problem.

UNIT 2
PAPER 01

Paper 01 comprised 45 multiple-choice items. The candidates performed satisfactorily. The mean score on this paper was 68.4 per cent and the standard deviation was 8.6.

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 7, 9; (b) 1, 2, 3, 5, 6, 7.

This question tested differentiation of various functions, exponential, logarithmic, and inverse trigonometrical, as well as parametric equations, and distinguishing a point of inflexion.

This question was generally well done by the majority of candidates. The average marks obtained were within the range 17 – 20 from a maximum of 25 marks.

(a) Most of the candidates gained full marks for this part of the question. A small number of candidates found it difficult to solve the quadratic equation obtained in terms of e^x. Some candidates attempted to take the natural log of each term with the obvious difficulties experienced.

(b) This part of the question was well done. All the candidates used second differentiation to prove the result. No candidate attempted to use implicit differentiation.

(c) (i) Generally, responses to this part of the question were good. Some candidates had difficulties with the Multiple Composite Functions. An application of the product rule over three terms is not a regular feature and more practice would be needed in this regard. $\frac{d}{dx}\sin^{-1}(2x)$ was not well done. Most candidates used the result of $\frac{d}{dx}\sin^{-1}(x)$ without paying attention to the composite $2x$. With the testing of this new topic it was not unexpected that lack of adequate practice would be evident.

(ii) a) This part of the question was well done. A few candidates attempted to set t in terms of x and y before differentiation. Clearly, they had an idea but failed to develop it successfully.

b) Very few candidates obtained full marks for this part of the question. The majority of candidates set $\frac{dy}{dx} = 0$ to find the point of inflexion. Those candidates who attempted to find $\frac{d^2y}{dx^2}$ merely found $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ and not multiplying by $\frac{dt}{dx}$. In fact distinguishing a point of inflexion appeared to be new to most of the candidates. A lot of practice is required in this regard.

Answer(s): (a) $x = \ln 7, x = 0$

(c) (i) $\ln x\left(\frac{2x}{\sqrt{1-4x^2}} + \sin^{-1} 2x\right) + \sin^{-1} 2x$

Question 2

Specific Objective(s): (c) 1, 3, 6, 8, 11, 12 (ii)

This question required candidates to evaluate an indefinite integral using integration by parts, solving a first-order differential equation using an integrating factor, the general
solution of a second-order differential equation with the principal integral being a trigonometric function, resolving partial fractions, and integration involving $\int \frac{f'(x)}{f(x)} dx$.

The majority of candidates attempted this question. The average range of marks obtained was 15 – 20 with a satisfactory number of candidates earning marks in the range 21 – 25 out of a maximum of 25 marks.

(a) (i) Approximately 30 per cent of the candidates who attempted this question obtained full marks. The substitution $u = \ln x$ was widely used. The practice of not stating the constant of integration continues to be a source of concern. Emphasis must be placed on this aspect for candidates to appreciate the importance of this constant and to earn full marks.

(ii) Some candidates had problems finding the correct integrating factor. They also failed to write the equation in the form $I\frac{dy}{dx} + Iy = I \ln x$. However, the majority of candidates seemed to have grasped the concept of using an integrating factor.

(b) (i) Most candidates successfully found the first and second derivatives of $m \cos x + n \sin x$. However, too many of these candidates made simple errors in calculating the values of m and n.

(ii) Candidates had no difficulties finding the complementary function correctly. Due to errors made in (i) some marks were lost overall.

(c) (i) The majority of candidates performed well in this part of the question.

(ii) Generally most of the candidates who were successful in Part (i) were able to integrate correctly. A very small number of candidates mistakenly found . . . $\int \frac{3x}{(x^2+1)} dx$ as arctan x. Candidates are guilty of omitting the constant of integration.

Answer(s): (a) (i) $\frac{1}{2}\{\ln (x)\}^2 + C$

(ii) $xy = \frac{1}{2}\{\ln (x)\}^2 + C$

(b) (i) $m = 2, \; n = 1$

(ii) $y = Ae^{3x} + Be^{x} + 2\cos x + \sin x$

(c) (i) $\dfrac{2}{x-1} - \dfrac{3x}{x^2+1}$

(ii) $2\ln\left|x-1\right| - \dfrac{3}{2}\ln\left|x^2+1\right| + C$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objectives: (b) 1, 3, 6, (e) 1, 2, 4

This question tested the candidates' abilities with respect to arithmetic progressions, the principle of mathematical induction, the intermediate value theorem for the existence of a real root, and the Newton-Raphson method.

The majority of candidates attempted this question. The range of marks obtained for this question was between 10 and 20 out of a maximum of 25 marks.

(a) (i) a) Most candidates used the approach of evaluating the sums for S and T by using formulae stated in the Formulae Booklet. However, they did not understand the concept tested and failed to gain marks for this part of the question.

b) Candidates, having failed to answer (a) correctly, proceeded to find the sum of the arithmetic series, S, using the formula stated in the Formulae Booklet. This type of candidate needs adequate practice to be proficient with algebraic manipulation and the deductions made from these manipulations.

(ii) Most candidates demonstrated a sound understanding of the principle of mathematical induction. However, some candidates are still unclear of the inductive process and failed at the step where the assumption that P_k is true is used to show that P_{k+1} is true for n = some k. More work on the principle of mathematical induction is required. Some candidates simply substituted $k + 1$ for k in the statement for P_k. Candidates also failed to express $(k + 2)(2k + 3)$ in the form $[(k + 1) + 1]\,[2(k + 1) + 1]$ to show that the statement P_{k+1} is of the form P_k for $n = k + 1$. It is clear, however, that candidates are becoming more adept at applying the principle of mathematical induction.

(iii) This part of the question required the use of the substitution of the formulae for $\sum_{r=1}^{n} r^2$ *and* $\sum_{r=1}^{n} r$, and simplifying to get the answer. A number of candidates mistakenly applied the principle of mathematical induction to prove the result. Those candidates who used substitution of the formulae obtained full marks.

(b) (i) Most candidates found f (0) and f (1), concluding that since there was a sign change that condition was sufficient for the existence of a real root in the interval (0, 1). Candidates were not aware that the function must be continuous in that interval to use the intermediate value theorem. Many candidates failed to obtain full marks due to this omission.

(ii) The majority of candidates were not able to use the concept of differentiation to show that the function was continuously increasing, hence the existence of only one real root. Some candidates very logically showed by way of two graphs that $f(x) = x^3$ and $g(x) = 3 - 6x - 3x^2$ had only one point of intersection. A few candidates used the roots of a cubic equation to show that the cubic equation had one real root and two complex roots.

(iii) More than 60 per cent of the candidates were able to obtain maximum marks for this part of the question. Very few candidates showed weaknesses in applying the Newton-Raphson method.

Answer(s): (b) (iii) 0.41

<u>Question 4</u>

Specific Objective(s): (a) 1, 2; (c) 1, 3; (e) 3, 4

This question tested the use of sequences defined by recurrence relations, simple algebraic proofs, and the binomial theorem. The average range of marks for this question was between 10 and 15 from a maximum of 25 marks. Generally the responses to this question were unsatisfactory.

(a) (i) Because of arithmetical errors, a few candidates did not earn full marks for this part of the question.

(ii) A number of candidates showed weaknesses in basic algebraic simplification and correct numerical answers.

(iii) a) b) Most candidates were unable to answer this part of the question logically. Simple algebraic proofs continue to be problematic and much more is required in this regard.

(b) The majority of candidates did well on this topic. Generally, most of them used an inspection method to determine the term independent of x. In fact, few candidates used the binomial expansion to determine the term required. Emphasis must be placed on the binomial theorem.

(c) A significant number of candidates found this part of the question difficult. Many of them simply used the calculator to find the difference. The use of the binomial expansion series for approximations seemed unfamiliar to most of the candidates. Practice in this regard is necessary.

Answer(s): (a) (i) $\frac{20}{11}, \frac{31}{16}$

(ii) $\frac{2(a_n-2)}{4+a_n}$

(b) $\frac{15!}{6!\,9!}(6^6)$

(c) 10.28620

SECTION C
(Module 3: Counting, Matrices and Complex Numbers)

Question 5

Specific Objective(s): (a) 1, 2, 7, 8, 9; (b) 2; (c) 1, 2, 4, 5, 11

This question tested selections using permutations, classical probability, complex numbers including De Moivre's theorem, and matrix algebra. The overall performance by candidates was satisfactory. The average range of marks was between 10 and 20 from a maximum 25 marks.

(a) (i) a) Most candidates responded well to this part of the question. A few candidates found distinguishing between combinations and permutations rather challenging.

b) The majority of candidates gave satisfactory responses to this part of the question.

(ii) This part of the question was well done.

(b) Candidates demonstrated a good understanding of probability theory, including the use of Venn diagrams and laws of probability.

(c) (i) A majority of candidates substituted 3 + 4i into the equation but had difficulties comparing coefficients since many of them made arithmetic errors in the expansions. Very few candidates used the principles of complex conjugate and the sum and product of roots of a quadratic equation. Not many candidates were able to obtain full marks.

(ii) The majority of candidates demonstrated an understanding of De Moivre's theorem. However, many of them made errors in the expansion, particularly the terms involving i^2. A number of candidates were unaware that they had to consider the real part of the expansion for $\cos 3\theta$. Very rarely did candidates define the complex number $\cos \theta + i \sin \theta$ as z, $\cos \theta - i \sin \theta$ as $\frac{1}{z}$ and used the principle of $\left(z + \frac{1}{z}\right) = 2\cos\theta$. More practice in this topic will improve candidates' understanding and performance.

Answer(s): (a) (i) a) $6^4 = 1296$

b) 360

(ii) $\frac{1}{3}$

(b) $\frac{13}{28}$

(c) (i) $h = -6, k = 25$

Question 6

Specific Objective(s): (b) 1, 2, 7, 8.

This question tested matrix algebra including solution of a variable for a singular matrix, multiplication of conformable matrices, finding the inverse of a non-singular matrix, and solution of a system of equations using matrix algebra. Approximately 20 per cent of the candidates gained full marks on this question. The average range of marks for this question was between 10 and 25 from a maximum of 25 marks.

(a) The majority of candidates performed well in this part of the question. Some candidates made errors in the cubic expansion for x and subsequently found it difficult to factorize the cubic equation correctly.

(b) (i) This part of the question was well done.

(ii) a) This part of the question was well done. Very few candidates made some arithmetic errors in multiplication.

b) A number of candidates failed to deduce the inverse of A from the result at a). Many of them went on to calculate A^{-1} as $\frac{1}{|A|}$ adj A. This showed a weakness in understanding the concepts involved. Much practice in this topic will improve performance.

(iii) This part of the question was well done by candidates who deduced or otherwise found A^{-1} correctly. However, some candidates failed to obtain full marks because they included the number of coaches in their answers.

Answer(s): (a) $x = 1, 2, -3$

(b) (i) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 34 \\ 49 \\ 71 \end{pmatrix}$

(ii) a) $AB = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ b) $\frac{1}{2}B = \begin{pmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{pmatrix}$

(iii) 24

UNIT 2
SECTION A
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 5, 6, 7, 8; (c) 5

This question examined an exponential and a logarithmic expression, integration by parts of a trigonometric function, and an exponential model. Overall this question was poorly done.

(a) Most of the candidates who attempted this part of the question did not demonstrate an understanding of exponential functions and of natural logarithms. This part of the question was poorly done.

(b) Candidates could not separate $\cos^3 x$ as $\cos x\,(1 - \sin^2 x)$ in order to substitute $\cos x\,dx$ for du. Many of the candidates only found $du = \cos x\,dx$. No candidate got marks beyond this point.

(c) (i) Many candidates failed to use the fact of $t = 0$ to find the answer to this part of the question.

(ii) Candidates were required to find the value of the constant k before proceeding to find the answer to this part of the question. However, substitution and subsequent solution proved beyond the ability of most of the candidates.

Answer(s): (a) $y = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

(b) $\sin x - \frac{1}{3}\sin^{3} x + C$

(d) (i) 70° C

(ii) 7.5 minutes

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

Specific Objective(s): (b) 3, 11, 12

This question tested the sum of a convergent series using the method of differences, and a model involving a geometric progression. The overall response was poor. Candidates seemed generally unprepared.

(a) (i) Candidates found it difficult to express the general term of the series. They appeared unfamiliar with patterns and sequences.

(ii) Follow through from (i) was not possible since the majority of candidates did not get the correct partial fractions to work with.

(iii) Most of the candidates did not respond to this part of the question. The few who did could not determine the nature of the series since answers to (i) and (ii) were either non-existent or wrong.

(b) (i) (ii) Overall they were very few and very poor responses to this part of the question. Analysis of the problem proved to be challenging for the candidates.

Answer(s): (a) (i) $\frac{1}{(2r-1)(2r+1)}$

(ii) $\frac{1}{2} - \frac{1}{2(2n+1)}$

(iii) $\frac{1}{2}$

(b) (i) $\$\left(100 + \frac{1}{10}r^{2}\right)$ (ii) $6 205

SECTION C
(Module 3: Counting, Matrices and Complex Numbers)

Question 3

Specific Objective(s): (b) 7, 8

This question tested complex numbers and matrix algebra. Candidates showed a fair understanding of these topics. However, some candidates had difficulty finding the inverse of the invertible matrix.

(a) (i) This part of the question was well done by the majority of candidates.

(ii) Very few candidates made arithmetical errors in this part of the question. Generally, this part of the question was well done.

(b) (i) A significant number of candidates did not demonstrate a clear understanding of the process required to find the inverse of an invertible matrix. No candidate attempted the row reduction of an augmented matrix of the identity matrix.

(ii) There were no correct responses to this part of the question.

Answer(s): (a) (i) $\frac{-13}{2} - \frac{9}{2}\mathrm{i}$

(ii) $5\sqrt{\left(\frac{5}{2}\right)}$

(b) (i) $\frac{1}{5}\begin{pmatrix} 8 & 7 & -6 \\ -2 & 2 & -1 \\ -5 & -5 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

PAPER 03 – INTERNAL ASSESSMENT

The Internal Assessment comprises three Module tests.

The main features assessed are the:

- Mapping of the items on the Module tests to the specific objectives in the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module tests and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised items from CAPE past examination papers.

2. Untidy 'cut and paste' presentations with varying font size were common place.

3. This year there was a general improvement in the creativity of the items, especially with regards to mathematical modelling. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.

4. The stipulated time for Module tests ($1\text{-}1^1/_2$hours) must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **MUST** be included.

5. Multiple-choice Questions will **NOT** be accepted in the Module tests.

6. Cases were noted where teachers were unfamiliar with recent syllabus changes, for example, complex numbers, 3-dimensional vectors, dividing a line segment internally or externally.

7. The moderation process relies on the validity of teacher assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some containing identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students' scripts did not correspond to the marks on the Moderation sheet.

8. Teachers **MUST** present evidence of having marked each individual question on the students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students' scripts.

9. To enhance the quality of the design of the Module tests, the validity of teacher assessment and the validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

(i) Design a separate test for each Module. The Module test MUST focus on objectives from that module.

(ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and a common marking scheme used. One sample of FIVE students will form the sample for the centre.

(iii) In 2009, the format of the Internal Assessment remains unchanged.

[Multiple Choice Examinations will NOT be accepted].

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of School and Territory, Name of Teacher, Centre Number.
- Unit Number and Module Number.
- Date and duration (1-1 $^1/_2$ hours) of Module Test.
- Clear instruction to candidates.
- Total marks allotted for the Module Test.
- Sub – marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of 1-1 $^1/_2$ hours.
- **CAPE past examination papers should be used as a guide ONLY**.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes MUST be submitted, holistic scoring is not recommended that is, **one mark per skill should be allocated.**
- FRACTIONAL DECIMAL MARKS MUST NOT BE AWARDED.
- The total marks for Module tests MUST be clearly stated on the teacher's solution sheets.
- A student's marks MUST be entered on the FRONT page of the student's script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The marks MUST be presented in the right hand side of the page.
- Diagrams MUST be neatly drawn with geometrical/mathematical instruments.

4. PRESENTATION OF THE SAMPLE

- Student's responses MUST be written on normal sized paper, preferably 8 $^1/_2$ ×11.
- Question numbers are to be written clearly in the left margin.
- The total marks for each question on students' scripts MUST be clearly written in the right margin.
- ONLY original students' scripts MUST be sent for moderation. **Photocopied scripts will not be accepted.**
- Typed Module tests MUST be in a legible font size (for example, size 12). Hand written tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:
  - A question paper.
  - Detailed solutions with detailed mark schemes.
  - The scripts (for each Module) of the candidates comprising the sample. The scripts MUST be collated by Modules.

- Marks recorded on the PMath 1 – 3 and PMath 2 – 3 forms must be rounded off to the nearest whole number.
- The guidelines at the bottom of these form should be observed. (see page 57 of the syllabus, no.6).
- In cases where there are five or more candidates, FIVE samples MUST be sent.
- In cases where there are five or less registered candidates, ALL samples MUST be sent.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2008

PURE MATHEMATICS
(Rest of the Region)

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2008

INTRODUCTION

This is the first year that the revised syllabus for Pure Mathematics is examined. The new format of Paper 01 is multiple choice (MC) and Papers 02 and 03 have retained the format with extended-response questions.

Generally, the performance of candidates was very satisfactory with a small number of excellent to very good grades. There still remained, however, too large a number of weak candidates who seemed unprepared for the examination. Approximately 3700 scripts were marked.

GENERAL COMMENTS

UNIT 1

The new topics in the revised Unit 1 syllabus are Cubic Equations, Indices and Logarithms, and L'Hopital's rule, with Complex Numbers moved to Unit 2. Of these new topics, candidates showed reasonable competence in Cubic Equations and Logarithms, but some seemed not to have been exposed to L'Hopital's rule. Among the old topics comprising Unit 1, candidates continue to experience difficulties with Indices, Mathematical Induction and Summation Notation ($\sum$). General skills at algebraic manipulation including substitution at all levels continue to pose challenges. A new area of difficulty has emerged, the topic of Trigonometric Identities. Strong performances were recorded in Differentiation, the Plotting of Graphs, Vectors, and Coordinate Geometry. This was encouraging. Some effort should be made in providing students with practice in connecting parts of the same question in order to facilitate efficient solutions.

DETAILED COMMENTS

UNIT 1
PAPER 01

Paper 01 comprised 45 multiple-choice items. The candidates performed satisfactorily. The mean score was 58 per cent with a standard deviation of 7.7.

UNIT 1
PAPER 02

<u>Question 1</u>

Specific Objectives(s): (a) 1, 2, 3, 5, 7, 8; (b) 1, 3, 4, 6; (f) 5 (ii)

The question tested properties of cubic equations, surds and the process of summation.

Overall, the question was attempted by over 90 per cent of the candidates. Various approaches were used in each part with a high degree of success.

(a) Few mistakes were made in this part; however, incorrect expansions of $(x + 1)(x - 1)(x - 3)$ were mainly responsible for those that occurred. Incorrect equating of coefficients was also a main source of error.

(b) This part of the question was generally well done, however, some candidates had some difficulties in rationalizing the surds. Many gained full marks in (ii) by using the result in (i).
Errors occurred in (ii) by incorrectly evaluating $\sqrt{12}$.

(c) (i) Many candidates used the principle of mathematical induction to obtain the result. Some candidates, in using this method, had difficulty manipulating the algebra involved. Others, who observed that results on the formula sheet were appropriate, had an easier passage to the final result.

(ii) Candidates were not as successful in this part as in (i). The most frequent error occurred in the separation of the summation

$$\sum_{r=31}^{50} r(r+1) = \sum_{r=1}^{50} r(r+1) - \sum_{r=1}^{30} r(r+1).$$

A common error was subtracting

$$\sum_{r=1}^{31} r(r+1) \text{ instead of } \sum_{r=1}^{30} r(r+1).$$

Answer(s): (a) $p = -1,\ q = -1,\ r = 3$

(c) (ii)

$$\sum_{r=31}^{50} r(r+1) = 34\,280$$

Question 2

Specific Objective(s): (f) 1, 5 (i); (c) 1, 3 (i), (ii).

This question tested quadratic equations, the sums and products of the roots of such equations, the solutions of quadratic equations and logarithms.

(a) This part of the question dealt with quadratic equations. Approximately 99 per cent of the candidates attempted this question and several obtained at least 10 of the 12 marks allocated to this part.

(b) There was mixed success with this part of this question which examined logarithms in (ii) and (iii). Many candidates did (i) successfully although there were some who did not obtain the correct value for x from $x^{1/3} = -1$. In Part (ii), some candidates did not discard the negative value of x while in (iii) some candidates unwisely used calculators although the question stated that calculators should not have been used.

Answer(s): (a) (i) $\alpha + \beta = -2,\ \alpha\beta = 5/2$

(ii) a) $\alpha^2 + \beta^2 = -1$

b) $\alpha^3 + \beta^3 = 7$

(iii) $8x^2 - 56x + 125 = 0$

(b) (i) $x = 64,\ -1$

(ii) $x = 2$

(iii) -1

UNIT 1
PAPER 02
SECTION B
(Module 2: Trigonometry and Plane Geometry)

Question 3

Specific Objective(s): (a) 4, 5, 9, 12; (b) 1.

This question tested the candidates' ability to use the gradients of line segments and to develop and apply trigonometric identities.

This question was not popular with most of the candidates. Most candidates attempting this question scored in the range of (0 - 4) marks, which was very disappointing.

(a) The majority of the candidates did not recognize the relationship between the gradient of the straight line and the tangent of the angle between the line and the positive direction of the *x*-axis. These candidates did not link the word "tangent" with "tan", finding instead points and lines (in many variations). The few who recognized "tangent of an angle", frequently found "$\tan\alpha - \tan\beta$" instead of "$\tan(\alpha - \beta)$" where $\alpha > \beta$. Some of those who started well seemed not to be aware that $\tan(\alpha \pm \beta) \neq \tan\alpha \pm \tan\beta$, which spoiled the work thereafter.

(b) The majority of candidates attempted (i) and were capable of finding the correct identities to replace $\sin 2\theta, \cos 2\theta$ *and* $\tan\theta$, but some had difficulty manipulating the identity to get to $\tan\theta$ in terms of $\sin 2\theta$ *and* $\cos 2\theta$.

In (ii) a significant number of candidates did not realize that "Express $\tan\theta$ in terms of $\sin 2\theta$ *and* $\cos 2\theta$" meant change the subject (or transpose) the formula given in (i). The word "hence" was not understood by the candidates to use previous work and thus in most cases this was not done.

Several **errors** were made in (iii) such as: $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ $\qquad \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\tan\theta = \frac{\sin 2\theta}{\cos 2\theta}$ $\qquad \cos\theta = 1 - \sin\theta$

(c) A large number of candidates did not use the fact that the angles of a triangle add up to 180^0. Instead, it was stated that A + B = C or in some cases, particular values of angles were used. Therefore, they did not recognize that the sine of an angle is equal to the cosine of its complement. In (c) (i) b), those candidates who actually attempted the question chose the correct factor formula but were unable to follow through for the second mark. Very few of the candidates recognised the link between (c) (i) b) and (ii). They did not recognise that A could be used as a double angle so they failed to use $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$.

Answer(s): (a) (i) $\tan\alpha = 3;\ \tan\beta = \frac{3}{4}$

(ii) $\tan(\alpha - \beta) = \frac{9}{13}$

(b) (ii) $\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$

Question 4

Specific Objective(s): (b) 1, 2, 3, 5, 7, 8.

This question examined the application of coordinate geometry to the properties of a circle, straight lines, tangents, normal, and intersections between straight lines and curves. For this question, Part (b) proved more challenging to the candidates than Part (a).

Part 4 (a) (i) was fairly well done. Seventy per cent of the candidates attempted the question and were able to get most of the eight marks allocated to it. The remaining 30 per cent of the candidates had difficulties with

(a) finding the midpoint of a line
(b) calculating the gradient of a line using the formula
(c) knowing that the perpendicular bisector passes through the midpoint
(d) knowing that the gradient of the perpendicular bisector is the negative reciprocal of the gradient of the line.

(ii) Of the few candidates that actually attempted this question, most were unable to recognize that the centre of the circle was the point of the intersection of the perpendicular bisector of any two chords of the circle. The alternative solution of substituting the points into the equation of the circle was also challenging for some candidates. Candidates substituted incorrect values and could not solve simultaneously the equations generated.

The main areas of concern in the students' approach were as follows:

(a) Some candidates could not write coordinates properly as (x, y).

(b) Some candidates were using the formula for finding the length of a line to calculate the midpoint of the line.

(c) When calculating the equation of the perpendicular line, several candidates erroneously submitted either the point P (-2, 0) or Q (8, 8), even when the midpoint was correctly found.

Most of the candidates solved Part 4 (b) (i) correctly. However, many of them did not recognise the significance of the repeated roots in relation to the tangent of the circle.

Some candidates also correctly used alternative solutions such as

(a) finding the perpendicular distance from the centre of the circle by means of the formula $\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$ and showing that this is equal to the radius of the circle

(b) showing that the gradient of the line from the centre of the circle to the tangent is the negative reciprocal of the gradient of the tangent and therefore the line and tangent are perpendicular to each other.

Part 4 (b) (ii) was very well done. In fact, most candidates were able to recover from (b)(i) as above.

Answer(s): (a) (i) $4y + 5x = 31$

(ii) $x = -1,\ y = 9$

(b) (ii) $x = 0,\ y = 1$

SECTION C
(Module 3: Calculus 1)

Question 5

Specific Objective(s): (a) 5, (b) 5, 6, 11-19

The question tested knowledge of limits, differentiation and integration. Curve sketching and the nature of turning points of a curve were also investigated.

(a) This part of the question dealt with the limit of a rational function in which both numerator and denominator are polynomials.

Candidates showed knowledge of the methods involved in finding the required limit but fell down on the mechanics of factorizing the polynomials correctly. The main difficulty occurred in the factorization of $x^3 - 27$. Many candidates stated that $x^3 - 27 = (x-3)(x^2-9)$.

(b) Very few candidates performed well on this part of the question. The main area of weakness was in differentiating the term $\frac{u}{t}$. As a consequence, many candidates had difficulty in deriving the appropriate equations required for the correct solutions.

(c) There were mixed performances on this part of the question. In finding the equation for y, many candidates omitted the constant of integration when integrating $\frac{dy}{dx}$, and this led to an incorrect equation for the curve C and an incorrect sketch. In spite of this, Part (ii) was reasonably well done.

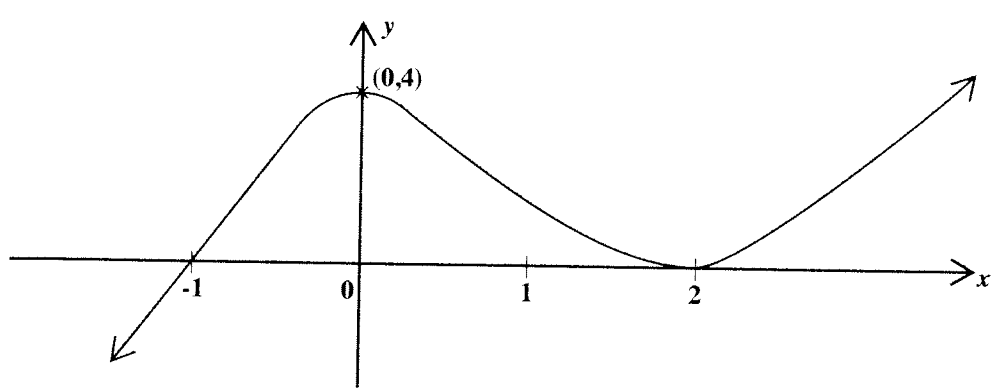
Answer(s): (a) $\frac{27}{7}$

(b) $u = \frac{4}{5}$, $v = \frac{-9}{5}$

(c) (i) $y = x^3 - 3x^2 + 4$

(ii) Stationary points are (0,4) and (2,0).
(0,4) is a maximum and (2,0) is a minimum.

(iii)



Question 6

Specific Objective(s): (b) 5, 6, 8, 9, 10 (c) 4, 6, 8 (i)

The question tested aspects of differentiation, integration and some applications to mensuration. The question was poorly done.

(a) This part of the question covers basic differentiation. In both parts, many candidates found difficulty in applying the chain rule to the process of differentiation. In too many cases, the function in (i) was replaced with $(2x^2 - x)^{1/2}$ or $x\left(2x^{1/2} - 1^{1/2}\right)$. In Part (ii), $\sin^2(x^3 + 4)$ was not interpreted correctly as a function requiring the chain rule for differentiation and terms were lost in the process.

(b) (i) Generally, candidates did not appear to recognize the need to apply the linearity property of integrals.

(ii) For those candidates who recognized the need to integrate the given function between limits 1 and 3, a few were unable to follow through to obtain the solution of k = 4. Many were unable to manipulate the fractions after substitution of the limits. Several candidates simply substituted the limits into the given function without integrating to find the required area.

(c) (i) Many candidates mistakenly used the volume of the sphere rather than that of the hemisphere.

(ii) Several candidates neglected to include the base area when finding the total surface area.

(iii) Of those candidates who answered this part of the question, many did not verify that A was indeed a minimum when r = 3.

Answer(s): (a) (i) $\frac{3x-1}{\sqrt{(2x-1)}}$

(ii) $6x^2 \sin(x^3 + 4)\cos(x^3 + 4)$

(b) (i) 3

(ii) $k = 4$

(c) (ii) r = 3

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (c) 1, 2, 3, 4; (d) 7; (f) 5 (i)

The question tested properties of quadratic equations and their roots, functions, indices and logarithms.

(a) There were several good attempts at this part of the question but few candidates obtained complete solutions.

(b) (i) This part was poorly done. Many candidates did not observe that a critical approach to the solution was to start with $f(0) = 6$. Others did not discern that substituting $x = 3$ in the given equation would provide a lead to obtaining $f(9)$.

(ii) This part was fairly well done.

(c) Both Parts (i) and (ii) were well done.

Answers: (a) (i) Roots are -3, -9

(ii) k = 27

(b) (i) $f(3) = 15 \text{ and } f(9) = 33$

(ii) $x = 6$

(c) (i) 30

(ii) 78

UNIT 1
PAPER 3 B
SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

Specific Objective(s): (a) 5, 13; (c) 7, 8, 9, 10.

This question tested candidates' ability to solve trigonometric equations for a given range; to apply the properties of vectors; to find the scalar (dot) product and the angle between the given vectors as well as the magnitude and direction of a vector.

(a) Candidates did not display sufficient knowledge of the trigonometric identities. The solutions presented were incomplete. Overall, this part of the question was poorly done.

(b) In most cases, candidates recognized the need to use the dot product, but some of them did not know that **a . b** = **0** for perpendicularity.

(c) While a number of candidates knew the formula to find the acute angle between the two vectors, few knew how to manipulate it correctly.

(d) Candidates were able to evaluate correctly the magnitude of F in most of the cases. Few correctly attempted to evaluate the angle of inclination requested in Part (b).

Answer(s): 2 (a) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

(b) (i) $t = \frac{12}{5}$

(ii) $\theta = 22.3^0$

(iii) a) $|F| = 3.60$

b) $\emptyset = 56.3^0$

SECTION C
(Module 3: Calculus I)

Question 3

Specific Objective(s): (b) 5, 6, 7(i), 16, 21; (c) 1, 2, 3, 4, 5(i), (ii), 9

3. The question tested knowledge of the differential and integral calculus, the equation of a normal to a curve and rate of change in calculus.

   (a) This part of the question was reasonably well done although a small number of the candidates experienced difficulties in differentiating the term $\frac{1}{x}$ in $y = x + \frac{1}{x}$.

   (b) Some candidates had problems with expressing the integrand as a sum of three separate terms. Very few candidates obtained full marks for this part.

   (c) Most candidates obtained a differential equation in V without the negative sign. Nevertheless a good understanding of the concept involved was exhibited in the solutions.

Answer(s): (a) (ii) $3y + 4x = 31$

(b) $-\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} + a\ constant$

(c) (i) $\frac{dV}{dt} = -30t$

(ii) Liquid lost is $75\ cm^3$.

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 may be regarded as satisfactory. A small number of candidates reached an outstanding level of proficiency. A number of candidates were inadequately prepared for the examinations.

Topics in Calculus, Simple Probability, Approximations to Roots of Equations, and Series were satisfactorily covered. The examination tested new topics which included Calculus of Inverse Trigonometrical Functions and the Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series for Rational and Negative Indices, Complex Numbers, De Moivre's Theorem for integral *n*, and the Locus of a Complex Number.

Weaknesses in algebraic manipulation and tasks involving substitution were manifestly evident. Candidates obviously found it difficult to solve problems which required these applications. It is imperative that more emphases be placed on these areas of weakness. Extensive practice in the use of substitution and algebraic manipulation is necessary if candidates are to be well-prepared to show improved performances in these topics. Candidates continue to demonstrate a lack of appreciation for questions which allow for "hence or otherwise". They fail to see existing links from previous parts of a question, and rarely seem disposed to using "otherwise," thus employing any other suitable method for solving the particular problem.

PAPER 01

Paper 01 comprised 45 multiple-choice items. The candidates performed fairly well with a mean score of 64 per cent and standard deviation of 8.6.

DETAILED COMMENTS

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 6, (b) 2, 5 (c) 1, 5, 6

This question tested the differentiation of exponential, trigonometric and logarithmic functions, resolution into partial fractions, and integration involving inverse trigonometric functions.

All candidates attempted this question with varying degrees of success.

(a) (i) Most of the candidates omitted the constant π in the derivative. Answers included in part, $e^{4x} \sin \pi x$ ...
A majority of the candidates treated e^{4x} as a constant rather than as a function of x. Consequently, some candidates did not obtain $4e^{4x}$ as the derivative of e^{4x}. Emphases must be placed on recognizing functions of a stated variable as against constants.

(ii) The majority of candidates attempting this question opted to use the chain rule $\frac{d}{du} \ln(u) \times \frac{d}{dx}\left[\frac{x^2+1}{\sqrt{x}}\right]$. However, they failed to apply the quotient rule correctly for $\frac{d}{dx}\left[\frac{x^2+1}{\sqrt{x}}\right]$, and were unable to secure full marks for this part of the question.

(b) Many candidates expressed y as $\frac{1}{3^x}$ and attempted to use the quotient rule which, for some, ran into difficulties. The majority of candidates used $\log_{10}$ or $\log_3$ and attempted to differentiate with respect to x. Some candidates **erroneously** expressed $y = \ln_3 x$ and attempted to differentiate with respect to x. A small number of candidates **erroneously** stated $\ln y = \ln 3^{-x}$ giving $-x = \frac{\ln y}{\ln 3}$.

It is clear that this method of differentiation was new to many candidates. Logarithmic and implicit differentiation were not part of many of the candidates' skills.

(c) (i) The majority of candidates demonstrated competence in this topic. Some errors in simple arithmetic were common. A few candidates had problems simplifying the terms of the fractions, thus making it difficult to answer Part (ii) successfully.

(ii) The majority of candidates were successful in obtaining the correct partial fractions from Part (i). However, the evaluation of

$$\cdots \int \frac{-5}{2(x^2+1)}\,dx$$

proved challenging for almost all the candidates. It seems that insufficient tutorials and practice in inverse trigonometrical calculus contributed to candidates' inability to complete the integration to this part of the question.

Candidates continue to omit the constant of integration from indefinite integrals. This results in loss of marks, and impacts negatively on their overall performances. The concept of the constant of integration must be fully explained so that candidates can be aware of the importance of including it in their resulting integrals.

Answer(s): (a) (i) $e^{4x}(4\cos\pi x - \pi\sin\pi x)$

(ii) $\frac{2x}{x^2+1} - \frac{1}{2x} = \frac{3x^2-1}{2x(x^2+1)}$

(c) (i) $\frac{3}{2(x-1)} + \frac{x-5}{2(x^2+1)}$

(ii) $\frac{3}{2}\ln|x-1| + \frac{1}{4}\ln|x^2+1| - \frac{5}{2}\arctan x + \text{const}$

Question 2

Specific Objective(s): (c) 5, 7, 8, 11.

This question required candidates to use an integrating factor, integration of exponential functions, and integration by parts for a definite integral.

The majority of candidates attempted this question. However, many of them were unable to secure maximum marks on all parts of the question for various reasons.

(a) The majority of candidates recognized the use of an integrating factor. However, many of them failed to evaluate this factor correctly. In addition, having found the integrating factor, many of them did not multiply the equation by the integrating factor. As a result, these candidates were not able to solve the differential equation completely.

(b) This question was satisfactorily done by most of the candidates who earned the maximum mark.

(c) The responses to this question revealed some weaknesses in identifying which of the terms to take as v and which as $\frac{du}{dx}$. This resulted in many candidates having to evaluate $\int \ln x\, dx$, since x^2 was taken as v. Those candidates who integrated correctly had some difficulty evaluating the integral using the stated limits. In general, candidates demonstrated a satisfactory understanding of integration by parts.

(d) (i) Many candidates continue to show weakness in working with given substitutions. Many of them failed to find $-dv = du$. They, in fact, substituted

$$v = 1 - u \text{ to get } \int \frac{1}{\sqrt{v}} du$$

Common **errors** included $\int -v^{\frac{-1}{2}}$, omitting dv, not stating the constant of integration, and not replacing $v^{\frac{1}{2}}$ with $\sqrt{1-u}$ in the final answer. **The use of substitution for integration must be extensively practised in order for candidates to show improved performances in this topic.**

(ii) The majority of candidates were able to replace $\cos x\, dx$ with du. However, the substitution $u = \sin x$ was not correctly used to transform the original integral into a manageable form using the given substitution. Candidates were unable to express $\sqrt{(1+\sin x)}$ in terms of u. As a result they were unable to obtain the cancellation of the term $\sqrt{(1+u)}$ to obtain $\frac{1}{\sqrt{1-u}}$ in order to make use of the answer at (i). Candidates were unable to obtain maximum marks for this part of the question.

Answer(s): (a) $y = \frac{1}{3}e^{2x} + \frac{c}{e^x}$

(b) $4y = e^{4x} + 3$

(c) $\frac{1}{9}(2e^3 + 1)$

(d) (i) $I = -2\sqrt{(1-u)} + C$

(ii) 2

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objectives: (a) 2, 5, 12, (b) 5, 7, 9, 10, (c) 13.

This question tested the candidates' abilities to use the recurrence relation of a sequence, apply the principle of mathematical induction, geometric progression, and the application of Maclaurin's expansion series.

(a) (i) This part of the question was well done by the majority of candidates. Simple substitution was required.

(ii) Few candidates demonstrated a sound understanding of the principle of mathematical induction. Moreover, the majority of candidates are still unclear about the inductive process and failed to show proper proof that the statement P_k and P_k+1 were true for n = some k. More work on the principle of mathematical induction is required. Some candidates simply substituted $k+1$ for k in the statement for P_k.

(b) The majority of candidates were able to obtain the equations in terms of a and r for the given conditions. However, weaknesses in algebra continue to be evident and many students failed to solve for a and r correctly. Few candidates gained full marks for this part of the question.

(c) (i), (ii), (iii) This part of the question was poorly done. Insufficient exposure and practice in using Maclaurin's theorem for expansions were evident. The Maclaurin's theorem is given in the Formulae Booklet issued to candidates at examinations and should have made it easier for them to answer this part of the question. Candidates also omitted the range of values of x for which the expansions are valid. This is one of the additional topics tested for the first time and it is evident that much more needs to be done by way of tutorials and practice.

Answer(s): (a) (i) 3, 4, 6, 9

(b) $a = 27,\ r = \frac{2}{3}$

(c) (i) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots \qquad -1 < x \le 1$

(ii) (a) $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \ldots \qquad -1 \le x < 1$

Question 4

Specific Objective(s): (b) 13, (c) 3, 4, (e) 1, 2, 4.

This question tested the concept of existence of roots for a continuous function, the Newton-Raphson method of approximation, the binomial expansion series for positive and negative rational indices, Maclaurin's theorem, and use of expansion series for calculating the fractional value of a surd.

(a) (i) The majority of candidates knew the principle of the Intermediate Value Theorem. However, very few of them stated that the function was a polynomial and more importantly that it was continuous.

(ii) This part of the question was well done. Some students misinterpreted the rubric and proceeded to find a numerical value for the root α.

(b) (i) All of the candidates opted to use the binomial expansion series for this part of the question. Arithmetical errors in calculating the coefficients of terms resulted in candidates losing marks. Very few students stated the range of values of x for which the expansion is valid. Emphasis should be placed on this aspect.

(ii) No candidate was able to deduce this expansion from (i) and proceeded to use the binomial expansion. As in (i), arithmetical errors, particularly signs of the coefficients, resulted in loss of marks. Candidates omitted the range of values of x.

(iii) Candidates who were able to complete (i) and (ii) correctly gained full marks on this part of the question.

(iv) Most candidates had difficulties using the given substitution and in reducing the value of $\sqrt{2}$ to the required answer. Surds continue to be challenging for many candidates. They have difficulties working numerical problems without the use of calculators or tables.

Answer(s): (b) (i) $1-\frac{x}{2}+\frac{3x^2}{8}-\frac{5x^3}{16}+\ \ldots, \quad -1<x<1$

(ii) $1-\frac{x}{2}+\frac{x^2}{8}-\frac{x^3}{16}+\ \ldots, \quad -1<x<1$

SECTION C

(Module 3: Counting, Matrices and Complex Numbers)

<u>Question 5</u>

Specific Objective(s): (a) 2, 3, 4, 6, (b) 1, 2, 7, 8.

This question tested the principle of combinations, matrix algebra, and complex numbers.

(a) (i) Responses to this part of the question were very satisfactory. Some candidates had difficulties distinguishing between combinations and permutations.

(ii) The majority of candidates gained full marks for this part of the question, having obtained (i) correctly.

(b) (i) (a) Generally, this part of the question was well done. Errors were mostly due to incorrect arithmetic.

(b) Candidates demonstrated good techniques for multiplying conformable matrices.

(ii) A significant number of candidates were not able to deduce A^{-1} from the previous results. Many of them attempted to find the inverse of A by the process $\frac{1}{|A|}$ adj A. Arithmetic errors did not allow some of them to obtain the correct answer. Some candidates also attempted to use row reduction of the augmented matrix $\left(\begin{array}{ccc|ccc} 3 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{array}\right)$ but had difficulties completing the process. A number of candidates were able to deduce A^{-1} form (i) b).

(iii) Most candidates demonstrated weaknesses in matrix algebra for this part of the question. Instances were seen where candidates merely stated $X = \frac{A-B}{A}$.

Answer(s): (a) (i) 70

(ii) 65

(b) (i) a) $\begin{pmatrix} 2 & 2 & -1 \\ 1 & 0 & 2 \\ 1 & -4 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(ii) $A^{-1} = \frac{1}{3}M = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & -1 \\ \frac{-1}{3} & 1 & \frac{-1}{3} \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & \frac{-2}{3} & 0 \\ -1 & 4 & -1 \\ 0 & \frac{2}{3} & 2 \end{pmatrix}$

Question 6

Specific Objective(s): (c) 4, 7, 8, 9, 10.

This question tested rationalization of a complex number, determining the value of a multiple of a complex number, use of a conjugate, and determining the loci of a complex number.

(a) (i) This part of the question was well done.

(ii) Arithmetical errors resulted in a few candidates stating an incorrect value for λ.

(iii) Candidates used more than one method to answer this part of the question. With (i) and (ii) some candidates used the binomial expansion. Some candidates used De Moivre's theorem. Generally this part of the question was well done.

(b) (i) Apart from some difficulties with algebra, candidates performed well in this part of the question.

(ii) Approximately 95 per cent of the candidates did not attempt this question. Candidates failed to see the link with (i) and were unable to determine the technique to be used. Seemingly, the area relating to loci of complex numbers was not extensively dealt with. Candidates should be exposed to much more of this, with adequate practice.

Answer(s): (a) (i) $\frac{1}{2}(1 - i)$

(ii) $\frac{1}{2}$

(iii) $\frac{-1}{4}$

(b) (ii) $C\left(\frac{1}{6}, 0\right)$

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (b) 2, 3, 5.

This question examined parametric differentiation and its application to a normal to a curve, and a modelling based on exponentials, differential equations, rate of change, and a graph of the model.

(a) (i) Most of the candidates who attempted this part of the question were unable to determine $\frac{dy}{dt}$ and $\frac{dx}{dt}$, hence could not evaluate $\frac{dy}{dx}$. In addition the value for x at $y = 18$ was not found.

(ii) Consequently, the correct equation of the normal was not determined. A majority of candidates did not apply the gradient of the normal as $\frac{-dx}{dy}$.

(b) (i) A majority of the candidates did not attempt this part of the question. The few who did demonstrated a lack of understanding of differentiation of natural logarithms.

(ii) a) This part of the question was poorly done since there was no follow-through from (i) to work with.

b) Candidates who attempted this part of the question substituted $180 = \frac{dV}{dt}$ instead of $V = 180$.

(iii) Only a few candidates attempted this part of the question. No candidate obtained full marks. Errors included no labels on axes and failing to use the fact that $t \geq 0$.

Overall this question was poorly done.

Answer(s): (a) (i) $\frac{1}{3}$

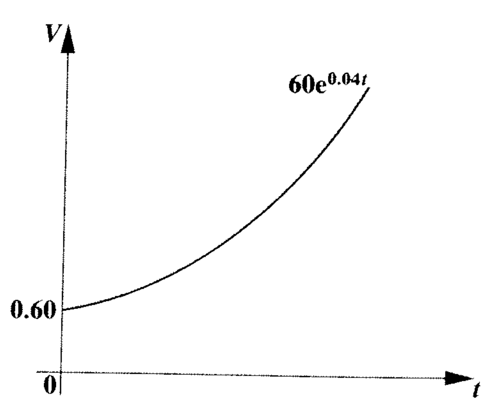
(ii) $3x + y - 99 = 0$

(b) (i) $2.4e^{0.04t}$

(ii) (a) 3.58

(b) 7.2

(iii)



Question 2

Specific Objective(s): (a) 2, 3, (b) 5, 8, 9, (c) 3, 4.

This question tested the arithmetic and the geometric progressions, the binomial expansion series for a negative index, and the Maclaurin's theorem for expansion of a trigonometric function.

(a) (i) a) Generally, this part of the question was poorly done. Candidates could not determine the geometric progression.

b) There was no follow-through after being unable to obtain a).

(ii) There was no follow-through after being unable to obtain the previous results.

(b) (i) (ii) Responses to these parts of the question were poor.

(c) (i) Some candidates attempted this part of the question but obtained the wrong answer due to arithmetical errors.

(ii) Candidates could not use the expansion given to express $\sec x$ as $\frac{1}{\cos x}$ algebraically. No candidate attempted differentiation to obtain the coefficients for the expansion of $\sec x$.

Answer(s): (a) (i) a) $5 \times 2^{r-1}$

b) $5(2^n - 1)$

(ii) $n = 8$

(b) (i) $S = (1 + 3 + 5 + 7 + \ldots) + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\right)$

(ii) $n^2 + 1 - \left(\frac{1}{2}\right)^n$

(c) (i) $1 + y + y^2 + y^3 + y^4 + \ldots$

(d) $1 + \frac{x^2}{2} + \frac{5x^4}{24} + \ldots$

Question 3

Specific Objective(s): (a) 8, 9, 11, 12, 13; (b) 1, 2, 5, 6.

This question tested matrix algebra, and classical probability.

(a) (i) Most candidates were able to state the augmented matrix. However, beyond this point there were serious challenges. They were unable to perform the necessary row reduction.

(ii) No candidate answered this part of the question.

(b) (i) Most of the candidates re-stated the probabilities given. No evidence of the use of the Venn diagram or the formula for solving the required probability was seen.

(ii) a) b) Responses to these parts of the questions were poor. Candidates seemed not to know the definitions of independent and mutually exclusive events.

Answer(s): (a) (i) -1

(ii) $x = 5 - 23z,\ y = 6z - 1$

(b) (i) 0.05

(ii) a) $P(A \cup B) \neq P(A) + P(B)$; not independent

b) $P(A \cup B) \neq 0$; not mutually exclusive

PAPER 03 – INTERNAL ASSESSMENT

The Internal Assessment comprises these Module tests.

The main features assessed are the:

- Mapping of the items tested to the specific objectives in the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module tests and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised items from CAPE past examination papers.

2. Untidy 'cut and paste' presentations with varying font sizes were common place.

3. This year there was a general improvement in the creativity of the items, especially with regards to mathematical modelling. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.

4. The stipulated time for Module tests (1-$1\,{}^{1}/_{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **MUST** be included.

5. Multiple-choice Questions will **NOT** be accepted in the Module tests.

6. Cases were noted where teachers were unfamiliar with recent syllabus changes, for example, complex numbers, 3-dimensional vectors, dividing a line segment internally or externally.

7. The moderation process relies on the validity of teacher assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students' scripts did not correspond to the marks on the Moderation sheet.

8. Teachers **MUST** present evidence of having marked each individual question on the students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students' scripts.

9. To enhance the quality of the design of the Module tests, the validity of teacher assessment and the validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

(i) Design a separate test for each Module. The Module test MUST focus on objectives from that module.

(ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and a common marking scheme used. One sample of FIVE students will form the sample for the centre.

(iii) In 2009, the format of the Internal Assessment remains unchanged.
[Multiple Choice Examinations will NOT be accepted.]

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of School and Territory, Name of Teacher, Centre Number.
- Unit Number and Module Number.
- Date and duration (1-1$^1/_2$ hours) of Module Test.
- Clear instruction to candidates.
- Total marks allotted for the Module Test.
- Sub – marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of 1-1$^1/_2$hours.
- **CAPE past examination papers should be used as a guide ONLY**.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes MUST be submitted, holistic scoring is not recommended, that is, **one mark per skill should be allocated.**
- FRACTIONAL DECIMAL MARKS MUST NOT BE AWARDED.
- The total marks for Module tests MUST be clearly stated on the teacher's solution sheets.
- A student's marks MUST be entered on the FRONT page of the student's script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The marks MUST be presented in the right hand side of the page.
- Diagrams MUST be neatly drawn with geometrical/mathematical instruments.

4. PRESENTATION OF THE SAMPLE

- Student's responses MUST be written on normal sized paper, preferably 8 $^1/_2$ ×11.
- Question numbers are to be written clearly in the left margin.
- The total marks for each question on students' scripts MUST be clearly written in the right margin.
- ONLY original students' scripts MUST be sent for moderation. **Photocopied scripts will not be accepted.**

- Typed Module tests MUST be in a legible font size (for example, size 12). Hand written texts MUST be NEAT and LEGIBLE.
- The following are required for each Module test:

  - A question paper.
  - Detailed solutions with detailed mark schemes.
  - The scripts (for each Module) of the candidates comprising the sample. The scripts MUST be collated by Modules.

- Marks recorded on the PMath 1 – 3 and PMath 2 – 3 forms must be rounded off to the nearest whole number.
- The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no.6).
- In cases where there are five or more candidates, FIVE samples MUST be sent.
- In cases where there are less than five registered candidates, ALL samples MUST be sent.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2009

PURE MATHEMATICS

PURE MATHEMATICS

MAY/JUNE 2009

GENERAL COMMENTS

This is the second year that the current syllabus has been examined in the new format of Paper 01 as Multiple Choice (MC) and Papers 02 and 03 in the typical essay-type questions. The syllabus is arranged into two Units, each consisting of three Modules:

Unit 1
- Module 1 - Basic Algebra and Functions
- Module 2 - Trigonometry and Plane Geometry
- Module 3 - Calculus I

Unit 2
- Module 1 - Calculus II
- Module 2 - Sequences, Series and Approximations
- Module 3 - Counting, Matrices and Complex Numbers

There were 5579 candidates who wrote the examinations for Unit 1 in 2009 compared to 4 995 in 2008 and for Unit 2, 2 701 compared to 2 690 in 2008. Performances varied across the entire spectrum of candidates with a significant number obtaining excellent grades. Nevertheless, there continues to be a number of candidates who seem unprepared to write the examinations, particularly for Unit 1. A more effective screening process needs to be instituted to reduce the number of poorly prepared candidates.

DETAILED COMMENTS

UNIT 1

The overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as Trigonometric Identities, Coordinate Geometry, Basic Differential and Integral Calculus and Surds. However, many candidates continue to find Indices, Limits, Continuity/Discontinuity and Algebraic Manipulation challenging. These topics and techniques should be given special attention if improvement in performance is to be achieved. Other areas that need consolidation are general algebraic manipulation of simple terms, expressions and equations, substitution, either as a substantive topic in the syllabus or as a tool for problem solving.

Paper 01 comprised 45 multiple-choice items, with 15 items based on each Module. The candidates performed satisfactorily with a mean score of 21 out of a possible 45. Paper 02 comprised six compulsory questions, two testing each Module. The mean mark on this paper was 51 out of a possible 150.

UNIT 1

PAPER 02

SECTION A

Module 1: Basic Algebra and Functions

Question 1

Specific Objective(s): (a)5; (b)5; (c)1, 3(iii), 5;(g).

This question tested knowledge of surds factors for expressions of the form a^n - b^n, simple skills in equalities and logarithms. Many candidates had difficulty with the algebraic manipulation of $x^4 - y^4$ and the change of base in Part (c).

(a) There were several good and complete answers to this part of the question. The mistakes most frequently encountered related to $\sqrt{4 \text{ x } 7} = 4\sqrt{7}$ and/or $\sqrt{7^2 \text{ x } 7} = 7^2\sqrt{7}$.

(b)(i) The observation that $x^4 - y^4 = (x^2 + y^2) = (x^2 + y^2)(x^2 - y^2)$ presented most difficulty for many candidates who tried the method factorization. Many of those who used long division succeeded in gaining full marks for this part of the question.

(ii) The substitution of $x = y + 1$ was poorly done by many of the candidates.
(iii) There were not many good attempts at this part.

The change of base concepts presented enormous difficulties for candidates. More practice in this area is recommended.

Answer(s):

(a) $\mathbf{k = 9}$,

(b)(i) $\mathbf{x^3 + x^2y + xy^2 + y^3}$,

(c) $\mathbf{x = \frac{1}{16}}$

Question 2

Specific Objective(s): (d) 1, 2, 7; (f) 3, 5(i).

This question examined properties of the roots of quadratic equations of functions and evaluation of a given function defined on intervals of the real numbers.

Part (a) of this question was very well done. However, in Part (b), candidates had difficulty applying the basic definition of a function, while in Part (c) the main weakness arose in recognizing the piece-wise nature of the function *f*. More practice in these topics is recommended.

Answer(s):

(a) $5x^2 + 8x + 8 = 0$

(b)(i) f=$\{(u, 1), (v, 2, (v, 3), (x, 1), (y, 3), (z, 4)\}$

(ii) a) $v \in A$ **has two images in B and** $w \in A$ **has no image in B.**

b) ***For g:*** $A \to B$, **remove from f:** $A \to B$ **one of the ordered pairs (v,2) or (v, 3) and map** $w \in A$ **to some** $b \in B$.
eg. $g = \{(u, 1), (v, 2), (x, 1), (w, 1)(y, 3)(z, 4)\}$

c) No. of functions $g = 4 \times 2 = 8$

(c)(i) $f(f(20)) = f(5) = \frac{5}{4}$,

(ii) $f(f(8)) = f(2) = -1$,

(iii) $f(f(3)) = f(0) = -3$,

SECTION B

Module 2: Trigonometry and Plane Geometry

Question 3

Specific Objective(s): (b) 1, 2, 3, 5, 7, 8.

This question examined in Part (a), the application of coordinate geometry to the properties of a circle, straight lines, tangents, normal and intersections between straight lines and curves. Additionally, Part (b) of the question examined the concept of finding the angle between two given vectors, position vectors and displacement vectors, as well as finding the area of the triangle.

The majority of the candidates attempted this question, and while a few of them would have attained full marks, a number of candidates had difficulties working out the coordinate geometry, especially the vector questions.

(a) Finding the radius and coordinates of the centre were easily answered. However, many candidates unnecessarily expanded the equation of the circle to find the coordinates of the centre. A number of candidates did not recognize that the gradient of the radius is actually the gradient of the normal at the point and hence did not get the equation of the tangent correct. Many students recognized they had to solve simultaneous equations for Part (iii) but used the equation of the tangent from Part (ii) instead of the equation of the circle, since they did not read the definition of *C* carefully.

(b) This part was attempted by the majority of the candidates who successfully used various methods to calculate the size of the angle between the vectors **p** and **q**. A number of students used $A = \frac{1}{2}bh$ to find the area of the triangle without checking to see if it was a right-angled triangle. However, some candidates used the correct formula $A = \frac{1}{2}\, pq \sin\theta$ but some used **p.q** rather than $|p||q|$. The majority of candidates found the vector PQ, but had difficulty finding the midpoint of **PQ** since they took OM as ½ PQ instead of OM = OP + PM or $\frac{1}{2}$ (OP + OQ). Candidates did not recognize that OR was equal and parallel to PQ. Even those candidates who did well in the majority of the question fell down at this point.

It was evident that aspects of the syllabus needed to be reinforced. More emphasis must be placed on the equation $(x - a)^2 + (y + b)^2 = r^2$, where (a, B) represents the centre of the circle. The use of diagrams in the teaching and answering of exercises on coordinate geometry and vectors should be encouraged in order to strengthen the responses in this area.

Answer(s):

(a) **(i)** **5units; (3, 4)**

(ii) $y = -\frac{3}{4}x + \frac{25}{2}$

(iii) **(-1,1); (3,9)**

(b) **(i) a)** **30°;**

b) **13 square units**

(ii) a) **i + 7j** **b) 4i + 2j**

Question 4

Specific Objectives(s): (a) 4, 5, 9, 12; (b) 1.

This question tested the candidates' ability to use and apply trigonometric functions, identities and equations.

A significant number of students attempted this question. A number of the candidates who attempted Part (a) attempted Part (ii) only. Part (b) and (c) proved to be quite popular with the candidates, with a significant number of candidates scoring the majority of marks in Part (b).

In Part (a) (i), there were few candidates who drew lines parallel to AD and CD respectively, to create the two right-angled triangles. Those who did were then able to use these two triangles to prove the result. Some candidates attempted methods such as sine and cosine rules without success. Most of the candidates who attempted Part (ii) of this question were able to obtain the correct values for r and $\propto$.

Some of the errors observed included:

- $r = \sqrt{(4 + 9)}$
- $r^2 = \sqrt{(4^2 + 9^2)} \Rightarrow r = 13$
- $\tan \propto = \frac{4}{9}$
- Maximum value is $\theta = \propto$ rather than the x-value

Part (b) was successfully completed by a significant number of candidates.

Some candidates, however, obtained incorrect solutions mainly due to

- Obtaining incorrect values for cos A and sin B
- Improper use of the relevant identities
- Incorrect substitution

Most candidates attempted Part (c) of the question. Many were able to successfully complete the first two steps of the proof, that is the expansion of tan (A + B), as well as recognizing $\tan \frac{\pi}{4} = 1$.

Many of the candidates failed to realize that some form of rationalization (use of (a + b) (a - b) $=a^2 - b^2$) had to be invoked to successfully complete the process.

Some candidates were successful using the t-approach. It was also observed that those candidates who were successful were adept at manipulating trigonometric identities.

Answer(s):

(a) (ii) $\sqrt{97}$

(b) (i) $\frac{63}{65}$, **(ii)** $\frac{56}{65}$, **(iii)** $\frac{7}{25}$

SECTION C

Module 3: Calculus 1

Question 5

Specific Objectives(s): (a) 1, 3, 4, 5, 8, 10; (b) 4; (c) 3, 4, 5 (ii), 6

This question covered topics on limits, continuity, differentiation from first principles and integration. The question was attempted by most of the candidates. The general performance was below average with only a limited number of candidates scoring more than 20 marks.

(a) In this part, several errors were made in factorizing $x^3 - 8$ which suggests that more practice is required on exercises of this sort.

(b) The graph of the function was done correctly by many candidates. A few recognized that there was a 'break' somewhere in the graph but did not know where it should be placed. Many candidates substituted -1 into $f(x) = 1 + x$ to find $\lim_{x \to 1^-} f(x)$. Very few candidates seemed to know the definition of 'continuity' and as a consequence did not find $f(1)$.

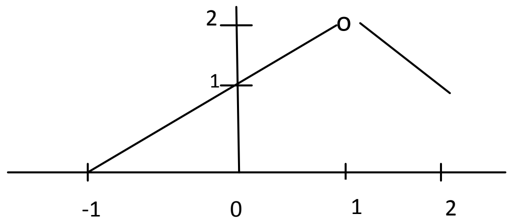
(c) Many candidates did not seem to know what 'differentiation from first principles' meant and some who knew were not able to complete the process successfully.

(d) Many candidates did not include kx and a constant of integration after integrating. Others attempted to find k before integrating.

Answer(s):

(a) -6

(b)(i)



(ii) a) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3 - x) = 3 - 1 = 2$

b) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1 = x) = 1 + 1 = 2$

(iii) $f(1) = 3 - 1 = 2 \Rightarrow f(x)$ **is continuous at** $x = 1$

(c) $\frac{dy}{dx} = -\frac{2}{x^3}$

(d) $f(x) = x^3 + 3x^2 - x - 6$

<u>Question 6</u>

Specific Objective(s): (b) 7(ii), 10, 14, 15; (c) 4, 5, 6 (i)

This question tested areas of the differential and integral calculus related to the definite integral and maximum/minimum problems.

(a) This part of the question required finding the first and second derivatives of a trigonometric function and the formation of a differential equation from such derivatives.

The question was very popular with an excellent success rate.

(b) Many candidates obtained parts of the integrals correctly but were unable to complete the question successfully because of errors in the algebraic manipulation of the terms.

(c) A few candidates found difficulty in obtaining the correct expression for *V* in (i). Others lost their way in solving $\frac{dv}{dx} = 0$ and using $\frac{d^2v}{dx^2}$ correctly.

Despite the weaknesses identified above there were several candidates who obtained full marks for this question.

Answer(s):

(b) $a = 4$

(c) (ii) $x = 2$

UNIT 1

PAPER 03/B - ALTERNATE TO INTERNAL ASSESSMENT

SECTION A

Module 1: Basic Algebra and Functions

Question 1

Specific Objective(s): (c) 1, 2, 3(ii), (iii), 5; (f) 3; (g) 1, 4

This question tested inequalities, the modulus of real numbers, algebraic expressions involving substitution, logarithms and mathematical modeling.

(a) Although a number of the candidates attempted this part of the question, many of them had difficulty manipulating the modulus sign and this weakness translated into the formation of inappropriate inequalities. There were, however, a few good answers to the problem.

(b) Many candidates did not use the substitution to its full advantage. Some others equated the expression in *y* to 3 000 and not to 3; nevertheless, there were some encouraging attempts presented by a few candidates.

(c) Some candidates found difficulty in establishing Part (i), while other candidates did not see the relevance of Part (i) to Part (ii). Outside of these instances, there were a few candidates who completed this part of the question successfully.

Answer(s):

(a) $\{\boldsymbol{x} \in \boldsymbol{R}: \mathbf{-2} < x < 0\}$

(b) $x = 1, 3, 2 \pm \sqrt{5}$

(c)(ii) **10**

SECTION B

Module 2: Trigonometry and Plane Geometry

Question 2

Specific Objective(s): (a) 6, 14; (b) 6, 7, 9

This question tested tangents to circles and properties of the locus of a point with coordinates described in parametric form.

(a) Many candidates showed the correct methodology in solving the problem but seemed unprepared to cope with the general point (p, q) on the circle. As a consequence, there were several unfinished solutions to this part of the question.

(b) Several candidates did not appeal to the basic properties of *sin x* and *cos x,* namely, $0 \leq |\sin x| \leq 1,\ 0 \leq |\cos x| \leq 1$ and $sin^2 x + cos^2 x = 1$ to solve the problems posed in this part of the question, and hence missed the simple approach to the solutions. Attempts at using calculus were made.

Answer(s):

(a)(iii) $p = -5,\ q = 1$ or $p = -2\frac{3}{5},\ q = \frac{1}{5}$

(b)(i) max $x = 5$, min $x = -1$
max $y = 8$, min $y = 0$

(ii) $\left(\frac{x-2}{3}\right)^2 + \left(\frac{y-4}{4}\right)^2 = 1$

SECTION C

Module 3: Calculus 1

Question 3

Specific Objective(s): (b) 7 to 10, 13 to 16, (c) 1 to 4

This question covered indefinite integrals and point of inflexion of, and normal to curves, as well as the notion of mathematical modeling.

(a) This part was not well done. The main hindrance to obtaining the correct solution stemmed from the candidates' failure to resolve the integrand into separate terms before attempting to integrate.

(b) The concept of a 'point of inflexion' seemed unfamiliar to many candidates. This resulted in several candidates not being able to find the values of *b* and *c* in (i), without which it was impossible to solve Part (ii) explicitly.

(c) Not many candidates attempted this part of the question, which depended on the notion of small increments, which is knowledge applied to the standard approach to the introduction of differentiation from first principles in calculus.

Answer(s):

(a) $\frac{t^2}{2} - \frac{1}{t^3} + \frac{1}{4t^4} + \textit{constant of integration}$

(b)(i) b = 3, c = 3
(ii) 3y = x +16

(c)(i) when r = 3, $\frac{dV}{dt} = 0.72\pi$
(ii) p=2

DETAILED COMMENTS

UNIT 2

In general, the performance of candidates on Unit 2 was very satisfactory. Although an increased number of candidates reached an outstanding level of proficiency, some candidates were inadequately prepared for the examinations.

The examination tested some of the newer topics in the revised syllabus and included Calculus of Inverse Trigonometrical Functions and Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series for Rational and Negative Indices, Complex Numbers and the Locus of a Complex Number.

Weaknesses in algebraic manipulation and tasks involving substitution were again evident and candidates found it difficult to solve problems which required these skills. It is imperative that more emphasis be placed on these areas of weakness. Extensive practice in the use of substitution and algebraic manipulation is necessary if candidates are to be well-prepared to show improved performances in these areas.

Paper 01 comprised 45 multiple choice items. The candidates performed fairly well with a mean score of 25 out of a possible 45. Paper 02 comprised six compulsory questions, two testing each Module. The mean mark on this paper was 54 out of a possible 150.

UNIT 2

PAPER 02

SECTION A

Module 1: Calculus II

<u>Question 1</u>

Specific Objective(s): (b) 2, 3, 4, 5, 6, 7

This question examined concepts in differentiation as they apply to trigonometric functions, inverse trigonometric functions, implicit functions and rational functions. Second derivatives emerged in the process leading to the formation of differential equations.

(a)(i) This part of the question was well done although many candidates did not use the identity $sin^2 3x + cos^2 3x = 1$ to simplify the given expression for y. As a consequence, many answers were not given in the simplest form. No penalty was applied for non-simplification.

(ii) Many candidates found difficulty in differentiating $cos\ x^2$. Several interpreted $cosx^2$ as $(cos\ x)^2$ or $(cos\ x)x$.

(iii) This part of the question was not well done. Too many candidates did not know how to cope with the implicit nature of the expression for y.

(b)(i) This part of the question was generally well done. However, amongst the candidates who did not perform satisfactorily, many equated $cos^{-1}x$ with $\frac{1}{cosx}$.

(ii) Candidates found this exercise manageable and readily recognized the relevance of the chain rule to the results. Mistakes were made in a), in differentiating $\sqrt{1-t}$ while in b) the second derivative $\frac{d^2y}{dx^2}$ proved to be a major challenge for many. A common mistake made in this case was $\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \times \frac{d^2t}{dx^2}$.

Answer(s):

(a) (i) $\frac{dy}{dx} = 10\ sin5x\ cos5x$ **(in its simplest form)**

(ii) $\frac{dy}{dx} = -\frac{x\ sinx^2}{\sqrt{cosx^2}}$

(iii) $\frac{dy}{dx} = x^x(1 + lnx)$

(b)(ii) $\frac{d^2y}{dx^2} = -\frac{\sqrt{1-t}}{4}$

Question 2

Specific Objective(s) (c) 7, 8, 13

This question examined knowledge of the trapezium rule and integration by parts in the internal calculus. Inverse trigonometric functions and approximations were also involved.

(a) This part of the question asked for the sketch of the particular function $\sqrt{1-x^2}$ on the interval $0 \leq x \leq 1$. Some candidates were unable to determine the correct quadrant or the circular property of the function.

(b) Several candidates could not find the width of the strips, but nevertheless, showed competent knowledge of the trapezium rule.

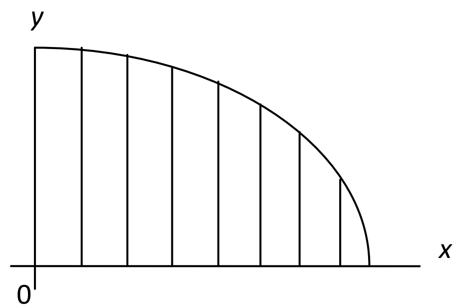
(c) (i) The majority of candidates were able to obtain the first 5 of the 9 marks allocated to this integration but could not proceed to completion of this part.

(ii) Several candidates did not see the link between (c)(i) above and failed to obtain the result for *I*.

(iii) Many answers for this part were stated in terms of degrees and not radians as expected.

(iv) The majority of candidates were unable to combine (c)(i) to (iii) to obtain the approximation to π. However, there were a few good answers to this question.

Answer(s):
(a)



The width of each strip = 0.2

(c)(iii) $\int_0^1 \sqrt{1-x^2}\; dx = \frac{\pi}{4}$

(iv) $\pi \cong 0.759 \times 4 = 3.036$

SECTION B

Module 2: Sequences, Series and Approximations

Question 3

Specific Objective(s): (a) 1, 2; (b) 1, 7, 11, 12

This question tested candidates' abilities to use the recurrence relation of a sequence to obtain values of the common ratio of a convergent geometric series, the method of differences for the summation of series and the sum to infinity.

(a) The majority of candidates answered Part (i) correctly but had serious difficulties in obtaining t_n in Part (ii).

(b) Few candidates coped well with this part of the question and among those attempting the question, some had severe challenges resolving the inequality produced.

(c) Part (i) was easily obtained by many of the candidates several of whom fell down as they proceeded through to Part (ii) in order to find S_n. Many resorted to partial fractions in order to cope with Part (ii)

It is recommended that extended practice in questions of this nature be undertaken to consolidate the fundamental concepts portrayed in this question.

Answer(s):

(a) (i) $t_2 = 16, t_3 = 21, t_4 = 26$
(ii) $t_n = 5n + 6$

(b) $-\frac{1}{3} < x < 7$

(c)(i) $f(r) - f(r+1) = \frac{1}{(r+1)(r+2)}$ **(ii)** $S_n = 2 - \frac{4}{n+2}$ **(iii)** $S_\infty = 2$

Question 4

Specific Objective(s): (b) 13; (c) 1, 3

The topics examined in this question were the binomial theorem, Maclaurin's theorem and power series expansions.

Overall, the candidates' performances in this question were below the expected level. Only approximately one-third of the candidates obtained more than 13 marks out of a possible 25. Areas of good performances involved Parts (a) (ii) and (b) (ii).

(a)(i) Candidates experienced difficulty in using the general binomial coefficient ^nC_r in problems of this kind.

(ii) Some good performances were registered in this section. Some of the candidates who answered poorly ignored the fact that terms in the separate expansions of $(1+ 2x)^5$ and $(1 + px)^4$ should have been multiplied instead of added to obtain the correct results.

(b)(i) The expansion of ln $(1 + x)$ appeared to be unfamiliar to many candidates. Without this basic expansion, ln $(1 + 2x)$ became much harder to obtain.

(ii) Several errors were made in deriving the various derivatives of *sin2x.*

(iii) Many candidates did not link this part to the earlier results obtained in the question and hence lost direction in trying to proceed. More practice is recommended.

Answer(s):

(a) (i) $n = 4,$

(ii) $p = -3 \text{ or } \frac{-11}{3}$

(b)(i) $\ln(1 + 2x) = 2x - 2x^2 + \frac{8}{3x^3} - 4x^4 + \ldots..$

(ii) $\sin 2x = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 - \ldots.$

(iii) $\ln(1+\sin 2x) = 2x - 2x^2 + \frac{4}{3}x^3\ldots..$

SECTION C

Module 3: Counting, Matrices and Complex Numbers

Question 5

Specific Objective(s): (a) 1, 2, 3, 7, 10; (c)1, 3, 4, 5

This question tested simple counting techniques, elements of probability and properties of complex numbers.

(a) There were several attempts at this part of the question with a high degree of success. The majority of candidates who attempted the question obtained full marks.

(b) A large number of the candidates who did this part of the question obtained full credit for their efforts. Some candidates, however, had difficulty in writing down the correct combinations. Many candidates tried to capitalize on the result in (b)(i) above but experienced challenges.

(c) Most candidates who attempted this part of the question were able to substitute and expand correctly. Some failed to achieve this end because of faulty algebraic manipulation of the expressions. Many candidates did not appreciate that the theory of quadratic equations applied and hence did not obtain the discriminant. Others who obtained the discriminant did not observe that the result of (c) (i) was relevant.

Answer(s):

(a) 50

(b)(i) $\frac{5}{22}$,

(ii) $\frac{6}{11}$

(c)(i) $u = 1 + 4i$ **or** $-1 - 4i$

(ii) $z = 2 + 3i$ *or* $1 - i$

Question 6

Specific Objective(s): (b) 1, 2, 6, 7

This question examined properties of determinants and matrices and solutions of simultaneous linear equations in three variables.

(a) There were many attempts at this part of the question with a high degree of success. However, poor algebraic manipulation was the cause of many errors in the solutions.

(b)(i) Several candidates who attempted this part of the question gained full marks.

(ii) Most candidates were able to express the system of equations in the required form. Many candidates saw the relevance of Part (ii) a) to the solutions of the system.
Several others used the ‘otherwise’ path and employed different approaches to solving the system of equations.

Answer(s):

(a) x = 2, 3 or 6

(b)(i) a) AB = 20I, **b)** $A^{-1} = \frac{1}{20}B$

(ii) a) $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 3 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 \\ 5 \\ 25 \end{pmatrix}$

b) x = 1, y = 2, z = 12

UNIT 2

PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)

SECTION A

Module 1: Calculus II

Question 1

Specific Objective(s): (c) 11, 12, and MM

One part of the question posed a mathematical problem against the background of a differential equation. Both parts examined the candidates' skills in solving such equations.

The success rate in this question was not very high, although many candidates attempted it. More exposure to such problems is required at the instructional level.

Answer(s):

(a) $\boldsymbol{xy = (x+1)(c - \frac{1}{2}e^{-x^2})}$, **c is constant**
(b) $\boldsymbol{y = e^{-x} - e^{4x} - sinx}$

SECTION B

Module 2: Sequences, Series and Approximations

Question 2

Specific Objective(s) 6, 8, 9 and MM

The question tested the principle of mathematical induction as well as arithmetic and geometric progressions.

The majority of candidates who attempted this question performed satisfactorily.

(a) Candidates knew how to verify the initial step in the proof for $n=1$, but some had difficulty with the induction step in proceeding from $n = k$ to $n = k + 1$.

(b) Most candidates used the formula for S_n to find the sum of all the terms, however, a few went the route of trying to calculate the value of the common difference and were unsuccessful. Despite the incidence of such cases, this part of the question was very well done.

(c) Many candidates failed to realize that the problem posed involved a geometric progression, nevertheless, some used the simple interest formula for each year and were able to reach the correct result. Most candidates did not approximate the answers to the nearest dollar.

Answer(s):

(b) Sum = -300;

(c)(i) $29 877, (ii) $ 273 743

SECTION C

Module 3: Counting, Matrices and Complex Numbers

Question 3

Specific Objective(s): (a) 1, 2, 4, 7, 9, 11; (c) 5, 7, 10

This question examined basic principles in counting methods, probability and the locus of complex numbers.

In general, candidates exhibited familiarity with the topics examined. At the level of detail, however, weaknesses were evident.

(a) Some candidates demonstrated an understanding of combinations. However, interpretation of exclusiveness was weak and led to incorrect solutions.

(b) Many candidates used a Venn diagram to reason out what was required, while some tried to use formulae but failed to obtain the correct answers.

(c) This part of the question presented an enormous amount of difficulty. Many were unable to obtain the circle in (i) and hence (ii) was not manageable in such circumstances.

Answer(s):

(a) **251**

(b)(i) $P(A \cup B) = 0.7$, **(ii)** $P(A \cap B') = 0.5$, **(iii) 0.6**

(c)(i) **locus is the circle** $(x-1)^2 + (y+1)^2 = 5^2$

(ii) **radius = 5, centre** $\equiv (1, -1)$

PAPER 03 – INTERNAL ASSESSMENT

This year 170 Unit 1 and 158 Unit 2 Internal Assessments were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases neither question papers with solutions nor mark schemes, were submitted. The majority of the samples submitted were not of the required standard. Teachers MUST pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the Internal Assessments.

The Internal Assessment is comprised of three Module tests. The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module test and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments-consistency of marking using the mark schemes
- Inclusion of mathematical modeling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised of items from CAPE past examination papers.

2. Untidy "cut and paste" presentations with varying font sizes were common place.

3. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.

4. The stipulated time for Module tests (1 to $1\frac{1}{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module test are too extensive or are inadequate.

5. The following guide can be used: 1minute per mark. About 75% of the syllabus should be tested and mathematical modeling MUST be included.

6. Multiple choice questions will NOT be accepted in the Module tests.

7. Cases were noted where teachers were unfamiliar with recent syllabus changes i.e.

    - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.
    - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been REMOVED for the CAPE syllabus (2008).

8. The moderation process relies on the validity of the teachers' assessment. There were few cases where students' solutions were replicas of the teachers' solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students' scripts did not correspond to the marks on the Moderation sheet.

9. Teachers MUST present evidence of having marked each individual question on the students' script before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students' scripts. To enhance the quality of the design of the Module tests, the validity of the teacher assessment and validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

I. Design a separate test for each Module. The Module test MUST focus on objectives from that Module.

II. In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.

III. One sample of FIVE students will form the sample for the centre. If there are less than five students ALL scripts will form the sample for the centre.

IV. In 2009, the format of the Internal Assessment remains unchanged.

MULTIPLE CHOICE EXAMINATIONS WILL NOT BE ACCEPTED AS INTERNAL ASSESSMENTS.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of the school and territory, Name of teacher and the Centre number.
- Unit Number and Module Number
- Date and Duration (1-$1\frac{1}{2}$ hours) of Module test.
- Clear instructions to candidates
- Total marks allotted for Module test.
- Sub-marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of (1-$1\frac{1}{2}$hours).
- **CAPE past examination papers should be used as a guide ONLY.**
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes **MUST** be submitted, that is, **one mark should be allocated per skill.** (not 2, 3, 4, etc marks per skill)
- **FRACTIONAL/DECIMAL MARKS MUST NOT BE AWARDED (i.e. DO NOT ALLOCATE $\left(\frac{1}{2}\right)$MARKS).**
- The total marks for Module test MUST be clearly stated on the teachers' solutions sheets.
- A student's marks **MUST** be entered on the front page of the student's scripts.
- Handwritten marks schemes **MUST** be NEAT and LEGIBLE. The unitary marks **MUST** be written on the right side of the page.
- **Diagrams MUST be neatly drawn with geometrical/mathematical instruments.**

4. PRESENTATION OF SAMPLE

- Student's responses MUST be written on letter sized paper $\left(8\frac{1}{2}\ x\ 11\right)$.
- Question numbers MUST be written clearly in the left hand margin.
- The total marks for EACH QUESTION on student's scripts MUST be clearly written in the left or right margin.
- ONLY ORIGINAL students' scripts MUST be sent for moderation.
- Photocopied scripts WILL NOT BE ACCEPTED.
- Typed Module tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:

  - ❖ A question paper.
  - ❖ Detailed solutions with detailed Mark Scheme.
  - ❖ The question paper, detailed solutions, Marks Schemes and 5 students' samples should be batched together for each Module.

- Marks are recorded on PMath1 – 3 and PMath2 – 3 forms and must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded.

- The guidelines at the bottom of these forms should be observed. (See Page 57 of the syllabus, No. 6).

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2010

PURE MATHEMATICS

GENERAL COMMENTS

This is the third year that the current syllabus has been examined in the format of Paper 01 as Multiple Choice (MC) and Papers 02 and 03 structured questions. Approximately 5 600 candidates wrote the Unit 1 and 2 800 Unit 2 examinations in 2010. Performances continued in the usual pattern across the total range of candidates with some obtaining excellent grades while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

UNIT 1

The overall performance in this unit was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Coordinate Geometry and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Limits and Continuity, Indices and Logarithms. Other aspects that need attention are summation as part of general algebraic manipulation of simple expressions, substitution and pattern recognition as effective tools in problem solving.

UNIT 2

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by many candidates.

Candidates continue to show marked weakness in algebraic manipulation. Much more emphasis must be placed on improving these skills. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrate a favour for problem solving by memorized formulae.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily with a mean score of 46.27 and a standard deviation of 17.02.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (b) 1, 3, 4; (c) 1−5; (f) 3; (g) 3.

The topics examined in this question covered the Remainder and Factor Theorems, simultaneous equations and logarithms, inequalities, quadratic equations and indices.

(a) The majority of candidates correctly applied the Factor Theorem to *f(x)*. Several opted to find the remainder by division but were unable to follow through successfully because of weaknesses in the algebraic manipulation required. Some were unable to factorize the equation for $f(p) = 0$ using the quadratic formula.

(b) Several candidates misinterpreted the basic laws of logarithms, replacing $\log(x - 1) + 2\log y$ with expressions such as $\frac{\log x}{\log 1} + \log 2y$. Such equivalences produced erroneous results and, in some cases, equations which the candidates could not solve. Elimination was tried in some cases without success.

(c) Many candidates multiplied through by $x + 1$ instead of $(x + 1)^2$ and had difficulty afterwards in solving the resulting inequality.

(d) The substitution $y = 2^x$ proved problematic for some candidates, many of whom interpreted 4^x as 2×2^x which led to incorrect answers. Other candidates did not complete the question, giving the answers for y only and not for x as required. More practice is needed to consolidate the areas of weakness identified above.

Solutions:

(a) $p = \frac{3}{2}$ or -1
(b) $x = 2, y = 3$
(c) $-\frac{8}{3} < x < -1$
(d) $x = 1$ or 2

Question 2

Specific Objectives: (a) 7; (d) 5−8.

The topics covered in this question related to summation notation and functions, together with the interpretation of the graphs of simple polynomial functions and the existence or non-existence of inverses.

(a) The majority of candidates answered (i) correctly. A common mistake made was to equate S_{2n} with $2 \times S_n$, while some candidates misinterpreted the question as a problem solving mathematical induction. Several candidates encountered simplification problems in (ii). A common error involved writing $-Sn$ as $-n^2/2 + n/2$ instead of $-n^2/2 - n/2$. In (iii), the correct quadratic equation in n, namely $3n^2 + n - 520 = 0$, escaped many candidates and some who derived it failed to solve it correctly to obtain $n = 13$.

(b) (i) Approximately 80 per cent of the candidates gave $x \geq 3$ as a possible solution. Some obtained the complete solution but failed to write in set notation. Several used the graph and gave the solution as $x \leq 0, x \leq 3$.

(ii) Less than 10 per cent of the candidates gave the correct solution. Many assigned values to k and attempted to solve the equation $x^2(3-x) - k = 0$, thereby completely ignoring the graph as a guide to the solution.

(iii) Many candidates showed familiarity with the terms injective, surjective and bijective as they applied to functions but seemed to struggle to separate the terms in respect of the given graphical representations for f and g. Approximately 80 per cent of the candidates used the horizontal line test (a) and (b).

Solutions:

(a) (i) $S_{2n} = n(2n + 1)$
(ii) $p = \frac{3}{2}$, $q = \frac{1}{2}$
(iii) $n = 13$

(b) (i) $\{0\} \cup \{x \geq 3\}$ (ii) $\{k: 0 \leq k \leq 4\}$

Section B

Module 2: Trigonometry and Plane Geometry

Question 3

Specific Objectives: (b) 4, 5; (c) 3, 9, 10

Overall, candidates performed poorly on this question with approximately 70 per cent scoring less than 12 of the maximum marks (25) and approximately 40 per cent scoring less than 4 marks. Generally, candidates scored higher in Part (a) than in Part (b) of the question. Attempts at Part (a) (i) by the candidates who achieved an acceptable score were usually fruitful. Many of the candidates gained full marks for finding the angle. In addition, many of them used the form $\mathbf{p}.\mathbf{q} = |p||q|\cos\theta$. However, a large percentage of candidates did not score maximum marks as they had a problem adding directed numbers accurately or they simply forgot the negative sign in front of the 84. Many candidates used the alternative method which included the graph and trigonometrical ratios. However, a few of them forgot to add the 90° which was required to find the entire angle.

A significant number of candidates experienced difficulty in the relatively easier form of finding the x co-ordinate of the vector in (a) (ii). Many started at $(6\boldsymbol{i} + 4\boldsymbol{j})$ $(x\boldsymbol{i} + v\boldsymbol{j}) = 6x + 4y = 0$ and then stopped. Astoundingly, many candidates declared that $\mathbf{p}.\mathbf{v} = 0$ was either parallel or inverse.

Overall, performance on (b) (i) was also disappointing. Many candidates started at the given equation for C_1 and ignored the given end points completely. Others simply substituted the given points into equation C_1 and then faltered. Some worked backwards and did not use the end points at all.

The responses to (b) (ii) were similarly disappointing. Only a small percentage of candidates attempted to eliminate the x^2 and y^2 hence obtaining $y = x + 5$ which could easily be substituted into C_1 or C_2. The majority who opted to obtain x or y as the subject ended up with an awkward round of algebra, often with limited success. Some of the candidates simply could not appreciate what to do with the equation $y = x + 5$ and proceeded with a completely different and inappropriate strategy.

General recommendations to teachers include:

1. Constant revision of algebraic simplification
2. The provision of opportunities for students to enhance their problem-solving skills

Solutions:

(a) (i) 165°
(ii) (a) $2k\mathbf{i} - 3k\mathbf{j}$, $k \in R$
(iii) **p**, **v** are perpendicular

(b) (ii) Points of intersection are (0, 5) and (-3, 2)

Question 4

Specific Objectives: (a) 4, 5, 13, 14

Most candidates attempted this question but performed poorly overall. Approximately 40 per cent of candidates scored less than 4 marks and approximately 65 per cent scored less than 12 marks.

(a) (i) Many candidates were able to gain marks for stating $3A = \cos^{-1}0.5$ and proceeding from there. However, only a small number of candidates arrived at the three solutions in the given range. The majority ended with the answer $A = \frac{\pi}{9}$ or 20°. In some cases, candidates attempted to find the solutions by using trigonometric identities. Most of them simply stalled thereafter.

(ii) Responses to this part of the question were surprisingly very good. Even some of the relatively lower-scoring candidates completed the exercise with a flourish, gaining the full six marks. Some showed great competence, utilizing standard identities, to prove this more difficult identity.

(iii) This part of the question challenged many candidates including many with high scores. They could not make the link to the earlier parts of the question, even when directed to do so. It was also noted that answers were not given to three significant figures (as indicated at the front of the question paper).

(b) (i) Many candidates were able to make substantial progress on this part of the question. Many of them knew the compound angle formulae and also derived the correct ratios for $\tan \alpha$ and $\tan \beta$, but some lacked the algebraic techniques needed to get the required answer.

(ii) Many candidates were clueless and had no idea how to solve this question. Indeed, there were many 'no responses' seen for this part of the question.

Solutions:

(a) (i) $A = \frac{1}{9}\pi,\ \frac{5}{9}\pi,\ \frac{7}{9}\pi$
(iii) $þ = \cos\frac{\pi}{9},\ \cos\frac{5\pi}{9},\ \cos\frac{7\pi}{9} \approx 0.940, -0.174, -0.766$

(b) (ii) $\max(\alpha - \beta) = \tan 0.125 \approx 0.124$ rad.

Section C

Module 3: Calculus 1

Question 5

Specific Objectives: (a) 1, 3−7, 9; (c) 1, 4, 6 (ii)

This question tested knowledge of limits, continuity and discontinuity, and integration.

There were some very good attempts, many of which gained full marks on the question. Nevertheless, some weaknesses in the preparation of the candidates were evident and there were cases of carelessness in pursuing routine procedures.

(a) (i) Many candidates experienced difficulty factorizing the denominator in this part and this curtailed success in obtaining the limit. Some used L'Hôpital's rule with great success.

(ii) Few candidates were successful in obtaining full marks on this part. Those who did used L'Hôpital's to great advantage.

(b) Some candidates misunderstood the symbols for left hand and right hand limits while others had difficulty with the concept of discontinuity. The word 'deduce' was largely ignored by many.

(c) (i) There were several good returns on this part of the question for the candidates who expanded the expression in the integrand correctly. Many of these candidates were able to follow through with the integration, termwise, of the expanded expression and the evaluation at each of the limit points $x = 1$ and $x = -1$, to obtain full marks for this part.

(ii) Several candidates performed well on this part of the questions. Many candidates who did not, failed to change the integrand completely from x's to u's.

Solutions:

(a) (i) $\frac{2}{9}$ (ii) 2

(b) (i) a) 5, b) 9

(c) (i) at $x = 1$, $-2\frac{2}{3}$; at $x = -1$, $2\frac{2}{3}$

(ii) $\frac{1}{3}(x^2 + 4)^{\frac{3}{2}} + k$ (constant)

Question 6

Specific Objectives: (b) 4, 7, 8, 9 (i), 10

This question covered differentiation, integration and calculus.

Several candidates attempted the question with varying degrees of success. The principles involved seemed to be familiar to most, yet some candidates did not receive maximum reward because of weaknesses in simple algebraic manipulations. More practice is recommended to reduce the incidence of such pitfalls.

(a) (i) Many candidates were strong on differentiating the basic trigonometric functions of sin and tan but some were unable to cope with the composite nature of the functions involved.

(ii) Several candidates had difficulty using the quotient rule often interchanging numerator with denominator in applying it. Nevertheless, there were some excellent answers returned on this question.

(b) (i) The use of the relevant integration theorem seemed not to be recognized by some candidates. In such cases, the candidates performed poorly to the extent that they did not seem to be aware that $\int_1^4 4\,dx$ had to be obtained.

(ii) It seemed that some candidates had little practice in using substitution to perform integration. However, several of those who knew the technique were able to obtain full marks on this part of the question. Greater attention should be paid to Specific Objectives 6 and 7 of Unit 1 Module 3, in preparing candidates.

(c) (i) This part of the question was well done. The majority of candidates obtained full marks.

(ii) The majority of candidates knew the methods to be used but many encountered problems with the algebra involved. Several methods were used to obtain the correct answer.

Solutions:

(a) (i) $3\cos(3x+2) + 5\sec^2(5x)$ (ii) $-\dfrac{(x^4+3x^2+2x)}{(x^3-1)^2}$

(b) (i) 33 (ii) 14

(c) (i) $P \equiv (-2,4)$, $Q \equiv (1,1)$; (ii) Area $= 4\frac{1}{2}$ units^2

Paper 03/B (Alternative Internal Assessment)

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 8, (d) 1, 7, (f) 4, 5 (ii)

This question examined the theory of cubic equations, functions and mathematical induction.

(a) (i) This part of the question was poorly done. There was little evidence of candidates demonstrating the theory of cubic equations. Some candidates merely stated the concepts of the sum of roots, product pair-wise and the product of roots of the cubic equations. However, they were unable to complete the calculations by substituting the correct values.

(ii) Without a correct follow through from (i) candidates could not find the required values of p and q.

(b) (i) Candidates understood the meaning of 'one-to-one' but were unable to show the required result mathematically. Instead, they substituted some values for x and concluded that f is one-to-one. No candidate used the horizontal line test.

(ii) A few candidates made correct expansions but failed to make the correct deductions. Generally, this part of the question was satisfactorily done.

(c) The majority of candidates knew that an assumption was necessary, followed by the inductive process. However, the inductive process and the algebra involved proved beyond their abilities. They merely stated conclusions without proof.

Solutions:

(a) (i) $\alpha = -2$

(ii) $p = 12; q = 44$

(b) $x = \frac{5}{2}$

Section B

Module 2: Trigonometry and Plane Geometry

<u>Question 2</u>

Specific Objectives: (b) 1, 2, 3; Content: (a) (ii), (b) (iii)

This question examined the intersection of lines, equations of straight lines and the area of a triangle using basic concepts of coordinate geometry.

(a) (i) This part of the question was well done.

(ii) This part of the question was well done. Some exceptions included the correct calculations for the gradient of a straight line given two points on the line.

(b) This part of the question was well done.

(c) Candidates had problems calculating the lengths of AC and DC which were required to find the area. The majority of candidates obtained partial marks for some related attempts to find the area.

Solutions:

(a) (i) A (0, 2), B (3, 0), C (6, -2)

(ii) CD: $3x - 2y - 22 = 0$ AD: $5x + y - 2 = 0$

(b) D (2, -8)

(c) Area = 26 square units

Section C

Module 3: Calculus 1

Question 3

Specific Objectives: (b) 6, 15, (c) 2, 3, 4, 5, (ii), (iii), 8 (i)

This question examined indefinite integration of composite trigonometric functions, area under the curve and applications of differentiation to maxima/minima situations. There was also some mathematical modelling included.

(a) This part of the question was poorly done. Approximately three per cent of the candidates obtained maximum marks. A common error among candidates was their inability to use the identity $\tan^2 x \equiv \sec^2 x - 1$ in order to integrate $\tan^2 x$ correctly. A significant number of candidates evaluated $\int (\cos 5x)\,\mathrm{d}x$ as $5\sin 5x$ or $-\frac{1}{5}\sin 5x$. Candidates continue to neglect the arbitrary constant of integration for indefinite integrals.

(b) (i) This part of the question was well done.

(ii) The majority of candidates used $\int_0^2 y\,\mathrm{d}x$ to find the area of the enclosed region. A few candidates recognized symmetry of the two regions and used $2\int_0^1 y\,\mathrm{d}x$ to find the area.

(c) (i) This part of the question was poorly done. Difficulties included being unable to state the perimeter in terms of r and x, and failing to equate the expression for the perimeter to 60.

(ii) Without the correct follow through from Part (i), many candidates could not obtain the area in terms of r only.

(iii) Differentiation of the expression for the area was poorly done. Candidates could not interpret the term $\left(1+\frac{\pi}{4}\right)$ as a constant. In addition, most of the candidates did not follow the instruction to find the **exact** answer.

Solutions:

(a) $\frac{1}{5}\sin 5x + \tan x - x + C$

(b) (i) $p = 1$; $q = 2$ (ii) Area = $\frac{1}{2}$ square units

(c) (i) $x = 30 - r - \frac{\pi r}{2}$

(iii) $r = \frac{60}{4+\pi}$

UNIT 2

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed fairly well with a mean score of 50.39 per cent and a standard deviation of 17.93.

Paper 02 – Structured Questions

Section A

Module 1: Calculus II

Question 1

Specific Objectives: (a) 9, (b) 1, 4, 5, 6, 7, (c) 1 (i), 5, 6, 7

This question examined a real-world situation involving an exponential equation, differentiation of an exponential function, differentiation of an inverse trigonometrical function, implicit differentiation including second derivatives and integration using partial fractions with suitable substitutions.

(a) (i) Approximately 85 per cent of the candidates who responded to this part of the question were able to deduce that the initial temperature occurred when $t = 0$. Full marks were obtained by all of the candidates who interpreted this question correctly. Some candidates demonstrated a lack of understanding of the term ‘initial temperature’.

(ii) Generally, a significant number of candidates attempting this part of the question found it difficult to interpret the term ‘stabilise’. The general response was below average. Approximately ten per cent of the candidates obtained the correct answer, deducing that the limiting value of the temperature for large increasing t was 65^0 C. It may be useful at the level of instruction to consider terms that may be easily interpreted by candidates who may not be science oriented.

(iii) This part of the question required candidates to solve the equation to find a value for t. Common errors seen were

a) $0 = 65 + 8e^{-0.02t}$

b) $\ln 70 = \ln 65 + \ln 8e^{-0.02t}$

c) $73 = 65 + 8e^{-0.02t}$, using the value 73 obtained in (a) (i)

d) $\ln 5 = 8 \ln e^{-0.02t}$, treating 8 as a power

e) $t = \dfrac{\ln 70 - \ln 65}{8(-0.02)}$

Approximately half of the candidates demonstrated a marked weakness in algebraic manipulation, particularly regarding logarithms. A small percentage of candidates who recognized that logarithms had to be applied used common logarithms instead of natural logarithms. Emphasis must be placed on the properties of exponential and logarithmic equations. This will allow candidates to gain confidence to solve these equations correctly.

(b) (i) For this part of the question approximately 30 per cent of the candidates expressed the equation $y = e^{tan^{-1}\ (2x)}$ as ln y = tan $^{-1}$ $^{(2x)}$. However, these candidates were not able to carry out the required implicit differentiation to obtain the correct answer. Common errors made included failing to apply the chain rule when differentiating the composite function tan $^{-1}$ $(2x)$. Some answers given were

$$\frac{dy}{dx} = \frac{1}{1+4x^2} \text{ x } e^{\tan -1\ (2x)} \times 2 \text{ and } \frac{dy}{dx} = \frac{1}{1+2x^2} \text{ x } e^{\tan -1\ (2x)} \times 2.$$

Very few candidates obtained full marks on this part of the question.

(ii) Approximately 15 per cent of the candidates attempted to use the result given in Part (b) (i) to show the result required in Part (b) (ii). However, they were unable to carry out the implicit differentiation required when using the form given in Part (b) (i). They failed to show that $\frac{d}{dx} 2y = 2\frac{dy}{dx}$, but instead showed $\frac{d}{dx} 2y = 2$.

Approximately 80 per cent of the candidates stated the equation $\left(1+4x^2\right)\frac{dy}{dx} = 2y$ as $\frac{dy}{dx} = 2e^{\tan\text{-}1(2x)} \text{x} \left((1+4x^2\right)^{-1}$ and proceeded to use the product rule correctly. These candidates obtained full marks with the correct algebraic manipulation to show the required result.

(c) (i) Approximately 50 per cent of the candidates used the substitution given, $u = e^x$, to express $\int \frac{4}{e^x+1} dx$ as $\int \frac{4}{u+1} dx$ in this part of the question and obtained the result 4 ln $(u + 1) + C$. They failed to replace dx with $\frac{1}{u}$ du. A number of candidates obtained the form $\int \frac{4}{u(u+1)}$ correctly. However, they failed to carry out the correct integration since they did not recognize that partial fractions were necessary at this point. A few candidates who carried out the correct procedure for the integration left their answer as $4 \ln u - 4 \ln\left(u+1\right) + C$.

(ii) Approximately 70 per cent of the candidates were able to obtain the expression $\int \frac{4e^{-x}}{1+e^{-x}} dx$. However, these candidates could not proceed with the correct integration.

They failed to recognise the systematic integration of the form $\int \frac{f'(x)}{f(x)}dx$. A significant number of those candidates who carried out the integration incorrectly obtained $4\ln\left(1+e^{-x}\right)+C$.

Solutions:

(a) (i) 73°C

(ii) 65°C

(iii) 23.5 hours

(c) (i) $4\ln(e^x) - 4\ln(e^x+1) + C$ or $4\ln\left(\frac{e^x}{e^x+1}\right)+C$

(ii) $-4\ln(1+e^{-x})+C$ or $4\ln\left(\frac{e^x}{1+e^x}\right)+C$

Question 2

Specific Objectives: (b) 2, 5, (c) 8, 10, 11

This question required differentiation of ln x using product and chain rules, reduction formula and the solution of a differential equation using an integrating factor.

(a) (i) Most candidates recognized that this part of the question required the use of the product and chain rules. However, application of the chain rule to differentiate the logarithmic function $(\ln x)^n$ proved to be difficult in many cases. Approximately 50 per cent of the candidates obtained full marks.

(ii) Generally, most of the candidates used integration by parts using the approach $\int(1)(\ln x)^n\,dx = x\ln - \int x\frac{d}{dx}(\ln x)^n\,dx$. No candidate used the result from (a) (i) to derive the reduction formula. This part of the question was well done by the majority of candidates.

(iii) This part of the question was poorly done. Many candidates demonstrated a lack of simple use of the derived reduction formula. No candidate successfully managed $I_1 = \int(\ln x)dx$. In addition, the weakness in algebraic manipulation was again evident in substituting successive values of n in the reduction formula, making correct use of brackets. Approximately ten per cent of the candidates obtained full marks.

(b) (i) A small percentage of candidates demonstrated the ability to find a suitable integrating factor and proceeded to show the required general solution of the differential equation.

Approximately ten per cent of candidates were able to state $I = \mathrm{e}^{\int \frac{2}{t+10} \mathrm{d}t}$ but were unable to complete this integration correctly. The remaining candidates did not demonstrate any knowledgeable attempt to find the integrating factor.

(ii) Using the general solution to the differential equation in (b) (i), some candidates were able to obtain the correct answer to this part of the question. However, a significant number of candidates did not understand the term 'initially' and substituted random values.

Solutions:

(a) (i) $n(\ln x)^{n-1} + (\ln x)^{n}$

(iii) $x(\ln x)^3 - 3x(\ln x)^2 + 6x(\ln x) - 6x + C$

(b) (ii) 24.2 kg

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (b) 2, 4, (c) 3

This question examined candidates' ability to define the r^{th} term of a sequence, obtain the n^{th} partial sum of a finite series, find the first term and common difference of an arithmetic progression and their application of the binomial theorem.

(a) (i) Approximately 70 per cent of the candidates had difficulty obtaining the r^{th} term of the sequence.

(ii) In this part of the question most candidates attempted to sum the given series as an A. P. or a G. P. Those candidates who were able to express the r^{th} term correctly used the standard formulae for $\sum_{r=1}^{n} r^2, \sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} c$ to find the required sum. Evidence of a weakness in algebraic manipulation resulted in some candidates not being able to simplify the answer.

(b) This part of the question was well done by approximately 75 per cent of the candidates. Some candidates incorrectly defined the equation required to express the 9^{th} term as 3 times the 3^{rd} term. As a result, incorrect values were obtained for the first term *a*, and the common difference *d*.

(c) (i) This part of the question was generally well done although some errors were made in simplifying the coefficients. Too many candidates did not state the correct range of values of x for which the expansion is valid.

(ii) A significant number of candidates did not attempt to rationalize the denominator by recognizing the resulting difference of squares. Many of those candidates who attempted to rationalize the denominator used $(1 - x)$ - $\sqrt{(1+2x)}$ as the 'conjugate' of $(1 + x) + \sqrt{(1+2x)}$. Other candidates used the result from Part (i) to expand the denominator and could not show the required expression. Only a few candidates obtained full marks.

(iii) Using the result from Part (ii), the majority of candidates expanded the expression $\frac{1}{x}(1 + x - \sqrt{(1+2x)}\ )$ up to and including the term in x^2. Consequently, the resulting expression could not be $\frac{1}{2}x(1-x)$ as required. It was clear that candidates did not take into account that the expansion was divided by x, thus requiring the expansion up to and including the term in x^3.

Solutions:

(a) (i) $(3r - 1)(2r + 1)$

(ii) $S_n = \frac{n}{2}(4n^2 + 7n + 1)$

(b) $a = d = 2$

(c) (i) $1 + x - \frac{x^2}{2} + \frac{x^3}{2} - \ldots - \frac{1}{2} < x < \frac{1}{2}$; $|x| < 1$

Question 4

Specific Objectives: (b) 1, 11, 12, (c) 1, (e) 1, 2, 4

This question examined candidates' ability to manipulate and prove expressions involving ${}^nC_r$, the sum of finite series using the method of differences and application of the Newton-Raphson procedure.

(a) (i) A small percentage of candidates obtained full marks for this part of the question. A significant number of candidates expressed ${}^nC_{r-1}$ as $\frac{n!}{(r-1)!(n-r-1)!}$. Many candidates stated the correct expression for ${}^nC_r + {}^nC_{r-1}$ but were unable to complete the algebraic manipulation to obtain the required result. Some candidates substituted numbers as a means of proof.

(ii) (a) Many candidates merely stated $f(r) - f(r+1) = \frac{1}{r!} - \frac{1}{(r+1)!}$. The algebraic manipulation and understanding of factorials required for showing the result were beyond the ability of these candidates.

(b) Most candidates recognized that the method of differences was needed to find the sum required. This part of the question was generally well done.

(c) A few candidates demonstrated an understanding of deducing the limiting value of this type of sum. Many of the candidates attempted to use the formula for the sum to infinity of a geometric progression but found it impossible to apply. A significant number of candidates did not respond to this part of the question.

(b) (i) Most of the candidates used the Intermediate Value Theorem but did not state the continuous property of the polynomial. Very few candidates obtained full marks.

(ii) Generally, this part of the question was well done. Some errors were made in substitution and there were incorrect calculations.

Solutions:

(a) (ii) b) $1 - \frac{1}{(n+1)!}$

c) $\lim_{n\to\infty} S_{_1} = 1$

(b) (ii) 0.725

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 5

Specific Objectives: (a) 1, 3, 11, 12, 13 (c) 1, 2, 4, 5

This question examined the concepts of counting, using permutations and combinations, basic probability theory and complex numbers.

(a) (i) Approximately half of the candidates wrongly applied the concept of all the letters of the word SYLLABUS as being unique, thus giving the solution as 8! Some candidates subtracted the repeated letters from the total number of letters and gave the solution as $\frac{6!}{2!2!}$.

(ii) This part of the question was poorly done and was not attempted by the majority of the candidates. Among the responses seen were ${}^6C_{5,}$ ${}^8C_5$ and $\frac{5!}{2!2!}$. Some candidates were awarded partial marks for applying the concept but not accounting for all of the combinations.

(b) (i) This part of the question was well done. Approximately 75 per cent of the candidates obtained full marks.

(ii) This part of the question was fairly well done although some candidates did not demonstrate an understanding of independence and mutual exclusivity of events. Many of the candidates attempted to define the terms ‘independence’ and ‘mutually exclusive’ and related the concepts to arbitrary sets of events rather than to the given set of events.

(c) (i) Generally, this question was fairly well done. Many candidates were unable to rationalize the denominator correctly, failing with the algebra involved.

(ii) A significant number of candidates deduced the root 1 + i but were unable to find by division, or otherwise, the third root. Some candidates used the theory of quadratic equations to deduce the third root by inspection.

Solutions:

(a) (i) $\frac{8!}{2!2!} = 10\ 080$

(ii) 30

(b) (i) 0.17

(ii) a) not mutually exclusive

b) not independent

(c) (i) 2 + 4i

(ii) 1 + i, – 3

Question 6

Specific Objectives: (b) 1, 3, 4, 5, 6, 7, 8

This question examined the augmented matrix, row-echelon reduction, consistency of a system of linear equations and inverting a matrix.

(a) (i) This part of the question was well done although some candidates merely wrote down the matrix for the system of equations.

(ii) In this part of the question, a common error seen was obtaining a row of zeroes in rows other than the last row. However, the question was generally well done.

(iii) A number of candidates did not understand the term 'consistent'.

(iv) This part of the question proved to be the most challenging to the majority of candidates. Most of them attempted to find a unique solution. Others expressed the variables x and y in terms of z but did not choose an arbitrary constant.

(b) (i) a) This part of the question was well done although some arithmetic errors were made. A number of candidates squared the elements of the matrix A instead of finding A × A.

b) This part of the question was well done.

(ii) This part of the question was well done.

(iii) A significant number of candidates used the cofactor method to find the inverse of A. However, they did not recognize the product AB was equal to $3I$.

Solutions:

(a) (i) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & -1 & -1 \\ 1 & 2 & 4 & k \end{array}\right)$

(iii) $k = 1$

(ii) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -3 & -1 \\ 0 & 0 & 4 & k-1 \end{array}\right)$

(iv) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2\lambda - 1 \\ 1 - 3\lambda \\ \lambda \end{pmatrix}$

(b) (i) a) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(iii) $\frac{1}{3}\begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(ii) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Paper 03/B – Alternative to Internal Assessment

Section A

Module 1: Calculus Ii

Question 1

Specific Objectives: (c) 1 (iii), 5, 11

This question examined partial fractions, solution of a logarithmic differential equation and proportional increase.

(a) This part of the question was poorly done. Candidates showed a marked weakness in manipulating fractions of the form $\frac{A}{x}+\frac{Bx+C}{x^2+1}$. Invariably, candidates expressed $\frac{1-x^2}{x(x^2+1)}$ as $\frac{A}{x}+\frac{Bx}{x^2+1}+C$ and $\frac{A}{x}+\frac{Bx}{x^2+1}+\frac{C}{x^2+1}$.

(b) (i) (a) This part of the question was poorly done in its entirety. A small number of candidates separated the variable successfully but could not complete the integration.

(b) Approximately 40 per cent of the candidates obtained full marks on this part of the question, using the follow through from (i) and making the correct substitution.

(ii) Candidates merely substituted $t = 2$ in the equation. This part of the question was poorly done.

Solutions:

(a) $\frac{1}{x}-\frac{2x}{x^2+1}$

(b) (ii) $2^{2/3} — 1$

Section B

Module 2: Sequences, Series and Approximations

Question 2

Specific Objectives: (b) 1, 10, 11, (c) 1, MM

This question examined the geometric progression, the limiting sum of an infinite series using Maclaurin's expansion and the applications of an exponential series.

(a) This part of the question was well done.

(b) This part of the question was hardly attempted. In fact, candidates seemed unfamiliar with the form of such series.

(c) (i) This part of the question was well done.

(ii) Only a small percentage of the candidates were able to state the correct value in terms of t.

(iii) A few candidates solved this part of the question using the index approach. A significant number of candidates used an arithmetic approach, calculating the depreciated value for successive years.

Solutions:

(a) $r = \frac{5}{6}$

(b) 3e – 1

(c) (i) \$13 125

(ii) \$15 000 $\left(\frac{7}{8}\right)^t$

(iii) \$ 4 510

Section C

Module 3: Counting, Matrices And Complex Numbers

<u>Question 3</u>

Specific Objectives: (a) 1, 2, 4, 7 (b) 1, 2, 6, 8, MM

This question examined selections with and without restrictions, classic probability and the solution of a system of linear equations using a matrix approach.

(a) (i) This part of the question was poorly done. Candidates applied combinations for calculations instead of reasoning.

(ii) The approach in Part (i) to finding a solution was repeated in this part of the question.

(b) This part of the question was poorly done since incorrect methods in Part (ii) resulted in incorrect answers being obtained.

(c) (i) This part of the question was well done.

(ii) This part of the question was well done.

(iii) Responses to this part of the question were poor. Candidates appeared to have no knowledge of the row-reduction method, the inverse method, or the solution of 3 linear equations with 3 unknowns.

Solutions:

(a) (i) 48

(ii) 100

(b) $\frac{1}{5}$

(c) (i)
$$\begin{aligned} 20x + 40y + 60z &= 1120 \\ 40x + 60y + 80z &= 1720 \\ 60x + 80y + 120z &= 2480 \end{aligned}$$

(ii)
$$\begin{pmatrix} 20 & 40 & 60 \\ 40 & 60 & 80 \\ 60 & 80 & 120 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1120 \\ 1720 \\ 2480 \end{pmatrix}$$

(iii) $x = 12;\ y = 10;\ z = 8$

Paper 03/1 – Internal Assessment

This year, 171 Unit 1 and 141 Unit 2 internal assessments were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases, neither question papers with solutions nor mark schemes were submitted. In an increasing number of cases, marks awarded were either too few or far too many. For example: an entire IA module test was worth 20 marks and in another case a simple polynomial division was awarded 78 marks.

The majority of samples submitted were not of the required standard. Teachers must pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the internal assessments.

The internal assessment is comprised of three module tests.

The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test and students' scripts)
- Quality of teachers' solutions and mark schemes
- Quality of teachers' assessments, that is, consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

GENERAL COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.
2. Untidy 'cut and paste' presentations with varying font sizes were common place.
3. Teachers are reminded that the CAPE past examination papers should be used **only** as a guide.
4. The stipulated time for module tests must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient.
5. The following guide can be used: one minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **must** be included.
6. Multiple-choice questions will **not** be accepted as the entire module test but the test may include some multiple-choice items.
7. Cases were noted where teachers were unfamiliar with recent syllabus changes. For example,

    - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.
    - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been removed from the CAPE syllabus (2008).

8. The moderation process relies on the validity of teachers' assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet.
9. Teachers must present evidence of having marked each individual question on students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of students' scripts. To enhance the quality of the design of the module tests, the validity of the teacher assessment and the validity of the moderation process, the internal assessment guidelines are listed below for emphasis.

Module Tests

(i) Design a separate test for each module. The module test **must** focus on objectives from that module.

(ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.

(iii) One sample of **five** students will form the sample for the centre. If there are less than five students **all** scripts will form the sample for the centre.

(iv) In 2010, the format of the internal assessment remains unchanged.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each module test.

- Name of school and territory, name of teacher, centre number
- Unit number and module number
- Date and duration of module test
- Clear instructions to candidates
- Total marks allocated for module test
- Sub-marks and total marks for each question **must** be clearly indicated

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each module test must be appropriate for the stipulated time.
- CAPE past examination papers should be used as a guide ONLY.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes MUST be submitted, that is, one mark should be allocated per skill (not 2, 3, 4 marks per skill)
- Fractional or decimal marks MUST NOT be awarded. (that is, Do not allocate $\frac{1}{2}$ marks).
- A student's marks MUST be entered on the front page of the student's script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The **unitary** marks MUST be written on the right side of the page.
- **Diagrams MUST be neatly drawn with geometrical/mathematical instruments**.

PRESENATION OF SAMPLE

- Students' responses MUST be written on letter sized paper ($8\frac{1}{2}$ x 11).
- Question numbers MUST be written clearly in the left hand margin.
- The total marks for EACH QUESTION on students' scripts MUST be clearly written in the left or right margin.
- ONLY ORIGINAL students' scripts MUST be sent for moderation.
- Photocopied scripts WILL NOT BE ACCEPTED.
- Typed module tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:

    - ❖ A question paper
    - ❖ Detailed solutions with detailed unitary mark schemes.
    - ❖ The question paper, detailed solutions, mark schemes and five students' samples should be batched together for each module.

- Marks recorded on PMath–3 and PMath2–3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded. The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no. 6.)

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2011

PURE MATHEMATICS

GENERAL COMMENTS

In 2011, approximately 5,855 and 2,970 candidates wrote the Units 1 and 2 examinations respectively. Performances continued in the usual pattern across the total range of candidates with some candidates obtaining excellent grades while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

The overall performance in Unit 1 was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Functions and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Modulus, Indices and Logarithms. Other aspects that need attention are manipulation of simple algebraic expressions, substitution and pattern recognition as effective tools in problem solving.

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by some candidates.

Candidates continue to show marked weakness in algebraic manipulation. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrated a favour for problem solving by using memorized formulae.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 55.10 and a standard deviation of 20.02.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (a) 5; (b) 1, 3, 4; (c) 1−3(iii); (d) 5; (e) 2; (f) 4; (g) 1

The topics examined in this question covered the Remainder and Factor Theorems, simultaneous equations, logarithms, inequalities, cubic functions and indices.

In general, both sections of the problem in Part (a) were handled well by the candidates. However, there were many cases in which the requisite skills for manipulation of the expressions were lacking.

Part (a) was attempted by almost all candidates and approximately 30 per cent scored between zero and five marks. Approximately 10 per cent of candidates scored between 21 and 25 marks.

In Part (a) (i) where manipulation of surds was the focus, some candidates did not recognize ($\sqrt{75}$ + $\sqrt{12}$)^2 – ($\sqrt{75}$ – $\sqrt{12}$)^2 as a difference of two squares and therefore missed out on a more efficient solution. However, those who saw this expression as a difference of two squares were also able to complete the subsequent manipulation required to arrive at the correct answers. Most candidates expanded the two bracketed terms then proceeded to further manipulation.

For Part (a) (ii), the manipulation of indices and the bases of the indices were the foci. Many candidates recognized that 3 was the smallest common base of the terms in the expression $27^{\frac{1}{4}} \times 9^{\frac{1}{8}} \times 81^{\frac{1}{8}}$, but several could not, complete the solution.

Approximately 90 per cent of the candidates responded well to Part (b) and were able to arrive at the correct answer. Identification from the graph was well done.

Many candidates were able to form the two simultaneous equations in Part (b) (ii). However, some of them were not able to solve the simultaneous equations.

Approximately 80 per cent of candidates who attempted Part (b) (iii) got the correct solution. Although most candidates were not able to factorize the polynomial, they were able to identify the x-value from the given graph.

Part (c) (i) focused on the solution of a quadratic equation involving logarithms. The equation which the candidates were required to solve was $\sqrt{\log_2 x} = \log_2 \sqrt{x}$ and the substitution $y = \log_2 x$ was provided as a possible means of solving the equation. In general, performance on this item was poor and candidates' attempt at solving the problem faltered at the substitution phase due to improper application of the laws of logarithms to transform the right hand side of the equation into a form that allowed the appropriate substitution. These candidates were unable to recognize that $\log_2 \sqrt{x} = \frac{1}{2} \log_2 x$ and hence failed to substitute $\frac{1}{2}y$ in its place. Candidates who completed the substitution correctly divided throughout by y instead of factorizing, thus losing one of the solutions.

Candidates found Part (c) (ii) the most difficult phase of the entire question. This item required solutions to the quadratic inequality $x^2 - |x| - 2 < 0$. Candidates performed poorly on this item due to a lack of understanding of how to deal with the modulus (absolute value) in such a context. In many cases, candidates just dropped both the modulus and the inequality signs and proceeded to solve $x^2 - x - 12 = 0$. In other cases, the inequality sign was retained after the removal of the modulus but only one resulting inequality was recognized ($x^2 - x - 12 < 0$). The candidates did not realize that another valid inequality was $x^2 + x - 12 < 0$, which also contributed to the overall solution.

Solutions:

(a) (i) 120 (ii) $3^2 = 9$

(b) (i) $p = 4$ (ii) $m = -1, n = -4$ (iii) $x = -2at$ Q

(c) (i) $x = 1, x = 16$ (ii) $1x\ 1 < 4$ $(-4 < x < 4)$

Question 2

Specific Objectives: (a) 6, 8; (d) 7; (f) 3, 5 (i)

This question tested knowledge of the roots of quadratic equations, the evaluation of a function at discrete points and mathematical induction.

Apart from Part (a), performance on this question was weak due, in a large number of cases, to faulty basic algebraic manipulation.

Part (a) dealt with the sum and the product of roots of a quadratic equation. The equation given was $x^2 - px + 24 = 0$ for $p \in \mathbf{R}$ and the problem was divided into two major parts.

Part (a)(i) required that candidates express

a) $\alpha + \beta$ and b) $d^2 + \beta^2$ in terms of p.

Most candidates were able to solve these problems which indicated that they understood the roots of quadratic equations. However, some candidates encountered difficulties in expressing $\alpha^2 + \beta^2$ in terms of sums and products of the roots of quadratic equations. Specifically these candidates did not recognize that αβ had to be subtracted from $(\alpha + \beta)^2$ to give the desired $\alpha^2 + \beta^2$ and instead attempted to use only $(\alpha + \beta)^2$.

In Part (a)(ii) candidates were generally able to solve the equation $\alpha^2 + \beta^2 = 33$ to obtain the value for p.

For this problem, the candidates were given the equation $f(2x + 3) = 2f(x) + 3$ along with a stipulation that $f(0) = 6$ and they were then asked to evaluate $f(x)$ at three specific points $f(3)$, $f(9)$ and $f(-3)$.

Although some candidates were able to provide correct solutions, this item was very poorly done in general. The main difficulty was non-recognition of the need to first solve $2x + 3 = a$, where a is the given point at which $f(x)$ was to be evaluated in each of the three cases, then substitute the value of a obtained into the right hand side of the equation as the value of the variable x. Generally, candidates tended to substitute the point at which the function was to be evaluated into $2x + 3$, then substituted the result into the right hand side of the equation. The values of a required to calculate (i) $f(3)$, (ii) $f(-3)$ were respectively $a = 0$, $a = -3$ emphasizing the power of substitution in this question.

The third part of this problem required that candidates solve $f(-3) = 2f(-3) + 3$. Candidates who were able to solve the first two parts of the problem were also able to so1ve this final part.

Part (c) of this question posed significant difficulties for candidates. Candidates needed to prove that the product of any two consecutive integers k and $k + 1$ is an even integer, which merely required candidates to state that for k and $k + 1$ as consecutive integers, one is even the other is odd so that the product $k\,(k + 1)$ must be even.

Part (d) was also poorly done by candidates primarily because they did not know, or did not understand, how to apply the steps required for a proof by induction. A small percentage of candidates who performed well on this part of the question also recognized the relevance of Part (c) to the solution of Part (d).

Solutions:

(a) (i) $\alpha + \beta = p$ (ii) $\alpha^2 + \beta^2 = p^2 - 48$

(b) (i) $f(3) = 15$ (ii) $f(9) = 33$ (iii) $f(-3) = -3$.

Section B

Module 2: Trigonometry and Plane Geometry

Question 3

Specific Objectives: (b) 1–7; C 1, 2, 9

This question examined vectors and the properties of the circle, the intersection of a straight line and a circle, and the parametric representation.

This question was generally not well done. The main errors encountered in 3 (a) (i) as highlighted below;

- Candidates substituted $|\boldsymbol{a}|$ and $|\boldsymbol{b}|$ into the vector expression $(\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{a} - \boldsymbol{b})$ rather than the vectors $a_1\boldsymbol{i} + a_2\,\boldsymbol{j}$ and $b_1\boldsymbol{i} + b_2\,\boldsymbol{j}$.

- Candidates who were able to find the dot product $a_1^2 + a_2^2 - (b_1^2 + b_2^2)$ correctly were in many cases unable to make the final substitution of $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ to obtain the final answer. In fact, $a^2 = 169$ and $b^2 = 100$ rather than $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ were frequently seen.

- Candidates who found the dot product by expanding

  $(a_1\boldsymbol{i} + a_2\boldsymbol{j} + b_1\boldsymbol{i} + b_2\boldsymbol{j}) \cdot (a_1\boldsymbol{i} + a_2\boldsymbol{j} - b_1\boldsymbol{i} - b_2\boldsymbol{j})$ were generally unable to simplify the resulting expression by using the fact that $\boldsymbol{i} \cdot \boldsymbol{i} = 1$ and $\boldsymbol{i} \cdot \boldsymbol{j} = 0$.

Part (a)(ii) was poorly done. Many candidates were able to correctly equate the coefficients of $\boldsymbol{i}$, $\boldsymbol{j}$ to obtain $2b_1 - a_1 = 11$ and $2b_2 - a_2 = 0$. They, however, did not recognize the need to use the previous results $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ to solve for a_1, a_2, b_1 and b_2.

Part (b) was not generally well done.

For Part (b)(i), many instances, candidates were unable to correctly identify the centre of the circle.

In Part (b)(ii), though the majority of candidates realized that a substitution was required to find the points of intersection of the line and the circle, many of them were unable to follow through to the correct answers. Errors frequently seen were

- Incorrect transposition of the linear equation resulting in an invalid substitution.
- Incorrect simplification after substitution leading to an invalid quadratic equation.
- Inability to correctly evaluate the roots using the quadratic formula.
- $x^2 = 8 \Rightarrow x = \sqrt{8}$ thereby omitting $x = -\sqrt{8}$.

For Part (b)(iii), though many candidates were able to put the Parametric equations in a valid Cartesian form $\left(\frac{x-b}{a}\right)^2 + \left(\frac{y-c}{a}\right)^2 = 1$, not all of them were able to follow through by comparing coefficients with the original equation given to determine a, b and c.

In Part (b)(iv), the majority of candidates were unable to determine the equations of C_2. The main error seen was that candidates misinterpreted the question because they did not appreciate the difference between the *line intersecting* the circle and the *line touching* the circle. As a result, many candidates used P and Q from Part (b)(ii) as the centres of possible equations of C_2. Very few candidates were able to recognize the need to use the equation of the perpendicular line through (0, 1), $y = -x + 1$, rather than the original line $y = x + 1$. Even in cases where candidates acknowledged the new line $y = -x + 1$, they often could not follow through to the final equation required. In some cases, candidates correctly identified the equation of the new circle C_2 as of the form $(x-a)^2 + (y-b)^2 = 16$. However, this information was rarely used to complete the question.

Solutions:

(a) (i) 69 (ii) $\boldsymbol{a} = 5\boldsymbol{i} + 12\boldsymbol{j}$, $\boldsymbol{b} = 8\boldsymbol{i} \pm 6\boldsymbol{j}$

(b) (ii) $(2\sqrt{2},\ 1+2\sqrt{2})$, $(-2\sqrt{2},\ 1-2\sqrt{2})$

(iii) $a = 4, b = 0, c = 1$ (iv) $[(x+2\sqrt{2})]^2 + [y-(1+2\sqrt{2})]^2 = 16$

Question 4

Specific Objectives: (a) 4, 5, 10, 11

This question tested candidates' ability to use and apply Trigonometric Functions, Identities and Equations.

In most cases for Part (a), candidates were able to correctly deduce the correct quadratic equation $8x^2 - 10x + 3 = 0$. However, very few of were able to follow through to obtain full marks because they

- Incorrect factorization leading to invalid roots.

- Did not recognize that these roots represented values of $\cos^2\theta$ and therefore it was required to find the square root to determine $\cos\theta$.

- Candidates worked in degrees rather than in radians as specified.

- Candidates neglected to find the second quadrant angle corresponding to the negative value of $\cos\theta$.

- Candidates changed $8\cos^4\theta$ to $8x^4$ instead of $8x^2$.

Part b (i) was generally well done. Candidates were able to obtain $BC = 8\sin\theta + 6\sin\theta$. However, there were many instances of candidates incorrectly giving BR as $6\sin\theta$ or
$6\sin(90° + \theta)$.

In Part (b)(ii) the majority of candidates were able to correctly equate the answer from Part (i) to 7 to obtain $8\sin\theta + 6\sin\theta = 7$. However, many candidates were unable to follow through to the correct value of $\theta = 7.6°$. Common errors seen included;

- taking $\dfrac{1-\cos 4\theta}{\sin 4\theta} = \tan 2\theta$ to mean $2 \times \dfrac{1-\cos 2\theta}{\sin 2\theta} = 2\tan\theta = \tan\theta$

- using $\dfrac{1-\cos 6\theta}{\sin 6\theta} = \tan 3\theta$ to mean $3 \times \dfrac{1-\cos 2\theta}{\sin 2\theta} = 3\tan\theta = \tan\theta$

- $\dfrac{1-\cos 4\theta}{\sin 4\theta} = \dfrac{1-1+4\sin^2 2\theta}{4\sin 2\theta\cos 2\theta}$

- $\dfrac{1-\cos 6\theta}{\sin 6\theta} = \dfrac{1-1+6\sin^2 3\theta}{6\sin 3\theta\cos 3\theta}$

- squaring both sides of the equation in an attempt to solve rather than using the form $R\cos(\theta - \alpha)$ or $R\sin(\theta + \alpha)$

- in some cases choosing to use the form $10\cos(\theta - 53.13) = 7$, candidates failed to realize that they had to use –45.6 rather than 45.6 as the value of $\cos^{-1}(0.7)$ obtaining $\theta = 53.13 + 45.57 = 98.7$ rather than $\theta = 53.13 - 45.57 = 7.56$

For Part (b)(iii), the majority of candidates were able to identify that 15 was *not* a possible value for $|BC|$. However, candidates did not always justify their answers with a valid reason.

Although in most cases candidates were able to substitute the correct identities $\cos 2\theta = 1 - 2\sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$, failure to use brackets in substituting resulted in incorrect simplification of the numerator in Part (c)(i).

The majority of candidates did Part (ii) as a ‘hence or otherwise’ rather than as a ‘hence’ opting to basically redo Part (c) (i) rather than deduce the correct results.

For Part (c), very few candidates were able to follow through to get the correct answer 'n'. Common errors seen included substituting $r = 1$ before using the previous results to reduce the summation to $\sum_{r=1}^{n} 1$. In fact, many candidates using this method, gave the final answer as 1 not recognizing that they had in fact done the summation.

Solutions:

(a) (ii) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}$

(b) (i) $|BC| = 8\sin\theta + 6\cos\theta$ or $|BC| = 8\sin\theta + 6\sin(90 - \theta)$, (ii) $\theta \approx 8°$,

(iii) No

(c) (iii) n

Section C

Module 3: Calculus 1

<u>Question 5</u>

Specific Objectives: (a) 3, 5, 7, 9; (b) 7, 21

This question tested candidates' knowledge of limits, continuity and basic elements of calculus.

In Part (a), both the L'Hopital and factorization methods were seen. Quite a significant number of candidates substituted positive (+ve) 2 rather than negative (–ve) –2, however, and were penalized for this.

Some candidates divided both the numerator and the denominator by x^2 and therefore lost direction.

The majority of candidates were able to score at least 5 out of 11 for Part (b).

For Part (b)(i), a significant number of candidates seemed not to know how to use a piece-wise function and determine the relevant part of the function for the given domain value. Many candidates substituted into both parts of the function.

For Part (b)(ii), elementary approaches to limits were seen where candidates used a table of values rather than direct substitution. Again some candidates simply substituted in both parts of the function.

Again in Part (b)(iii), some candidates substituted in both parts of the function. Several candidates gave the answer as $-2b + 1$.

For Part (b)(iv), few candidates were able to use the condition for continuity at a point to correctly find the value of b. Many candidates did not respond to this part of the question.

Teachers must reinforce, that

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

is the necessary condition for continuity at the point a.

In Part(c)(i), some candidates had difficulty translating the given information into mathematical statements. Many of the more successful candidates were not able to solve both simultaneous equations to find the solutions for $\boldsymbol{p}$ and $\boldsymbol{q}$.

For Part (c)(ii), the incorrect gradient of the normal was seen. A few candidates applied the formula $y - y_1 = m(x - x_1)$ incorrectly.

Candidates who were able to do Parts (c)(i) and (ii), in most cases calculated the length of MN correctly for Part (c)(iii).

Solutions:

(a) $-\frac{1}{5}$ (b) (i) 5 (ii) 5 (iii) $2b + 1$ (iv) $b = 2$

(c) (i) $p = 10, q = -13$ (ii) $7y + x = 15$ (iii) MN = 14

Question 6

Specific Objectives: (b) 4, 7, 8, 9; (i), 10

This question tested differentiation, integration and calculus.

Several candidates attempted the question with varying degrees of success. The principles involved seemed to be familiar to most, yet some candidates did not receive maximum reward because of weaknesses in simple algebraic manipulations. More practice is recommended in applying the principle involved in part (b) of this question.

In Part (a)(i), some candidates did not know that they should have differentiated to find the stationary points, while a few candidates were unable to differentiate correctly. Some candidates were unable to solve the equation x2 = 4, although many got only ‘2’ as the solution and others got ± 4. A number of candidates were unable to substitute correctly.

For Part (a) (ii), to find the gradient, many candidates treated the function as a straight line.

In Part (a)(iii), a number of candidates chose the wrong function to integrate.

Many candidates did not use the correct limits of integration, several of them used 2 or 4 as the upper limit. Some candidates failed to recognize that area cannot be negative.

Some candidates did not understand the concept of proving, hence, they simply rewrote the question in Part (b)(i). This was also done for Part (b)(ii). Some candidates chose to integrate the product $x\sin x$ in the same manner you would integrate $x + \sin x$ would be integrated.

A few candidates chose to use the method of integration by parts.

Solutions:

(a) (i) A ≡ (–2, 16), B ≡ (2, –16)

(ii) $12y = x$ is the equation of the normal

(iii) Area = 36 sq. units

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (c) 1–4; (d) 1, 7

This question examined the theory of logarithms, functions and exponentials.

Among the small number of candidates there were a few good attempts at Part (a). However, the term 2^{2-x} was not correctly interpreted in the majority of cases.

In Part (b)(i), the notion of one-on-one functions was not properly understood.

There were a few encouraging attempts in Part (b)(ii) but poor algebraic manipulation spoiled some of the efforts at completing the solutions correctly.

Poor or inappropriate substitution was evidenced in the few attempts at Part (c). More practice is recommended in this area.

Solutions:

(a) $x = 0, x = 2$ (b) (ii) $x = -4$ (c) (i) \$35 million (ii) \$4 million

Section B

Module 2: Trigonometry and Plane Geometry

Question 2

Specific Objectives: (b) 1, 2, 3; Content: (a) (ii), (b) (iii)

This question examined the intersection of lines, equations of straight lines, circles and tangents to circles, using basic concepts of coordinate geometry. Vectors and trigonometric identities were also included.

There were some good attempts at Part (a) although some weaknesses were evident in finding the coordinates of the point P in Part (a)(i) and the point Q in Part (a)(ii).

Part (b) was well done by using established formulae for $2A$.

Not many candidates who attempted Part (b)(ii) completed it correctly. The main source of difficulty was the incorrect manipulation of trigonometric formulae.

Solutions:

(a) (i) $P \equiv (3, 1)$ (ii) $Q \equiv (1, -3)$ (iii) $4y = 3x - 5$

(b) (ii) $\theta = \pi/3,\ 2\pi/3,\ \pi$.

Section C

Module 3: Calculus 1

Question 3

Specific Objectives: (a) 3, 5, 7; (b) 8, 9 (i), 15, 16, 18; (c) 1, 5 (i), 3

This question examined limits, differentiation and the reverse process of integration and applications of differentiation to maxima/minima situations. There was also some mathematical modeling included.

In Part (a) several of the few candidates who made an attempt did not factorize $x^3 - 4x$ correctly, the consequence of which was an incorrect limit.

Some candidates had difficulty differentiating $\frac{x}{3x+4}$ as a quotient in Part (b)(i). However, a few candidates did it quite competently as the product $x\,(3 + 4x)^{-1}$.

A few candidates saw the connection between Part (b)(ii) and Part (b)(i). Most of those who were able to make the connection were able to complete the solution competently.

There were some good attempts at Part (c)(i).

Not many of the candidates who did Part (c)(i) were able to complete part (c)(ii).

Solutions:

(a) 8

(b) (i) $\frac{4}{(3x+4)^2}$ (ii) $\frac{4x}{3x+4}$ + constant

(c) (ii) S_{min} at $r = \left(\frac{5}{\pi}\right)^{\frac{1}{3}}$.

UNIT 2

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed fairly well with a mean score of 60.73 per cent and a standard deviation of 9.29.

Paper 02 – Structured Questions

Section A

Module 1: Calculus II

<u>Question 1</u>

Specific Objectives: (b) 1, 3, 4, 5

This question examined implicit differentiation, differentiation of combinations of polynomials, exponentials, trigonometric functions, application of the chain rule to obtain the tangent of a curve given by its parametric equations, and the second derivative.

Part (a)(i), was well done by the majority of candidates. Full marks were obtained by almost all candidates. Common errors included incorrect transposition of $\frac{dy}{dx} = \ldots$

Full marks were obtained for Part (a)(ii) by approximately 99 per cent of the candidates.

In Part (a)(iii), few instances of errors in using the concept of differentiation of composite functions were seen. Common errors included $\frac{d}{dx}\cos x = \sin x$. Generally, most candidates obtained full marks for this part of the question.

For Part (b)(i), a small percentage of candidates had difficulty applying the concept of differentiation of the composite function $\sin\frac{1}{x}$, particularly $\frac{d}{dx}\left(\frac{1}{x}\right)$, although, the correct application of the product rule was applied. However, only a small percentage of candidates were unable to obtain full marks for this part of the question.

Generally, Part (b)(ii) was well done. A very small percentage of candidates was unable to apply the concept of implicit differentiation correctly, with the result that they were unable to show the final answer as required.

Part (c)(i) was well done. Some arithmetic errors were made in substituting for $t = 4$ resulting in the incorrect value of the gradient of the tangent. However, candidates were able to get follow through marks for Part (c)(ii).

Part (c)(ii) was well done. Candidates who made errors calculating the correct gradient in Part (c)(i) were not penalized having earned follow through marks.

Solutions:

(a) (i) $\dfrac{dy}{dx} = \dfrac{1-x}{1+y}$.

(ii) $\dfrac{dy}{dx} = (-\sin x)e^{\cos x}$

(iii) $\dfrac{dy}{dx} = 8 \sin 16x - 6 \sin 12x$

(c) (i) $\left(\dfrac{dy}{dx}\right)_{t=4} = \dfrac{15}{4}$

(ii) $15x - 4y = 12$

<u>Question 2</u>

Specific Objectives: (c) 1, 3, 4, 8, 10

This question required candidates to derive a reduction formula and use it for a partial expansion of the product of an exponential function of *x* and the derived reduction formula; derive partial fractions and the integration of a rational function involving a trigonometric substitution and an inverse trigonometric function.

Approximately 20 per cent of the candidates were unable to obtain the correct answers to Part (a)(i). Many candidates could not deduce that $\int_0^0 F_n\,(0)\,dx = 0$. Some difficulty was also experienced in evaluating $\int_0^x F_0\,(x)\,dx$ correctly, particularly using the limits of integration, obtaining $e^{-x} - 1$ instead of $1 - e^{-x}$.

In Part (a)(ii), poor algebraic skills resulted in many candidates being unable to complete integration by parts and to show the correct reduction formula for $F_n(x)$. In particular, some candidates were unable to simplify $\dfrac{n}{n!} \equiv \dfrac{n}{n\,(n-1)!} = \dfrac{1}{(n-1)!}$ to enable the expression of $F_{n-1}\,(x)$.

Very few candidates were able to complete Part (a)(iii), having failed to determine $F_0(x)$ and $F_n\,(0)$ correctly. Partially correct answers were facilitated using the given result in Part (a)(ii).

In Parts (b)(i)(ii), there was evidence of candidates applying the concepts of repeated factors and the form of a linear numerator for a quadratic factor to find the partial fractions required. Some candidates were able to find the required partial fractions and proceeded to integrate the resulting rational functions. A few candidates successfully completed that part of the integration which involved an inverse trigonometric function. *This part of the question was zero-weighted and adjustments were made to the final marks so that candidates were not disadvantaged.*

Solutions:

(a) (i) $F_n\,(0) = 0$, $F_0\,(x) = 1 - e^{-x}$

(b) (i) $\frac{2x^2+3}{(x^2+1)} \equiv \frac{2}{x^2+1} + \frac{1}{(x^2+1)^2}$

(ii)

$$\int \frac{2x^2+3}{(x^2\ +1)^2}dx = \frac{5}{2}\ tan^{-1}(x) + \frac{x}{2(x^{2+1)}} + \ constant$$

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (a) 5; (b) 2, 4, 5

This question examined candidates' abilities to establish the properties of a sequence by applying Mathematical Induction; expand $(1 + ax)^n$, for $n = -1$; identify that a given series follows an arithmetic progression. Overall performance on this question was unsatisfactory.

A small percentage of candidates obtained full marks for Part (a)(ii). Generally, candidates seemed unfamiliar with proof by induction of sequences. Many of them knew that they had to prove the assertion for $n = 1$ and to proceed to taking arbitrary $k + 1$ for $n + 1$. However, having obtained $x_{k+2} = x_{k+1}^2 x + \frac{1}{4}$, they could not proceed further. Some candidates lost marks by using strict equality signs, ignoring the restriction $x_n < \frac{1}{2}$.

Only a few candidates were successful in obtaining full marks on Part (a)(ii). Those who were able to express $x_{n+1} - x_n$ as a perfect square made progress to completely solve the problem.

Part (b)(i) was well done. The majority of candidates demonstrated a sound knowledge of partial fractions and were able to obtain a correct solution. Those candidates who did not secure the full three marks were mainly faulted by arithmetic and algebraic errors.

Part (b)(ii) was well done by approximately half of the candidates. Generally, candidates used two different methods to solve this problem, namely, the binomial and Maclaurin's expansions. Those who failed to secure the full four marks committed a range of arithmetic and algebraic errors.

Part (b)(ii)a) was poorly done. More than 50 per cent of the candidates merely stated the ranges $|x| < 1$ and $|x| < \frac{1}{2}$ without proceeding to the correct answer $\left(-\frac{1}{2} < x < \frac{1}{2}\right)$.

Part (b)(iii)b) challenged the majority of the candidates' including those who successfully completed the previous parts of the question. They could not make the link to the earlier parts of the question. A significant number of candidates did not respond to this part of the question.

Candidates responded well to Part (b)(iv). The concepts of the difference between S_{n+1} and S_n resulting in the n^{th} term and subsequently $T_n - T_{n-1} = d$ were known to most candidates. However, poor algebraic manipulations resulted in candidates' unsuccessful efforts to prove the required solution

Solutions:

(b) (i) $A = B = 1$

(ii) $1 + x + x^2 + x^3$; $1 + 2x + 4x^2 + 8x^3$

(iii) a) $-\frac{1}{2} < x < \frac{1}{2}$

b) $1 + 2^n$

(iv) $u_n = S_n - S_{n-1} = 6n - 7$; $d = u_n - u_{n-1} = 6$

Question 4

Specific Objectives: (b) 9, 11, 12, 13

This question examined candidates' ability to manipulate a geometric progression and determine the first term and common ratio; obtain a series expansion of a fraction involving a denominator of $e^x + e^{-x}$; find the sum and limit of a finite series using the method of differences. Overall, candidates' performance on this question was very unsatisfactory. Approximately 40 per cent of the candidates either offered no responses or scored no marks.

For Part (a)(i) a large percentage of the candidates obtained 2 of the 4 marks available by establishing the equations $a + ar + ar^2 = \frac{26}{3}$ and $a^3r^3 = 8$. Some candidates used the equations $\frac{a(1-r^3)}{1-r} = \frac{26}{3}$ and $a^3r^3 = 8$. Poor algebraic skills prevented the majority of these candidates from eliminating a and thus finding the required equation in terms of r.

For Part (a)(ii) a), a significant number of candidates could not simplify the equation given in Part (a)(i) to solve for r. A few of the candidates who found two values for r, ($r = 3$) or ($r = \frac{1}{3}$), did not follow the constraint $0 < r < 1$.

Candidates who did not use the constraint for r abandoned Part (a)(ii)b).

A small percentage of candidates obtained full marks for this Part (a)(ii)(c).

Part (b) required candidates to find a series expansion for a fraction involving the denominator $e^x + e^{-x}$. Although a few candidates were able to recall and use the expansion for e^x and e^{-x}, the majority of them employed Maclaurin's theorem for the expansion of the denominator without success. Successive differentiation proved problematic and the exercise was abandoned. Approximately 20 per cent of the candidates were able to obtain the result $\frac{2}{e^x + e^{-x}} = \frac{1}{1 + \frac{x^2}{2} + \frac{x^4}{24}} + \ldots$ The required expansion of the denominator, using the binomial expansion with $\left(\frac{x^2}{2} + \frac{x^4}{24}\right) = X$ in the expansion of $(1 + X)^{-1}$ was beyond the ability of a majority of the candidates.

Part (c)(i) was well done.

In Part (c)(ii), a large percentage of the candidates incorrectly found

$3\sum_{r=1}^{n}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$ giving the

resulting incorrect sum of $3\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$.

Following the error made in Part (c) (ii), those candidates obtained the wrong limiting sum of the series in Part (c)(iii).

Solutions:

(a) (ii) a) $r = \frac{1}{3}$

b) $a = 6$

c) $S_\infty = 9$

(b) $1 - \frac{x^2}{2} + \frac{5x^4}{24} + \ldots$

(c) (i) $\frac{2}{r(r+1)(r+2)}$

(ii) $\frac{3}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$

(iii) $S_\infty = \frac{3}{4}$

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 5

Specific Objectives: (a) 2, 4, 7; (c) 2, 3, 5

This question examined the concepts of arrangements of n distinct objects; the selection of r distinct objects from n distinct objects; the probability of an event occurring; the complex roots of a quadratic equation and the square roots of a complex number.

The overall performance by most of the candidates was satisfactory in parts of the question. As evident in previous problems which required algebraic manipulation, most candidates were at a severe disadvantage in using algebra to show required results. Algebraic simplification continues to prove problematic to most candidates.

Approximately 75 per cent of the candidates was able to obtain full marks for Part (a)(i). Some candidates substituted numbers to show the required result.

For Part (a)(ii), the algebra required to show the required result was beyond most of the candidates. Half-hearted attempts were made to simplify the initial definitions of the left hand and right hand sides of the equations. Many candidates resorted to substituting numbers to balance the equation.

In Part (a)(iii), candidates simply used the numbers given in the equations and calculated the arithmetic results. No attempts were made to use the results of Parts (a)(i) and (ii).

Part (b)(i) was well done. Some arithmetic errors resulted in some candidates being unable to obtain full marks.

Part (b)(ii) was well done by approximately half of the candidates. Arithmetic errors and some loss of reasoning resulted in many candidates not obtaining full marks.

In Part (c)(i) a), a significant number of candidates found the square roots of $-2i$ using the approach $(x + iy)^2 = -2i$. This resulted in some of these candidates making algebraic errors and subsequently obtaining incorrect roots. Some candidates misunderstood the question and attempted to show that $(1 - i) \times (1 + i) = -2i$.

For Part (c) (i) candidates who found the square roots of $-2i$ using the method described in Part (c)(i) a) were able to get the correct answer. There was no evidence that candidates simply applied the concept that the square root of a complex number $(x + iy)$ is $\pm (a + ib)$.

Part (c)(ii), most candidates who used the quadratic formula to solve this equation could not establish the link with $b^2 - 4ac = -2i$ and use the results of Part (c)(i). Very few candidates were able to obtain full marks for this part of the question.

Solutions:

(b) (i) 96

(ii) $\frac{3}{8}$

(c) (i) b) $-(1 - i)$

(ii) $2 + 2i$, $1 + 3i$

Question 6

Specific Objectives: (b) 1, 2, 6, 8

This question examined matrices and systems of linear equations. Particularly tested were operations with conformable matrices and manipulation of matrices using their properties; evaluation of determinants for 3×3 matrices; solutions of a consistent system; solution of a 3×3 system of linear equations.

Overall this question was well done. A notable number of candidates obtained marks ranging from 15 to 20.

Part (a)(i) was well done and, arithmetic errors apart, candidates obtained full marks. Candidates generally answered Parts (a), (b) and (c) of the question by making the required changes and using the algorithmic approach. No evidence was seen that any candidate used the properties of matrices to obtain their answers.

All parts of Part (b) were well done and full marks were obtained by almost all candidates.

All parts of Part (c) were well done and full marks were obtained by almost all candidates. In Part (c)(iv), some candidates having shown that (1, 1, 1) was a solution for the system of equations, were unable to find the general solution for the system of equations, despite some attempts. It was not recognized that the system represented parallel planes thus resulting in infinitely many solutions.

Solutions:

(a) (i) $|\mathbf{A}| = 5$

(ii) a) $|\mathbf{B}| = |\mathbf{A}| = 5$, The value of a determinant is unaltered when the columns and rows are completely interchanged.

b) $|\mathbf{C}| = |\mathbf{A}| = 5$, The value of the determinant is not changed if any row (column) is added or subtracted from any other row (column).

c) $|\mathbf{D}| = 5^3\,|\mathbf{A}| = 625$. If all rows are multiplied by λ, the determinant is multiplied by λ^3.

(b) (i) $\mathbf{AM} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = 5\mathbf{I}$

(ii) $\mathbf{A}^{-1} = \frac{1}{5}\ \mathbf{M} = \frac{1}{5}\begin{pmatrix} 12 & -1 & 5 \\ 2 & -1 & 0 \\ -9 & 2 & -5 \end{pmatrix}$

(c) (i) $$\underset{\mathbf{A}}{\begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 2 \\ -1 & -3 & -2 \end{pmatrix}} \underset{\boldsymbol{x}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{\mathbf{b}}{\begin{pmatrix} 5 \\ -10 \\ -11 \end{pmatrix}}$$

(ii) $\mathbf{A}^{-1}\,\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} 5 \\ -10 \\ -11 \end{pmatrix} \Rightarrow \boldsymbol{x} = \mathbf{A}^{-1}\ \mathbf{b}$

(iii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

(iv) b) (x, y, z) = λ(1, 0, –1) + μ(0, 1, –1) + (1, 1, 1)

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Calculus II

<u>Question 1</u>

Specific Objectives: (b) 3; (c) 12

This question examined differentiation of parametric equations, rate(s) of increase/decrease and the general solution of a second order differential equation.

Performance, overall, was generally poor. The majority of candidates seemed to be unprepared for this question.

In Part (a)(i), some measure of successful differentiation of y and x with respect to t was seen. However, candidates could not determine $\frac{dy}{dx}$ in terms of t and to proceeded to equate $\frac{dy}{dx} = \tan\,\theta$.

Candidates did not respond to Part (a)(ii), having not completed Part (a)(i).

Some attempts were made to answer Part (a)(iii). Problems encountered by candidates involved incorrect transpositions of x and y and identifying with the correct trigonometric identities.

Part (b) did not elicit many responses. Those candidates who attempted to solve the auxiliary equation used the wrong roots to express the complementary function. The solution for the particular integral was beyond the abilities of almost all the candidates.

Solutions:

(a) (i) rate of decrease = $\frac{24}{31}$

(ii) radians per second

(iii) $\left(\frac{x-4}{3}\right)^2+\left(\frac{y-5}{2}\right)^2=1$

(b) $y = Ae^{-x} + \mathrm{B}e^{4x} - 2x^2 + 3x - \frac{13}{4}$

Section B

Module 2: Sequences, Series and Approximations

<u>Question 2</u>

Specific Objectives: (b) 3, 13; (e) 1, 2

This question examined the existence of a real root in a given interval, finding an approximation using a given iterative method, expansion of a logarithmic and exponential function using Maclaurin's theorem and determining the n^{th} term of a sequence of terms.

It was evident that candidates were underprepared for most of this question. Overall, performance was poor.

For Part (a)(i), most candidates were able to establish a change of sign over the given interval. Without stating continuity of the function over this interval, candidates concluded that a real root existed over the interval.

Part (a)(ii) was done satisfactorily.

Some candidates showed some understanding of Maclaurin's theorem and were able to obtain full marks for Part (b)(i).

There were no favourable responses to Part (b)(ii).

Part (c)(i) was well done.

In Part (c)(ii), candidates were not able to make a deduction to obtain the equation.

There were no meaningful responses to Part (i)(iii). Candidates appeared to be guessing about a suitable approach to this part of the question.

Solutions:

(a) (ii) 0.904

(b) (i) $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots \ (-1 \le x < 1)$

$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \ldots$ for all real x

(c) (i) $p_1 = 1000(1.20) - 100 \quad p_2 = 1.20[1000(1.20) - 100] - 100$

(ii) $p_{n+1} = (1.20)p_n - 100$

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 3

Specific Objectives: (b) 1, 8; (c) 4, 6, 7

This question examined simple operations on a conformable matrix, solutions of a system of equations and operations on a complex number. Overall, performance was poor.

Most candidates were able to obtain marks for Part (a)(i) of the question. The common problems evidenced were arithmetic and in some cases failing to identify I (identity matrix).

In Part (a)(ii), candidates were unable to deduce $\mathbf{A}^{-1}$ in the given form since they could not identify the identity matrix.

For Part (a)(iii), those candidates who attempted to find the solution of the system of equations completely ignored the link from Part (a)(ii). As a result, arithmetic errors inhibited their ability to obtain the correct solutions.

In Part (b), candidates demonstrated an understanding of the method(s) to be used. However, poor algebra resulted in incorrect answers.

Most candidates did not attempt Part (c). The few candidates who attempted it did not show a fair understanding of the modulus and argument of a complex number.

Solutions:

(a) (iii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

(b) $z + \frac{1}{z} = \frac{1}{10}\ (7 + 9\mathrm{i})$

(c) $r = \sqrt{\frac{13}{10}},\ \tan\ \theta = \frac{9}{7}$

Paper 031 – School-Based Assessment (SBA)

This year, 174 Unit 1 and 145 Unit 2 SBAs were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases, neither question papers with solutions nor mark schemes were submitted. Mark schemes for questions and their subsequent parts were not broken down into unitary marks. In an increasing number of cases, the marks awarded were either too few or far too many for the skills tested. (Example: an entire SBA module test was worth 20 marks and in another case on one test paper a simple probability question was awarded 27 marks and a matrix question was awarded 24 marks).

In Unit 1, the majority of the samples submitted were not of the required standard. Teachers **must** pay particular attention to the following guidelines and comments to ensure effective and reliable submission of SBAs.

The SBA is comprised of three module tests. The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of teachers' assessments — consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

FURTHER COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.

2. Untidy 'cut and paste' presentations with varying font sizes were commonplace.

3. Teachers are reminded that the CAPE past examination papers should be used *only* as a guide.

4. The stipulated time for module tests *(*1–1 hour 30 minutes) must be strictly adhered to as students may be at an undue disadvantage when module tests are too extensive or too insufficient.

5. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling *must* be included.

6. Cases were noted where teachers were unfamiliar with recent syllabus changes that is,

   - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.

   - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations were removed from the Unit 1 CAPE syllabus (2008).

7. The moderation process relies on validity of the teachers' assessments. There were few cases where students' solutions were *replicas* of the teachers' solutions — some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. The SBA must be administered under examination conditions at the school. It is not to be done as a homework assignment or research project.

8. Teachers must present evidence of having marked each individual question on students' scripts before the marks scored out of the possible total is calculated at the top of the script. The corresponding whole number score out of 20 must be placed at the front of students' scripts.

Module Tests

- Design a separate test for each module. The module test must focus on objectives from that module.

- In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.

- A sample of five students will form the sample for the centre. If there are less than five students, *all* scripts will form the sample for the centre.

- In 2011, the format of the SBA remains unchanged.

To enhance the quality of the design of module tests, the validity of teachers' assessments and the validity of the moderation process, the SBA guidelines are listed below for emphasis.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. **COVER PAGE TO ACCOMPANY EACH MODULE TEST**

   The following information is required on the cover of each module test.

   - Name of school and territory, name of teacher, centre number
   - Unit number and module number
   - Date and duration of module test
   - Clear instructions to candidates
   - Total marks allocated for module test
   - Sub-marks and total marks for each question **must** be clearly indicated

2. **COVERAGE OF THE SYLLABUS CONTENT**

   - The number of questions in each module test must be appropriate for the stipulated time.
   - CAPE past examination papers should be used as a guide ONLY.
   - Duplication of specific objectives and questions must be avoided.
   - Specific objectives tested must be from the relevant unit of the syllabus.

3. **MARK SCHEME**

   - Detailed mark schemes MUST be submitted, that is, one mark should be allocated per skill (not 2, 3, 4 marks per skill)
   - Fractional or decimal marks MUST NOT be awarded. (that is, do not allocate $\frac{1}{2}$ marks).
   - A student's marks MUST be entered on the front page of the student's script.
   - Hand written mark schemes MUST be NEAT and LEGIBLE. The **unitary** marks MUST be written on the right side of the page.
   - **Diagrams MUST be neatly drawn with geometrical/mathematical instruments**.

   PRESENATION OF SAMPLE

   - Students' responses MUST be written on letter sized paper (8 ½ x 11) or A4 (8.27 x 11.69).
   - Question numbers MUST be written clearly in the left hand margin.
   - The total marks for EACH QUESTION on students' scripts MUST be clearly written in the left or right margin.
   - ONLY ORIGINAL students' scripts MUST be sent for moderation.
   - Photocopied scripts WILL NOT BE ACCEPTED.
   - Typed module tests MUST be NEAT and LEGIBLE.
   - The following are required for each Module test:

     - ❖ A question paper
     - ❖ Detailed solutions with detailed unitary mark schemes.
     - ❖ The question paper, detailed solutions, mark schemes and five students' samples should be batched together for each module.

   - Marks recorded on PMath–3 and PMath2–3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded. The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no. 6.)

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2012

PURE MATHEMATICS

GENERAL COMMENTS

In 2012, approximately 5500 and 2800 candidates wrote the Units 1 and 2 examinations respectively. Performance continued in the usual pattern across the total range of candidates with some candidates obtaining excellent grades, while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

The overall performance in Unit 1 was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Functions and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Modulus, Indices and Logarithms. Other aspects that need attention are manipulation of simple algebraic expressions, substitution and pattern recognition as effective tools in problem solving.

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by some candidates.

Candidates continue to show marked weaknesses in algebraic manipulation. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrated a favour for problem solving by using memorized formulae.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 29.5 and a standard deviation of 20.55.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (a) 5; (b) 1, 2, 3, 4; (g) 1, 4

The topics covered in this question were the Remainder and Factor Theorems, Operations on Surds and Modulus Inequalities.

The majority of candidates showed good performance in Parts (a) (i) and (ii). Some instances of errors in substitution were observed. These included $f(1) = -6$, $f(-1) = 6$ and $f(-1) = 0$. A small number of candidates failed to complete the factorization of $f(x)$.

Part (b) was generally well done. Some computational errors included incorrectly expanding $\left(\sqrt{x} + \sqrt{y}\right)^2$ which resulted in the loss of marks by some candidates, failure to form two equations in x and y and failure to equate the terms to solve x and y. General algebraic weaknesses were also evident.

The majority of candidates removed the modulus in Part (c) (i) by squaring both sides of the inequality obtaining a quadratic inequality in x. Some candidates were able to use a graphical approach to identify the range of values of x for which the inequality was satisfied. Common errors included, (i) stating the critical values of $|3x - 7| = 5$ and $(3x - 2)(x - 4) \leq 0 \Rightarrow x \leq \frac{2}{3}, x \leq 4.$

In Part (c) (ii), a few candidates used the quadratic inequality obtained by squaring both sides of the inequality $|3x - 7| + 5 \leq 0$ and used the characteristic of the discriminant, $b^2 - 4ac < 0$, to prove the desired result. A number of candidates deduced that $|3x - 7| \geq 0$ for all real values of x.

Solutions

(a) (i) $p = -7, q = 1$ (ii) $\mathrm{f}(x) = (x - 1)(x + 2)(2x + 5)$

(b) $x = 10, y = 6$ or $x = 6, y = 10$

(c) (i) $\frac{2}{3} \leq x \leq 4$

Question 2

Specific Objectives: (c) 1, 3; (d) 1, 7; (f) 3

This question tested applications of a composite quadratic function, solution of the resulting quartic equation equal to a linear equation, the relationships between the sum and product of the roots of a quadratic equation, applications of the laws of logarithms in base 10 to simplify a sum of quotients without the use of calculators or tables and the finite sum of a quotient of logarithms in base 10.

Part (a) (i) required a composite function ff (x). This was generally well done. However, many candidates failed to simplify the resulting expression. This resulted in a number of candidates being unable to successfully complete Part (a) (ii). Poor algebraic skills did not allow for the correct solution of $x^4 - 7x^2 - 6x = 0$. Some candidates seemed perplexed that the equation did not contain a term in x^3.

Parts (b) (i) and (ii) were generally well done. A small number of candidates were unable to state $\alpha^2 + \beta^2$ correctly in terms of $\alpha + \beta$ and $\alpha\beta$. Performance on Part (b) (iii) was generally good with the exception of a small number of candidates who found the algebra for $\frac{2}{\alpha^2} + \frac{2}{\beta^2}$ expressed in terms of $\alpha + \beta$ and $\alpha\beta$ beyond their capabilities. Too many candidates did not write the required quadratic equation but instead stated the expression $x^2 - 2x + 64$.

Part (c) (i) was generally well done. Candidates recognized the required laws of logarithms to use and were able to simplify the logarithmic expression.

In Part (c) (ii), a significant number of candidates demonstrated their inability to use the sigma notation. The answer $\log_{10}\left(\frac{99}{100}\right)$ was seen in many cases. Some candidates were able to express $\sum_{r=1}^{99}\left(\frac{r}{r+1}\right)$ as $\sum_{r=1}^{99}\left(\log_{10} r - \log_{10}(r+1)\right)$ but could not apply the concept of the sigma notation. Generally this part of the question had limited successes.

Solutions

(a) (i) $\text{ff}(x) = x^4 - 6x^2 + 6$ (ii) $x = -2, -1, 0, 3$

(b) (i) $\alpha + \beta = \frac{3}{4},\ \alpha\beta = \frac{1}{4}$ (ii) $\alpha^2 + \beta^2 = \frac{1}{16}$

(iii) $x^2 - 2x + 64 = 0$

(c) (i) -1 (ii) -2

Section B

Module 2: Trigonometry and Plane Geometry

<u>Question 3</u>

Specific Objectives: (a) 4, 5, 9, 10, 12, 13

This question tested trigonometric identities, compound and multiple angle formulae, the factor formulae and solutions of trigonometric equations, use of trigonometric identities, compound and multiple angles formulae and the factor formulae.

Most candidates attempted Part (a) (i). Many of them failed to show the desired result due to poor manipulation of the identities given and subsequent simplification. A number of candidates failed to deduce that the factor formula was required to show the desired result in Part (a) (ii). Those who attempted other approaches were not able to complete the result. In Part (a) (iii), many candidates did not heed the directive ‘Hence…’ thus enabling them to use the result at Part (a) (ii). The factor formula could have been easily used heeding the directive ‘or otherwise’. A small number of candidates attempted the expansions of sin 6θ and sin 2θ in terms of sin θ but experienced algebraic difficulties to complete the solution.

In Part (b), the majority of candidates recognized the need to substitute $\frac{\cos^2\theta}{\sin^2\theta}$ for $\cot^2\theta$. In some cases candidates divided the equation $2\cos^2\theta + \cos\theta\sin^2\theta = 0$ by $\cos\theta$ thus losing the result $\cos\theta = 0$. A number of candidates solved the quadratic equation in $\cos\theta$ to obtain $\cos\theta = 1 \pm \sqrt{2}$. However, candidates were penalized for stating $\cos\theta = 1 + \sqrt{2}$. Generally, only a small number of candidates were able to successfully complete this part of the question.

Solutions

(a) (iii) $\theta = \frac{\pi}{8}, \frac{3\pi}{8}$

(b) $\cos\theta = 0 \quad \cos\theta = 1 - \sqrt{2}$.

Question 4

Specific Objectives: (b) 8, 9; (c) 1, 3, 7, 9, 10

This question tested candidates' ability to give the Cartesian equation of a curve defined in trigonometric parameters, the intersection of a curve with a straight line, expressions of coordinate points in vector form and finding the angle between two vectors using the dot product method.

Part (a) (i) was generally well done by the majority of candidates. There were a few cases of candidates being unable to use the appropriate trigonometric identities to eliminate the parameter. Algebraic errors were evident in Part (a) (ii) where many candidates wrote $\left(\sqrt{10x}\right)^2 = 10x^2$ and $\left(\sqrt{10x}\right)^2 = \sqrt{10}x$. Attempts at subsequent solution of $\frac{y^2}{9} - \frac{x^2}{9} - 1 = \sqrt{10x}$ resulted in confusion and were often abandoned. The question required candidates to find the points of intersection. Many candidates solved the quadratic equation in x and stopped short of finding the corresponding values of y. In addition, a number of candidates did not state the coordinates of the points of intersection.

The majority of candidates successfully completed Parts (b) (i) to (iv). Some arithmetic errors were seen in computing the angle in degrees.

Solutions

(a) (i) $\frac{y^2}{9} - \frac{x^2}{9} = 1 \Rightarrow y^2 = x^2 + 9$ (ii) $(1, \sqrt{10}), (9, 3\sqrt{10})$

(b) (i) $p = -3i + 4j, \; q = -i + 6j$ (ii) $-2i - 2j$

(iii) 27 (iv) 27.41^0

Section C

Module 3: Calculus 1

Question 5

Specific Objectives: (a) 3–5, 7, 10; (b) 5, 11

This question tested the concepts of limit of a function, limit theorems, differentiation of simple functions, continuity and discontinuity and rate of change.

For Part (a) (i), the majority of candidates understood that it was necessary to show that for discontinuity the denominator must be zero. A few candidates solved $x^2 - 4 = 0$ as $x = 2$ only. Generally this part of the question was well done. The majority of candidates had no difficulties successfully completing Part (a) (ii) by using the result at Part (a) (i) and making the relevant cancellation. A few candidates were unable to factorize $x^3 + 8$ as a sum of cubes. The composite function $\frac{2x^3 + 4x}{\sin 2x}$ in Part (a) (iii) seemed to have left candidates in a state of confusion, apparently being drilled in limits involving either rational functions involving factors that cancel or trigonometric functions that are variations of $\frac{\sin x}{x}$. Poor algebraic skills prevented many candidates from simplifying the composite function and using the theorems of limits of sums, differences and quotients. A small number of candidates used L'Hopital's rule and successfully found the correct limit. In Part (b) (i) a), the majority of candidates found the correct limit as 2. It was relatively simple to find the value of p for continuity having found the correct limit for $x > 1$ in b). The answer to Part (b) (ii) was merely deduced from Part (b) (i) a).

The majority of candidates was able to substitute $t = 1$ in the equation for M in order to find the first equation in terms of u and v. A number of candidates made errors in differentiating $\frac{v}{t^2}$ correctly and as a result found incorrect values for u and v. Some candidates did not deduce that a rate of change meant to find $\frac{dM}{dt}$. Generally, a significant number of candidates failed to find the correct values of u and v.

Solutions

(a) (i) $x = \pm 2$ (ii) -3

(iii) 2

(b) (i) a) 2 b) $p = -2$

(ii) $\mathrm{f}(1) = 2$

(c) $u = 2$ $v = -3$

<u>Question 6</u>

Specific Objectives: (b) 5, 10, 13 – 16; (c) 8 (i)

This question tested first and second derivatives using the chain rule and the product/quotient rules, evaluation of a definite integral, location of stationary points, nature of stationary points and sketching a cubic curve.

Part (a) (i) was satisfactorily done. However, some candidates were not able to show the desired result because of poor algebraic skills and subsequent simplification. Due to poor use of the unsimplified result of Part (a) (i), several candidates were not able to show the required result for Part (b) (ii). The continued applications of the chain rule and the product/quotient were beyond the majority of candidates. A very small number of knowledgeable candidates used implicit differentiation and successfully completed this part of the question.

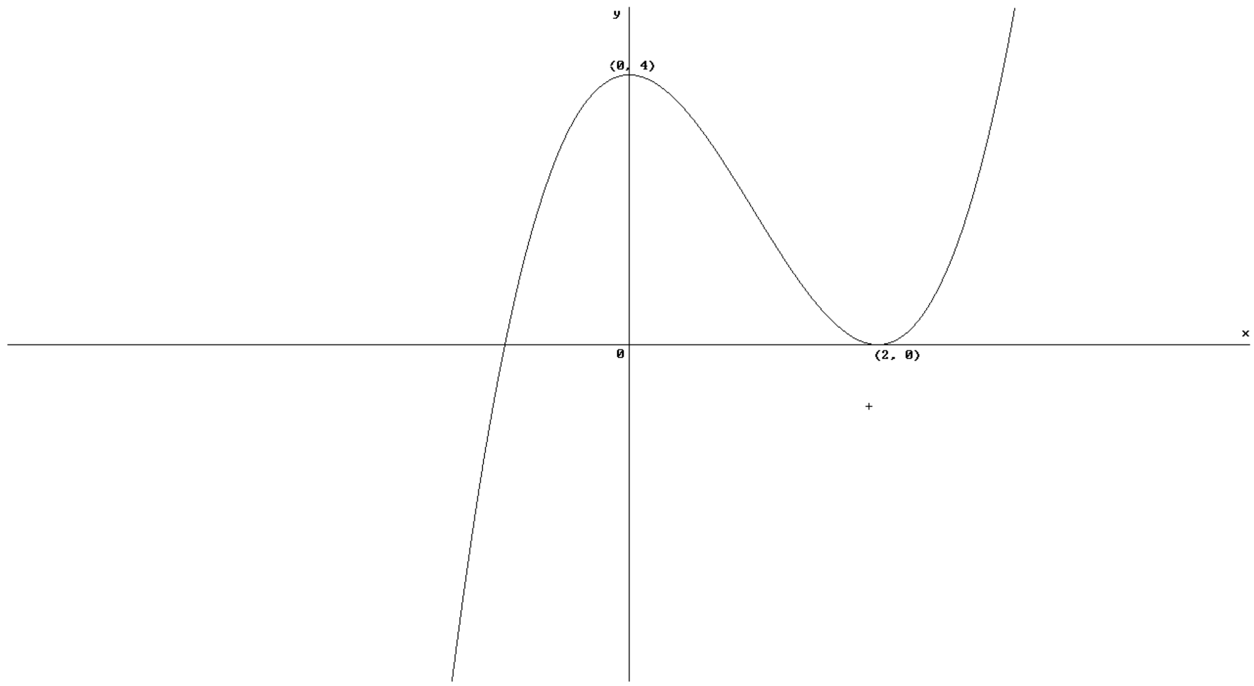
Part (b) (i) required a definite integration. The majority of candidates performed well. Successful solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ in (ii) assisted the majority of candidates in being awarded maximum marks. Too many candidates substituted the values of x found for $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ in the equation for $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 6x$ to find the y-ordinates of the stationary points. Most candidates knew the concept of using the sign of the second derivative to determine the nature of the stationary points in Part (b) (iii). A number of candidates failed to factorize $x^3 - 3x^2 + 4 = 0$ correctly. Consequently, those candidates were unable to find the correct x-intercepts of the curve in Part (b) (iv). Following the failures at Parts (b) (ii) and (iv), a number of candidates were unable to sketch the curve C showing the critical points correctly.

Solutions

(b) (i) $y = x^3 - 3x^2 + 4$ (ii) (0, 4) and (2, 0)

(iii) $(0, 4)_{\text{maximum}}$ $(2, 0)_{\text{minimum}}$ (iv) $(-1, 0)$, $(2, 0)$

(v) see plot



Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 8; (c) 1, 3, 5; (f) 4, 5 (ii)

This question tested the roots of a cubic equation, mathematical induction for divisibility, applications of the laws of indices and the laws of logarithms and the solution of a logarithmic equation involving a change of base.

Parts (a) (i) and (ii) were satisfactorily done. Candidates were vague on Part (b). Apart from proving that the statement is true for $n = 1$ and hence the assumption that the statement is also true for $n = k$, $k > 1$, poor algebraic skills prevented a significant number of candidates from proving that the statement is true for $n = k + 1$. In the majority of cases, this part of the question was badly done. The majority of candidates was unable to express a logarithm in index form and use that form to change the base of a logarithm, thus (c) (i) was poorly done. Consequently, Part (c) (ii) was not done by the majority of candidates.

Solutions:

(a) (i) $\alpha = -2$ (ii) $p = 28$

(c) (ii) $x = 2$ or $x = 4$

Section B

Module 2: Trigonometry and Plane Geometry

Question 2

Specific Objectives: (a) 11, 14; (b) 2, 5, 6, 7; (c) 4, 5, 8

This question tested the properties of a circle, a tangent to a circle at a given point, intersection of a curve with a straight line, expression for $a \cos \theta + b \sin \theta = r \cos (\theta - \alpha)$, maximum value of $a \cos \theta + b \sin \theta$, unit vector and a displacement vector.

Candidates performed with limited successes in Parts (a) (i) to (iii). Algebraic and arithmetic errors resulted in loss of marks by some candidates.

Most candidates demonstrated a lack of knowledge of the trigonometric forms in Parts (b) (i). Consequently, performance on Parts (b) (ii) was poor.

Most candidates in attempting Part (c) seemed unaware that it was required to find the unit vector to $\overrightarrow{\mathbf{PQ}}$ before multiplying $\overrightarrow{\mathbf{OR}}$ by $\sqrt{5}$. Generally this part of the question was done poorly.

Solutions

(a) (i) centre (3, –1) radius = 5 units (ii) $\text{tangent}_{(7,\ 2)}$ $4x + 3y - 34 = 0$

(iii) Q (–1, –4)

(b) (i) $\mathrm{f}(\theta) = 6 \cos\left(\theta + 30^0\right)$ (ii) $\mathrm{f}(\theta)_{\text{maximum}} = 6$

(c) $a = \frac{1}{\sqrt{2}}$ $b = \frac{3}{\sqrt{2}}$

Section C

Module 3: Calculus 1

Question 3

Specific Objectives: (a) 3 to 6; (b) 1, 5, 7, 11

This question tested limits using limit theorems, gradient at a point and rate of change.

Overall, candidates performed satisfactorily on this question. More than 50 per cent of the candidates scored at least seven of the 20 available marks and more than 30 per cent scored at least 10 of the 20 available marks. Most candidates were able to use the algebraic expression given and to correctly substitute the given value of x to obtain the limit in Part (a) (i). Using the result from Part (a) (i) the majority of candidates was able to obtain the correct limit in Part (a) (ii).

Candidates demonstrated a fair level of understanding of a gradient function at a point. Good performance was shown in Part (b).

Part (c) posed severe challenges for many candidates. The algebraic expressions for V in terms of t only and V in terms of x only in Parts (b) (i) and (ii) respectively were poorly done by many candidates. The majority of them could not see a relationship between the height of the water and the radius at that instant.

Solutions

(a)(i) $\frac{1}{4}$ (ii) $\frac{1}{12}$

(b) 24

(c) (i) $V = 10t$ (ii) $V = \frac{1}{3}\pi x^3$

(iii) ≈ 0.24 cm/s

UNIT 2

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 32.11 and a standard deviation of 18.27.

Paper 02 – Structured Questions

Section A

Module 1: Calculus II

Question 1

Specific Objectives: (b) 1, 3 to 7

This question tested first and second derivatives of a product of a polynomial and an exponential function, stationary points, differentiation of parametric equations including an inverse trigonometric function, gradient at a point and equation of a tangent at a point to a curve defined by parametric equations.

In Part (a) (i) a), candidates demonstrated a sound understanding of the first and second derivatives using the product rule although many of the answers were left unsimplified.

Candidates were able to determine the values of x-coordinates correctly for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ respectively in Parts (a) (i) b) and c). Using the second derivative and the significance of the sign change, the majority of candidates was able to distinguish the maxima and minima points as required in Part (a) (ii). However, a significant number of candidates seemed not to know the conditions for points of inflection, using the properties of the second derivative to identify these points. This part of the question had very limited successes.

Candidates demonstrated a sound understanding of the chain rule to obtain the first derivative of parametric equations required in Part (b) (i). However, the majority of candidates opted to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ which involved differentiating an inverse trigonometric function along with differentiating a variable involving a fractional index. Many errors in the evaluation of this differential resulted in the incorrect expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. A number of candidates used the fact that $\sin^{-1}\left(\frac{1}{2}\right) = 30^0$ instead of $\frac{\pi}{6}$.

As a result of the inability by many candidates to complete Part (b) (i) successfully, it was not possible to derive the correct equation of the tangent as required in Part (b) (ii).

Solutions

(a) (i) a) $\dfrac{dy}{dx} = xe^x(2 + x)$ and $\dfrac{d^2y}{dx^2} = e^x\left(2 + 4x + x^2\right)$

b) $x = 0$ or $x = -2$ c) $x = -2 \pm \sqrt{2}$

(ii) $x = 0$ gives a relative minimum point, $x = -2$ gives a relative minimum point,

$x = -2 \pm \sqrt{2}$ gives relative inflection points

(b) (i) $\dfrac{dy}{dx} = 4(t-1)\sqrt{t(1-t)}$

(ii) $4y + 4x = \pi - 3$

Question 2

Specific Objectives: (c) 1 (iii), 3, 4, 6, 8, 10

This question tested partial fractions, indefinite integral using partial fractions, the reduction formula and definite integration of a trigonometric function.

Some candidates did not recognize the partial fractions of the form $\dfrac{A}{x-1} + \dfrac{Bx + C}{x^2 + 1}$ in Part (a) (i). However, this part of the question was generally satisfactorily done despite some arithmetic errors which resulted in the wrong values for the constants A, B and C.

In Part (a) (ii) many candidates did not express $\displaystyle\int\left(-\frac{1}{x-1} + \frac{2x-1}{x^2+1}\right)dx$ as $\displaystyle\int -\frac{1}{x-1}\,dx + \int \frac{2x}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx$ which would have facilitated simple integration. Approximately 20 per cent of the candidates evaluated $\displaystyle\int \frac{f'(x)}{f(x)}\,dx$ correctly and approximately 10 per cent of the candidates evaluated $\displaystyle\int \frac{1}{x^2+1}\,dx$ correctly. A number of candidates did not show evidence of good integration skills. A common error was the omission of the constant of integration for indefinite integrals.

There were no difficulties with Part (b) (i). However, Part (b) (ii) was very challenging to most candidates. Most candidates understood that application of integration by parts was required to find the required result. However, the sequence of the integration techniques and the resulting algebra were beyond the capabilities of most candidates. The candidates who attempted this part of the question failed to recognize and use the link at Part (b) (i). This part of the question was poorly done.

Given the substitution $m = 1$ allowed approximately 10 per cent of the candidates to show the desired result in Part (b) (iii). A number of candidates did not include the limits of integration for $-\cos x \cos 3x$.

Approximately 60 per cent of the candidates evaluated Part (b) (iv) correctly, without any reference to the preceding results.

Solutions

(a) (i) $-\dfrac{1}{x-1} + \dfrac{2x-1}{x^2+1}$ (ii) $\ln\left(\dfrac{x^2+1}{x-1}\right) - \tan^{-1}(x) + C$

(b) (iv) $\dfrac{1}{2}$

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (a) 2, (b) 1, 4, 6, 9, 13

This question tested geometric progression, sequences, proof by mathematical induction and Maclaurin's series expansion.

The majority of candidates did not find it difficult to complete Part (a) (i) correctly. A few candidates made errors evaluating indices.

Successful completion of Part (a) (ii) by those candidates who found the correct values for a and r followed easily from Part (a) (i). However, a number of candidates made errors in evaluating $177146 = \dfrac{2(3^n - 1)}{3-1}$. Common errors included $2\left(3^n - 1\right) = 6^n - 2$ or $2 \times 3^{n-1}$.

A number of candidates were unable to express the rth term of the sequence in Part (b) (i). Proof by mathematical induction was poorly done in Part (b) (ii) by the majority of candidates. A number of candidates showed some evidence of knowing how to begin the proof but lacked the algebraic skills and the sophistication necessary to complete the proof. Apart from proving the statement true for $n = 1$ and attempting to show that it is true for $n = k + 1$, the resulting algebraic substitutions were poorly done. As a result, a number of candidates were not able to complete the proof.

Some candidates who probably memorized Maclaurin's expansion for cos x simply substituted $2x$ for x and obtained the result. A number of candidates however used differentiation and Maclaurin's theorem to obtain the result for Part (c) (i).

Candidates who attempted to express $\sin^2 x$ in terms of $\cos 2x$ for Part (c) (ii) invariably used the wrong identity and could not obtain the expansion of $\sin^2 x$ correctly.

Solutions

(a) (i) $a = 2, \quad r = 3$ (ii) $n = 11$

(b) (i) $u_r = r(r+2),\ r \in \mathrm{N}$

(c) (i) $1 - 2x^2 + \frac{2}{3}x^4 + \ldots$ (ii) $x^2 - \frac{1}{3}x^4$

<u>Question 4</u>

Specific Objectives: (c) 1, 2, 3, 4; (e) 1, 4

This question tested the concepts of factorials, binomial expansion, location of roots and the Newton-Raphson iterative method.

Parts (a) (i) and (ii) were well done by the majority of candidates.

In Part (a) (iii), a small number of candidates used the complete expansion of $\left(x^2 - \frac{3}{x}\right)^8$ before extracting the term in x^4. Some arithmetic errors with the negative sign in the terms involving $\left(-\frac{3}{x}\right)^n$ were evident. Generally, this part of the question was well done, with the majority of candidates demonstrating a good understanding of the binomial theorem.

The majority of candidates who attempted Part (a) (iv) stopped at the expansion of $(1+x)^{2n}$ up to and including the 4th term. No candidate recognized that ${}^{2n}C_n$ is the coefficient of the term in x^n. Consequently, candidates could not proceed to make the link with Part (a) (ii) and hence were unable to show the desired result. Beyond the expansion of $(1+x)^{2n}$ no candidate obtained marks other than those awarded for the partial expansion. The analysis and algebra beyond this point was beyond the grasp of all the candidates who attempted this part of the question.

For Part (b) (i), the majority of candidates concluded that a root exists in the given interval, using the concept of a sign change. Only a very small number of candidates stated that the function is continuous over the given interval. This failure resulted in the majority of candidates being penalized.

The majority of candidates who attempted Part (b) (ii) carried out the required number of iterations with some cases of arithmetic errors. In the absence of the final answer specified to a given number of decimal places or significant figures, most candidates used varying approximate values for x_2, x_3, x_4 and x_5. In many cases the value of the approximation required was not consistent.

Solutions

(a) (i) $\dfrac{n!}{(n-r)!\, r!}$ (iii) 5670

(b) (ii) $T \approx 1.002$

Section C

Module 3: Counting, Matrices and Complex Numbers

<u>Question 5</u>

Specific Objectives: (a) 2, 4, 7; (b) 1, 7, 8

This question tested arrangements with and without repetitions, selections with restrictions, probability, matrix multiplication and subtraction and the solution of a system of linear equations.

Many candidates demonstrated a penchant for using the formula for permutations in Parts (a) (i) and (ii) rather than analysing the problem with particular regard to the restrictions. Satisfactory performance was duly awarded.

In Parts (b) (i) and (ii), candidates demonstrated a sound grasp of determining selections with or without restrictions. This part of the question was well done.

Apart from some arithmetic errors candidates performed well in Part (c) (i). Those candidates who were successful in Part (c) (i) were able to show the desired result in Part (c) (ii).

Candidates who failed to show the result in Part (c) (ii) made attempts to find the inverse of A as required in Part (c) (iii) using the cofactor method. A small number of candidates attempted the row-reduction method. Poor algebraic skills prevented a number of candidates from obtaining the specified result using the result at Part (c) (ii). Generally, this part of the question had limited successes.

Except for those candidates who were successful in Parts (c) (ii) and (iii), a number of candidates attempted to solve the system of linear equations with 3 unknowns by the elimination method. Arithmetic errors apart, some correct solutions were obtained for Part (c) (iv). Candidates who failed to show the result at Part (c) (i) did not reason that they could obtain the solutions at Part (c) (iv) using B^{-1} in terms of A from the equation at Part (c) (ii).

Solutions

(a) (i) 480 ways (ii) 1372 ways

(b) (i) $\frac{1}{77}$ (ii) 144 ways

(c) (i) $B = \begin{pmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{pmatrix}$ (iii) $A^{-1} = -\frac{1}{9}\begin{pmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix}$

<u>Question 6</u>

Specific Objectives: (c) 1, 2, 3, 5, 6, 9, 11

This question tested representation of complex numbers on the Argand diagram, the principal argument of a complex number, the square roots of a complex number, finding the complex roots of a quadratic equation and application of de Moivre's theorem to prove a trigonometric identity.

Part (a) (i) was well done by the majority of candidates. Severe constraints due to poor algebraic skills resulted in poor performance on Part (a) (ii). Candidates failed to use the relationship between the complex numbers A and B and the representation of these points on the Argand diagram to find the argument of A + B. A number of candidates attempted to rationalize $\frac{1+\sqrt{2}+i}{1-i}$ and find $\tan^{-1}\left(1+\sqrt{2}\right)$ approximating the answer to $\frac{3\pi}{8}$.

The majority of candidates who attempted Part (b) (i) obtained full marks. However, a number of candidates could not link the result at Part (b) (i) to the solutions of the quadratic equation in Part (b) (ii). Nevertheless, most candidates demonstrated a sound grasp of the techniques involved to find complex roots of a quadratic equation.

The majority of candidates who attempted Part (c) understood de Moivre's theorem and its application to proof of trigonometric identities. This part of the question was well done.

Solutions

(a) (i) Points $(0, 1)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(b) (i) $z = \pm\frac{1}{\sqrt{2}}(1 + i)$ (ii) $z = 2 + 3i,\ 1 + 2i$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Calculus II

Question 1

Specific Objectives: (b) 2, 4, 5; (c) 8, 13

This question tested logarithmic and implicit differentiation involving polynomials and trigonometric functions, the trapezium rule and definite integration by parts to find the area under a given curve.

The majority of candidates showed a marked weakness in the applications of logarithmic differentiation required in Part (a). A few candidates attempted to complete the differentiation by using the product and quotient rules. However, they failed to apply the chain rule for the composite functions correctly. This part of the question was poorly done.

Some weak attempts were made at sketching the curve in Part (b) (i). Most candidates showed a straight line between the limits stated.

Weak performances were also recorded in Part (b) (ii). A number of candidates failed to use the trapezium rule correctly. In addition, some arithmetic errors resulted in wrong values of the required approximation.

Most candidates demonstrated a fair understanding of integration by parts required in Part (c) (i). However, many of them appeared at a loss when required to repeat the process for $\int 2x \sin x \, dx$.

Limited successes were recorded for this part of the question.

Solutions

(a) $\frac{dy}{dx} = \frac{x(x-1)^{\frac{1}{3}}}{1+sin^3x}\left[\frac{1}{x} + \frac{1}{3(x-1)} - \frac{3\,sin^2x\cos x}{1+sin^3x}\right]$

(b) (ii) 1.115 square units

(c) (i) $sin^2\,sinx\, +\, 2xcosx\, -2sinx\, + C$

(ii) $\frac{\pi^2}{4} - 2$

Section B

Module 2: Sequences, Series and Approximation

Question 2

Specific Objectives: (c) 2, 3, 4; (d) 1; (e) 3, 4

This question tested the binomial theorem, the Newton-Raphson iteration method and errors of measurement.

Most candidates performed well in Part (a) (i). In Part (a) (ii), a number of candidates could not determine that it was necessary to substitute x = 0.1 in the expansion found in Part (a) (i). Arithmetic errors resulted in some incorrect answers.

A number of candidates failed to observe the range of values of x for which the graphs of f (x) and g (x) were to be sketched in Part (b) (i). However, most candidates knew the shapes of the curves to be sketched.

Part (b) (ii) was generally well done, except for some arithmetic errors and failure to observe that the approximation was required to *four decimal places*.

The majority of candidates who attempted Part (b) (iii) knew the concept of errors of measurements and generally this part of the question was done satisfactorily.

Solutions

(a) (i) $1+\frac{5}{4}x + \frac{5}{8}x^2 + \frac{5}{32}x^3 + \frac{5}{256}x^4 + \frac{1}{1024}x^5$

(ii) 1.131

(b) (ii) 0.3419

(iii) 0.1226

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 3

Specific Objectives: (a) 2, 7; (b) 1, 2, 7

This question tested the number of arrangements with and without repetitions, selections with restrictions, multiplication of conformable matrices and inverting a 3×3 matrix.

Candidates attempting Part (a) (i) did not follow the requirement of repetitions. Rather, most of the candidates obtained distinct permutations of the letters given. Part (a) (ii) was poorly done by the few candidates who attempted it.

Candidates demonstrated a sound understanding of the cofactor method for inverting a matrix required in Part (b) (i). Arithmetic errors resulted in some candidates losing marks due to inaccuracy. Part (b) (ii) was generally well done.

Part (b) (iii) was poorly done. Candidates could not make the links with Parts (b) (i) and (ii) to determine the matrix B. Poor algebraic skills prevented candidates from manipulating the links in Parts (b) (i) and (ii) to solve Part (b) (iii). This part of the question was poorly done.

Solutions

(a) (i) 24 times (ii) 576

(b) (i) $A^{-1} = \begin{pmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 \\ 7 \\ 12 \end{pmatrix}$

(iii) $B = \begin{pmatrix} 6 & -6 & 2 \\ -6 & 10 & -4 \\ 2 & -4 & 2 \end{pmatrix}$

Paper 031 – School Based Assessment (SBA)

For the year 2012, 191 Unit 1 and 149 Unit 2 SBAs were moderated. It is increasingly difficult to successfully complete the moderation for samples for the SBAs submitted by some centres since many teachers continue to submit solutions *without unitary* mark schemes. In a number of cases, *neither question papers nor detailed worked solutions with unitary mark schemes* were submitted. Mark schemes for questions and their subsequent parts were not broken down into unitary marks. In an increasing number of cases, marks awarded were either too few or far too many. (Example: an entire SBA module test was 20 marks in total and in another case only one syllabus objective assessed within a module test was allocated 40 marks).

The majority of the samples submitted were not of the required standard. Teachers must pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the SBAs.

The SBA is comprised of three module tests.

The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant unit.
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test questions, detailed mark schemes and students' scripts)
- Quality of the teachers' solutions and mark scheme
- Quality of teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

FURTHER COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.
2. Teachers are reminded that CAPE past examination papers should be used *only* as a guide. They should not constitute any part of or an entire module test.
3. Untidy 'cut and paste' presentations with varying font sizes were common and as a result very unsatisfactory. Module tests must be neatly handwritten or typed.

4. The stipulated time for module tests $(1\,\mathrm{hr} - 1\,\mathrm{hr}\,30\,\mathrm{mins})$ must be strictly adhered to as students may be at an undue disadvantage when the times allotted for module tests are too extensive or insufficient.
5. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling *must* be included.
6. Multiple choice questions will **NOT** be accepted in the module tests.
7. Cases were noted where teachers were unfamiliar with recent syllabus changes, that is,
   - Complex numbers and the Intermediate Value Theorem are tested in Unit 2.
   - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been *removed* from the CAPE syllabus (2008).
8. The moderation process relies on the validity of teachers' assessment. There were a few cases where tampering of the scripts and subsequent questionable mark changes occurred. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. There were a few cases where students' solutions were replicas of teachers' solutions — some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. The SBA must be administered under examination conditions at the centre. It is not to be done as a homework assignment or research project.
9. Teachers *must* present evidence of having marked each question on students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of students' scripts.
10. Teachers *must* indicate any changes/omissions that were made to question papers, solutions and mark schemes and scripts. Teachers should also inform the examiner about the circumstances regarding missing script(s).
11. Students' names on the computer generated form must correspond to the names on PMATH 1-3 and PMATH 2-3 forms and students' scripts.
12. The maximum number of marks for each assessment should be the same for all students.

To enhance the quality of the design of the module tests, the validity of teachers' assessments and the validity of the moderation process, the SBA guidelines are listed below for emphasis.

Module Tests

- Design a separate test for each module. The module test *must* focus on objectives from that module.
- In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.
- One sample of *five* students will form the sample for the centre. If there are fewer than five students, *all* scripts will form the sample for the centre.
- In 2012, the format of the SBA remains unchanged.

Guidelines for Module Tests and Presentation of Samples

1. Cover Page To Accompany Each Module Test

   The following information is required on the cover of each module test:

   - Name of school and territory, name of teacher, centre number
   - Unit number and module number
   - Date and duration $(1\,\text{hr} - 1\,\text{hr}\,30\,\text{mins})$ of module test
   - Clear instructions to students
   - Total marks allotted for module test
   - Sub-marks and total marks for each question *must* be clearly indicated.
2. Coverage Of The Syllabus Content
   - The number of questions in each module test must be appropriate for the stipulated time of $(1\,\text{hr} - 1\,\text{hr}\,30\,\text{mins})$.
   - *CAPE past examination papers should be used as a guide ONLY***.**
   - Duplication of specific objectives and questions must be avoided.
   - Specific objectives tested must be from the relevant unit of the syllabus.
3. Mark Scheme
   - Unitary mark schemes *must* be done on the detailed worked solution. (that is, one mark should be allocated per skill assessed, not 2, 3, 4 etc marks per skill)
   - Fractional/decimal marks must not be awarded (that is, do not allocate $\left(\frac{1}{2}\right)$ marks
   - The total marks for module tests *must* be clearly stated on teachers' solution sheets.

- A student's final mark out of 20 *must* be entered on the front page of the student's script.
- Hand written mark schemes *must* be *neat* and *legible*. The unitary marks *must* be written on the right side of the page.
- Diagrams *must* be neatly drawn with geometrical/mathematical instruments.

4. <u>Presentation Of Sample</u>

- Students' responses *must* be written on letter-sized $\left(8\frac{1}{2}\times 11\right)$ or A4$\left(8\frac{1}{2}\times 11.69\right)$ paper.
- Question numbers *must* be written clearly in the left hand margin.
- The total marks for *each* question on students' scripts MUST be clearly written in the left or right margin.
- Only original students' scripts *must* be sent for moderation.
- Photocopied scripts *will not be accepted*.
- Typed module tests *must* be *neat* and *legible*.
- The following are required for each module test:
    - ❖ A question paper
    - ❖ Detailed solutions with detailed unitary mark schemes
    - ❖ The question paper, detailed solutions, unitary mark schemes and five students'samples should be batched together for each module.
- Marks recorded on PMATH 1-3 and PMATH 2-3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded.
- Form PMATH 2-4 is for official use *only* and should not be completed by the teacher. However, teachers may complete the relevant information: Centre Code, Name of Centre, Territory, Year of Examination and Name of Teacher(s).
- The guidelines at the bottom of the PMath forms should be observed. (See page 57 of the syllabus, no. (6).

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2013

PURE MATHEMATICS

GENERAL COMMENTS

In 2013, approximately 4,800 and 2,750 candidates wrote the Unit 1 and 2 examinations respectively. Overall, the performance of candidates in both units was consistent with performance in 2012. In Unit 1, 72 per cent of the candidates achieved acceptable grades compared with 70 per cent in 2012; while in Unit 2, 81 per cent of the candidates achieved acceptable grades compared with 83 per cent in 2012. Candidates continue to experience challenges with algebraic manipulation, reasoning skills and analytic approaches to problem solving.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 6 to a maximum of 45. The mean mark for the paper was 64.58 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 149 out of 150. The mean score was 52.26.

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 2, 4; (b) 1, 3, 5; (c) 2, 3, 4; (d) 3, 8

The topics tested in this question included the use of truth tables, binary operations, proof by mathematical induction and the factor theorem. Overall, candidates demonstrated competence in this question with approximately 90 per cent of them attempting it and obtaining at least 16 marks. A number of candidates were also able to obtain the maximum score.

In answering Part (a), candidates used a variety of styles to represent the inputs and outputs such as ‘1 and 0’, ‘True and False’, ‘x and √’. The majority of candidates attempted Part (a) (i) and was successful. In Part (a) (ii), some candidates misinterpreted $\sim(p \wedge q)$ and used it as if it were $\sim p \wedge \sim q$.

Part (b) was misinterpreted by many candidates. They substituted $x = 2, -2$ instead of $y = 2$ in the given function. Other candidates treated $y \oplus x$ as $y + x$, solved for y by replacing x and hence substituted y as -2. They also had difficulty factorizing and solving the quadratic equation.

For Part (c), a majority of candidates were able to achieve the first four marks allocated. However, most candidates did not apply the induction steps correctly. Some were more familiar with questions involving the sigma notation and incorporated the sigma notation in their solution. A few candidates did not use the smallest natural number, 1, to begin the proof by induction and instead used 0 and 2. Other candidates were able to recognize the $k + 1^{\text{th}}$ term but they were unable to simplify the term 5^{k+1} because they did not realize that it could be expressed as $5^k \times 5^1$. The conclusion also posed a challenge to many candidates. Candidates should be reminded that the conclusion should relate to the hypothesis. The general format for the conclusion could be as follows: Since $P(1)$ is true and $P(k) \rightarrow P(k+1)$, the proposition $P(k)$ is true for all positive integers 'k'.

Most candidates obtained full marks in Part (d). Some candidates substituted the value of 'p' into the function to prove that $(x+1)$ is a factor as opposed to using the factor to find 'p'. In general, long division was used to show that the remainder is zero under division by $(x+1)$. This method could also have been used for Part (d) (ii) to obtain the quadratic equation, followed by factorizing the quadratic equation to obtain the other two factors and then equating each factor to zero to solve the cubic function. In some cases, candidates were able to factorize the cubic function correctly but were unable to identify the roots.

Solutions

(a) (i) (ii)

p	q	p → q	p ∧ q	~ (p ∧ q)
T	T	T	T	F
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(b) $x = 8$ or $x = 1$

(d) (ii) $f(x) = (x - 8)(x - 2)(x + 1)$ (iii) $x = 8,\ x = 2,\ x = -2$

Question 2

Specific Objectives: (e) 2, 3, 4; (f) 2, 3; (g)

This question tested the concept of a one-to-one function on the domain of real numbers; the inverse of a linear and an exponential function; the inverse of the composite of a linear and exponential function; quadratic inequalities and the modulus function.

Part (a) failed to attract answers from the majority of candidates. Among the methods used for the proof of a one-to-one function was (i) the graphical approach, (ii) a deductive approach, (iii) differentiation and (iv) proof by induction. Candidates who attempted to use the graphical test used a vertical line test instead of the horizontal line test, not indicating the domain clearly on the graph used and in some cases graphed the quadratic function without indicating the specific domain over which the given function is one-to-one. Candidates who attempted to show that $\mathrm{f}(a) = \mathrm{f}(b) \Rightarrow a = b$ could not show the correct algebraic simplification and subsequent deductive proof. Those candidates who attempted differentiation to show the required result simply could not proceed beyond merely differentiating $x^2 - x$. Proof by induction was beyond the ability of those candidates who attempted to use this approach.

In Part (b), the results of $\mathrm{f}^{-1}(x)$ and $\mathrm{fg}(x)$ were easily shown. However, a significant number of candidates failed to find the expression for $g^{-1}(x)$. Common errors included the inability to take $\log_e$ for the change of variable and in cases where $\log_e$ was taken candidates incorrectly cancelled the logs on each side of the equation. In some cases candidates used $\log_e (x - 2)$ as $\log_e x - \log_e 2$. Evidence was seen where some candidates mistakenly interpreted $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ as $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{f}(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{g}(x)$.

In Part (c) (i), candidates used the quadratic graph to find the correct range of values of x. Candidates who used the results $(x + 2)(3x - 2) \le 0$ incorrectly reasoned that

$x + 2 \le 0 \Rightarrow x \le -2$ and $3x - 2 \le 0 \Rightarrow x \le \frac{2}{3}$.

Some candidates used methods including squaring both sides of the equation and thus finding values of x for which the resulting quadratic equation was equal to zero. However, they failed to test the values of x found and were not able to obtain the mark given for showing or stating that $x = -\frac{7}{4}$ was inadmissible. Candidates who used the concept of $x + 2 = 3x + 5$ and $-(x + 2) = 3x + 5$ could not reason the correct value of x for which the equation was true.

Solutions

(b) (i) a) $\mathrm{f}^{-1}(x) = \dfrac{x-2}{3}$ $\quad \mathrm{g}^{-1}(x) = \dfrac{1}{2}\ln x$ $\quad$ b) $\mathrm{fg}(x) = 3\mathrm{e}^{2x} + 2$

(c) (i) $-2 \le x \le \dfrac{2}{3}$ $\quad$ (ii) $x = -\dfrac{3}{2}$ only

Section B

Module 2: Trigonometry, Geometry and Vectors

Question 3

Specific Objectives: (a) 2, 3, 4, 5, 6

This question tested trigonometric identities of multiple angles; solving trigonometric equations involving multiple angles; expressing a cos x + b sin x in the form r cos (x + α); determining maximum and minimum values of trigonometric expressions; and proof of simple trigonometric equations.

In Part (a) (i), most candidates performed satisfactorily. Challenges encountered included the inability to use the identity given to obtain a quadratic equation in terms of tan θ. Candidates who obtained the correct equation $\tan\theta\left(1-\tan^2\theta\right)=0$ wrongly divided the equation by $\tan\theta$ thus losing the roots of the equation $\tan\theta = 0$. Candidates also deduced $\tan^2\theta = 1 \Rightarrow \tan\theta = 1$. This error resulted in candidates being unable to get the solutions for $\tan\theta = -1$.

Part (b) (i) was generally well done. Some errors included solving $\alpha = \tan^{-1}\left(\dfrac{3}{4}\right)$ and $\alpha = \tan^{-1}\left(-\dfrac{4}{3}\right)$. Part (b) (ii) a) was satisfactorily done. Errors made by candidates included stating that the maximum value of $5\cos\left(\theta+\alpha\right)$ is 1.

However, Part (b) (ii) b) was not satisfactorily done. Most candidates seemed unable to deduce the maximum value of a reciprocal function, particularly when another term is added to the denominator.

Responses to Parts (b) (iii) a) and (b) were very poor. Candidates did not demonstrate knowledge of the fact that the sum of the interior angles, A, B and C of the triangle given, was π radians. Evidence of candidates expanding sin (B + C) and being unable to link that expansion with sin A was observed. Some candidates attempted to substitute numerical values for angles A, B and C with no success.

Solutions

(a) (ii) $\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

(b) (i) $5\cos\left(\theta + 0.927^c\right)$ (ii) a) $\mathrm{f}(\alpha)_{\max} = 5$ (ii) b) $\left(\frac{1}{8 + \mathrm{f}(\theta)}\right)_{\min} = \frac{1}{13}$

<u>Question 4</u>

Specific Objectives: (b) 1, 2, 5, 6; (c) 4, 5, 6, 8, 9

This question tested geometry of the circle; parametric equations to a Cartesian equation; three dimensional vectors; and equations of lines in the Cartesian of a plane.

Part (a) (i) was generally well done. Some candidates expressed the equation of the circle using completion of the square to deduce the coordinates of the centre and the radius. A number of candidates used the equation $x^2 + y^2 + 2\mathrm{f}x + 2\mathrm{g}y + c = 0$ to deduce that the coordinates of the centre were $(-\mathrm{f}, -\mathrm{g})$ and the radius $\sqrt{\left(\mathrm{f}^2 + \mathrm{g}^2 - c\right)}$. Some errors were made when expressing the coordinates of the centre as (f, g).

In Part (b) (ii) a), many candidates deduced that the normal to the circle lies along the diameter while other candidates worked though the equation $y - y_P = \frac{y_P - y_C}{x_P - x_C}\left(x - x_C\right)$.

In Part (b) (ii) (b), some candidates used the gradient of the normal to find the equation of the tangent. Those candidates who used the gradient of the tangent, $-\left(\frac{x_P - x_C}{y_P - y_C}\right)$ could not interpret the meaning of the resulting gradient $-\frac{3}{0}$. It was not unusual to see candidates stating the gradient as 0. The fact of the tangent being parallel to the y-axis was not understood by a significant number of candidates. Further, a number of candidates drew a graph of the circle and indicated the tangent parallel to the y-axis but were unable to state the equation as $x = 6$.

In Part (b), various methods were used by candidates. Apart from expressing $t = \frac{y + 4}{2}$ and substituting for $x = \left(\frac{y + 4}{2}\right)^2 + \frac{y + 4}{2}$, some candidates substituted the parametric equations given into the equation required to be shown. Common errors resulted from poor algebraic simplification. For example $\left(\frac{y + 4}{2}\right)^2 = \frac{y^2 + 16}{4}$.

Generally Part (c) (i) was done satisfactorily. Some candidates made the basic error that the vector $\overline{\mathbf{AB}} = \overline{\mathbf{OA}} + \overline{\mathbf{OB}}$ and $\overline{\mathbf{BC}} = \overline{\mathbf{OB}} + \overline{\mathbf{B}C}$. Part (c) (ii) appeared challenging to many candidates. It was generally understood that the dot product was required. However, most candidates used the $\overline{\mathbf{OA}}$ and $\overline{\mathbf{OB}}$ with the vector $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$ to attempt to show the perpendicular property without success. A small number of candidates understood that they were required to show perpendicularity between the vectors $\overline{\mathbf{AB}}$ and $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$ and between the vectors $\overline{\mathbf{BC}}$ and $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$. It was interesting to observe some candidates using the vector cross product to show the perpendicular vector $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$.

In Part (c) (iii), candidates quoted the vector equation of the plane, but were not able to use the correct resulting point and the normal vector to the plane to complete the equation $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$. Those candidates who used the correct values were unable to express the Cartesian equation of the plane as required.

Solutions

(a) (ii) a) normal$_{(6,\,2)}$: $y = 2$ b) tangent$_{(6,\,2)}$: $x = 6$

(c) (i) $\overline{\mathbf{AB}} = -2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ $\overline{\mathbf{BC}} = -2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ (iii) $2y + z = 0$

Section C

Module 3: Calculus I

<u>Question 5</u>

Specific Objectives: (a) 3, 7, 9, 10; (b) 3–8, 12

This question tested limits; continuity; differentiation using the quotient rule; parametric differentiation; and finding the area enclosed by two curves using integration.

In Part (a) (i), the majority of candidates gave satisfactory performances although only a small number of candidates presented their statements in acceptable mathematical language. However, Part (a) (ii) was poorly done since a number of candidates made guesses on the question of continuity and were unable to justify their responses mathematically.

In Part (b), most candidates showed a good understanding of the quotient rule for differentiation. However, there were cases in which candidates used the product rule by expressing the denominator as a multiplying term. Some candidates committed errors in their differentiation of the term $(x^2 + 2)^3$. They mainly differentiated the cubic term and neglected the differential of x^2. Weaknesses in algebraic manipulation prohibited many candidates from obtaining the required simplified result.

Almost half of the candidates who responded to Part (c) demonstrated a lack of knowledge of differentiation of parametric terms. Some candidates opted to convert the equation to Cartesian form and to proceed with the differentiation. However, the term in y^2 made it difficult for them to successfully complete the correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Only a small number of candidates were able to obtain the correct result.

Generally, the responses to Part (d) were satisfactory. Most candidates were familiar with the concept tested in this part of the question. Some errors included candidates subtracting the area enclosed by the curve $y = 4x$ from the area enclosed by the curve $y = x^2 + 3$ incorrectly. Very few candidates used the integral of $4x - (x^3 + 3)$ but preferred to find the area using the difference of two areas.

Solutions

(a) (i) lim = 4 (ii) f (x) is not continuous since f (x) is not defined at $x = 2$

(c) $\frac{2}{3}\cot\theta$ (d) (i) P (1, 4) Q (3, 12) (ii) $\frac{4}{3}$ units^2

<u>Question 6</u>

Specific Objectives: (c) 3, 4, 6, 8, 9 (b)

This question tested indefinite integration using substitution; the theorem of the integral of sums being equal to the sum of integrals, determining maxima using differentiation; and determining the constants of integration given initial conditions.

Part (a) (i) tested integration using substitution. The substitution $x = 1 - u$ was given and the majority of candidates demonstrated a good understanding of having to express dx in terms of du. However, in proceeding to complete the substitution of $x\ (1 - x)^2$ in terms of u, the majority of candidates stated the expression as $x\ u^2$. Hence, they could not continue integration in this form since the variable x was not expressed as $1 - u$. Many candidates who made the correct substitution and successfully integrated in terms of u failed to express their answer in terms of x.

Part (a) (ii) was generally well done. Some errors seen included:

$\int 4 \sin 5t\ \mathrm{d}t = -4\cos 5t$, $4\cos 5t$ and $20\cos 5t$.

Part (b) was generally well done. However, in Part (b) (i), a significant number of candidates were unable to find the correct formula for the area of a simple plane figure. Candidates who used the correct formula for the area were not able to simplify the expression thus allowing for easy differentiation. As a result, a majority of the candidates failed to obtain the correct differential to proceed to find the value of x. However, candidates demonstrated the knowledge that it was necessary to solve $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$.

Part (c) (i) required candidates to find the first and second differentials of y, explicitly given in terms of x. The terms to be differentiated involved a product of x and a trigonometric term in x. The majority of candidates failed to apply the product rule in differentiating $-x \sin x$. The common results shown were $-x \cos x$, $x \cos x$ and $\sin x$. This error was compounded by adopting the same approach for the second differential. Consequently, candidates were unable to show the required answer. Overall, candidates performed poorly on this part of the question.

Part (c) (ii) required that candidates find the values of two constants of integration for an explicit function of y in terms of x given the boundary conditions. This required substitution of the values of y for given values of x. A significant number of candidates failed to recognize this simple procedure and appeared to think integration was required. Those candidates who recognized the methods required for solving this part of the question made algebraic and arithmetic errors in their substitutions.

Solutions

(a) (i) $-\frac{1}{12}(1-x)^3(1+3x)+C$ (ii) $5 \sin t - \frac{4}{5} \cos 5t + C$

(b) (ii) 84 metres approx.

(c) (ii) $y = -x \sin x - 2 \cos x + \frac{1}{\pi} x + 3$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 1, 3; (b) 2, 4; (c) 1, 2; (d) 1, 2, 4, 6; (f) 3

This question tested the converse, inverse and contrapositive of a conditional statement; solving logarithmic and exponential equations; and graphing a modulus and a linear function of x.

For Part (a), candidates generally demonstrated a lack of knowledge in these topics. A small number of candidates attempted to use truth tables to show the required results but performed poorly.

In Part (b), candidates demonstrated understanding of expressing a sum of two logs as a single log. However, they failed to express 3 as $\log_2 2^3$ which would have allowed them to proceed with the solution of the correct quadratic equation $(x+3)(x+2)=8$.

Very few candidates attempted Part (c). Those who did substitute $A = 0$ could not proceed to express the resulting equation $3e^{4t} - 7e^{2t} - 6 = 0$ in a convenient quadratic form to solve for t.

In Part (d), the majority of candidates graphed the line f $(x) = 2x + 3$ correctly but did not graph g $(x) = |2x + 3|$ correctly.

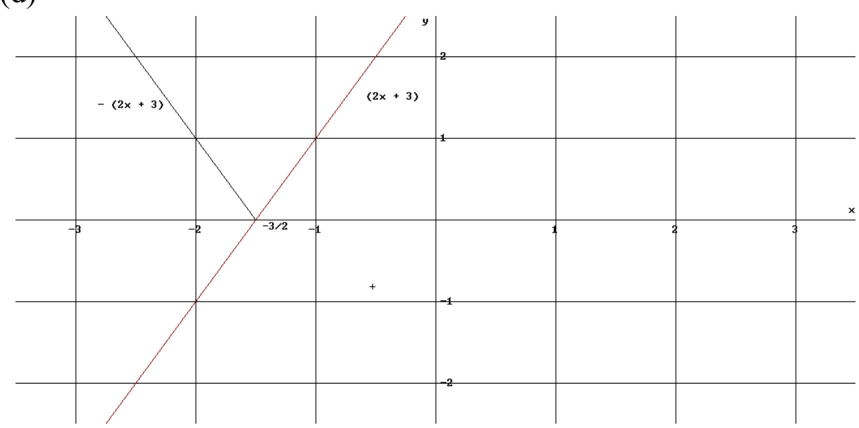
Solutions

(a) (i) $(q \vee \sim p) \rightarrow (p \wedge q)$

(b) $x = 0.37$ (2 d.p.)

(c) $t = \dfrac{1}{2} \ln 3$

(d)



Section B

Module 2: Trigonometry, Geometry and Vectors

<u>Question 2</u>

Specific Objectives: (a) 1, 3, 4, 7, 8; (b) 3, 4, 6; (c) 1, 7, 10

This question tested the exact value of cos $3A$ given exact values of sin A and cos B; the solution of a trigonometric equation involving a mix of cos 2θ and sin θ in a given range; and coordinate geometry involving the intersection of two circles.

In general, the responses to Part (a) were unsatisfactory. Candidates attempted to convert $\sin A = \frac{4}{5}$ and $\cos B = -\frac{3}{5}$ into degrees and substitute for $\cos 3A = 3 \cos 3$ (their value of A). It was clear that candidates did not have the required knowledge of this topic.

The responses to Parts (b) and (c) were also unsatisfactory. Candidates could not correctly express $\cos 2\theta$ in terms of $\sin \theta$ and the arithmetic errors made by candidates resulted in incorrect values for x and y in Part (c). However, most candidates demonstrated an understanding of using the equation two curves to find point(s) of intersection.

Solutions

(a) $-\frac{117}{125}$ (b) 6.031 radians (c) $(1.46, -1)$ and $(-5.46, -1)$

Module 3: Calculus I

Question 3

Specific Objectives: (a) 2, 4, 5; (b) 1, 3, 9–12; (c) 2, 5, 7, 8 (b)

This question tested limits, differentiation from first principles; finding minimum and maximum stationary points; and integration to find the volume of revolution about the x-axis. Overall, the majority of candidates gave no response to this question. A few unsuccessful attempts were made for Part (c) with some candidates managing to find $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{f}(x)$.

Solutions

(a) (i) a) 1 (b) 2 (ii) not continuous since it is not defined at $x = 2$

(b) $-\dfrac{1}{2\sqrt{2}\sqrt{(x)^3}}$

(c) $\frac{1}{3}$ and $-\frac{3}{2}$

(d) $\frac{100}{3}$ units^3

UNIT 2

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 6 to a maximum of 44. The mean mark for the paper was 69.18 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 150 out of 150. The mean score was 47.52.

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Objectives: (a) 7, 8, 12, 13; (b) 1–5, 8

In Part (a), the majority of candidates completed the differentiation satisfactorily. Some candidates who completed the differentiation were unable to differentiate the natural log of the term $\ln(x^2y)$ correctly. Some common results for differentiating this term include $\frac{1}{x^2y}$ and $\frac{2x\frac{dy}{dx}}{x^2y}$. A few candidates applied the natural log laws to separate the terms before differentiating and many were successful with the differentiation using this approach.

In general, the candidates showed a lack of understanding of partial derivatives in Part (b). Many inserted additional terms. Common responses included:

$\frac{\partial f}{\partial y} = 3z^2 - e^{4x}\cos 4z - 6y - y$ and $\frac{\partial f}{\partial z} = 6yz + 4\sin 4ze^{4x} - 4\cos 4ze^{4x}$.

Part (c) was generally well done, with candidates gaining the majority of marks. Almost all candidates were able to identify that only the real part of the complex number was needed for the solution.

In Part (d), many candidates recognized that $\tan^{-1}$ was needed to find the argument. However, most obtained:

$$\arg(z) = \tan^{-1}(-1)$$
$$= -\frac{\pi}{4}$$

and completely ignored the use of the Argand Diagram. In addition, a majority of candidates obtained $|z| = \sqrt{2}$ but left out the exponent, 7, when doing their final calculation and hence they did not write the modulus as $\left(\sqrt{2}\right)^7$.

Solutions

(a) Undefined but marks awarded for finding the derivative and explaining why the gradient could not be found.

(b) $\dfrac{\partial z}{\partial y} = \dfrac{3(z^2 - 2y)}{2(3yz - 2e^{4x} Sin4z)}$

(d) (i) $z = \sqrt{2^7} e^{i7\left(\frac{3\pi}{4}\right)}$

<u>Question 2</u>

Objectives: (c) 1–3, 6, 8, 9, 11

Most candidates attempted Part (a) (i) using integration by parts. However, some candidates either differentiated or integrated the parts incorrectly while others had difficulty manipulating the signs. Some candidates used the identity $cos2\theta = 1 - 2sin^2\theta$ to simplify the integral to $\int sinx\, dx - 2\int sin^3 dx$, but most of them could not manipulate $\int sin^3 dx$.

In Part (a) (ii), the majority of candidates knew how to substitute the values into their answer from Part (a) (i) above and easily obtained full marks.

In Part (b), it was evident that many candidates did not understand what was meant by *four intervals*. Several candidates interpreted four intervals as four ordinates instead of four trapezia. Hence, they used $n = 3$. In other cases, candidates used $n = 5$ instead of $n = 4$. This resulted in a variety of incorrect responses. Further, most candidates did know what to do when the curve went below the x-axis. They did not take the modulus of the part of the curve that was below the x–axis, that is, $|f(-0.75)| = |-0.5625| = 0.5625$. Consequently, their responses to the problem were often smaller than the expected area between the curve and the x-axis.

The two common problems which arose in Part (c) were candidates not recognizing that partial fractions was not required and not competently dealing with irreducible quadratic factors and repeated roots. The vast majority therefore missed out on the opportunity to solve the problem easily by working on the right-hand side. Many candidates did not get the basic form of the expansion correct. Some did not recognize that (x^2+4) is a factor of $(x^2+4)^2$. They made their denominator $(x^2+4)(x^2+4)^2$ which made the question more complicated and left them unable to complete the solution.

In Part (c) (ii), candidates experienced several difficulties. Many did not know how to use the substitution $x = 2tan\theta$. Some candidates determined $\frac{1}{2}\int \frac{1}{sec^2\theta} d\theta$, but could not go any further. Those who were able to manipulate the trigonometric function and integrate it to obtain $-\frac{1}{8}\sin 2\vartheta + \frac{3}{4}\vartheta + C$ experienced great difficulty changing the variable in $\sin 2\vartheta$ back to x. They did not realize that they could have used the identity $\sin 2\theta = 2\sin\theta\cos\theta$ and a right-angled triangle to get their final answer in terms of x.

Solutions

(a) (i) $-\frac{2cos^3 x}{3} + cos\ x + c$ or $-\frac{1}{6}\cos 3x + \frac{1}{2}\cos x + c$ (ii) $-\frac{1}{3}$

(b) 4.22 square units

(c) (ii) $\frac{3}{4}tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\left(\frac{x}{x^2+4}\right) + c$

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objective(s): (a) 2–4; (b) 3, 6, 7, 9

This question tested the concepts of mathematical induction, telescoping and the Taylor series.

In Part (a), it was very clear that most candidates lacked understanding of the process of induction since they were unable to deduce what was to be proved. Many candidates attempted to prove the recurrence relation via mathematical induction, but they ignored the inequality. For those who recognized the inequality as the induction hypothesis and proved the base case, many found it difficult to carry out the inductive procedure.

Part (b) was generally well done. In Part (i) (b), most candidates applied the method of differences in the summation of a series although a few candidates opted to use mathematical induction to prove the equality with limited success. In Part (b) (ii), some candidates saw the connection to Part (b) (i) (b) and were able to complete the solution competently. However, in many cases the correct limit notation was not used.

Overall, candidates performed below expectations in Part (c) (i). Several candidates started correctly but experienced difficulty completing the series. Some candidates did not differentiate sine and cosine functions correctly while others confused Taylor with Maclaurin's series. In Part (c) (ii), candidates saw the connection with the solution of Part (c) (i) and attempted to substitute $\frac{\pi}{16}$ into their solution. However, many equated $\left(x - \frac{\pi}{4}\right)$ to $\frac{\pi}{16}$ and used $x = \frac{5\pi}{16}$ instead. Others failed to arrive at the correct solution because of faulty algebraic manipulation and arithmetic errors.

Solutions

(c) (i) $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3$

(ii) 0.977

<u>Question 4</u>

Specific Objective(s): (c) 1–4; (d) 1, 2, 6

This question examined the use of the binomial theorem to approximate a surd and to compute a coefficient of a term in an expansion; the Intermediate Value Theorem in testing for the existence of a root in an equation; and the 'interval bisection' method in finding successive approximations to a root in an equation.

Candidates generally found Part (a) (i) manageable and showed good understanding of the binomial expansion. However, a significant number of candidates simplified $\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)}$ incorrectly as $(1+x)^2 + (1-x)^2$. Some candidates also had difficulty obtaining the correct binomial coefficients due to arithmetical errors, particularly the signs of coefficients.

In Part (a) (ii), most candidates substituted $x = \frac{1}{16}$ into the binomial expansion rather than first substituting $x = \frac{1}{16}$ into $\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)}$ in order to determine how to modify the expansion to give the desired result. Simplifying surds also continues to be an area of difficulty for candidates. For example, several candidates wrote $\sqrt[4]{\frac{17}{16}} = \frac{\sqrt[4]{17}}{\sqrt[4]{16}} = \frac{\sqrt[4]{17}}{4}$, which is incorrect.

Part (b) examined candidates' ability to extract the coefficient of x^5 from a binomial expansion. Most candidates opted to expand both expressions first then multiply them. Some deviated from this and only wrote down the terms that had the desired power of x. On the other hand, certain candidates used more advanced approaches. They were able to apply the binomial expansion to the product of the two binomials and obtain the correct coefficient of x^5. In Part (b) (ii), most candidates appeared to be unfamiliar with interval bisection and continued to calculate midpoints using $b_n = 3$ or found an approximation using the Newton–Raphson method. Most candidates who demonstrated some knowledge of the topic produced at least two successive iterations. However, one commonly observed problem was the incorrect application of the stopping criterion. Interval bisection is a geometrical approach to finding a root; therefore, the use of diagrams in the teaching this topic should be encouraged in order to strengthen the responses in this area.

Solutions

(a) (i) $2 - \frac{3}{16}x^2$ (ii) 3.9986

(c) (i) $f(2) < 0;\ f(3) > 0\ hence$ continuity (ii) 2.92

Section C

Module 3: Counting, Matrices and Differential Equations

<u>Question 5</u>

Objectives: (a) 2, 4, 6, 16, 17, (b) 4–6.

In Part (a) (i), although the majority of the candidates was able to provide the required solution, some candidates did not know how to construct a tree diagram. In many cases, candidates used the actual letter of the word BRIDGE to show the outcomes rather than classifying the outcomes into vowel and consonant before recording the outcome on the tree. They were then unable to determine the required probability. In some cases although candidates correctly determined the number of outcomes, they were unable to calculate the associated probabilities.

In Part (b), a few candidates did not understand the use of the word *system* or what was meant by a system of equations having a "unique" solution. This question did not restrict candidates to using the row reduction approach and as a result, some candidates used the determinant method to arrive at their conclusion.

Performance on Part (c) (i) was unsatisfactory. Most candidates did not seem to understand how to move from the conditional probabilities given to the total probability required. Several candidates gave values greater than 1 which indicated that they lacked understanding of the basic concept of a probability. Some also used the values of 45 per cent, 30 per cent and 25 per cent and not the fractional form in their calculations. Consequently, they obtained incorrect responses.

Solutions

(b) (i) The system is not consistent since we have $0x+0y+0z=9$. *(The justification will depend on how the reduction is executed.)*

(ii) The solution is unique. The matrix can be reduced to $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 25/6 \\ 0 & 0 & 1 & 9/2 \end{array}\right)$.

Since only the leading diagonal elements are non-zero, the solution is unique.

Alternatively, $|A|=12$ which is not 0. The system of equations can therefore be solved.

(c) (i) 0.7975 (ii) 0.2665

Question 6

Objectives: (c) 2, 3 (c)

The first part of this question appeared to be the most difficult for candidates who were unable to determine the correct integrating factor and thus failed to arrive at the correct integral. In Part (a) (ii), several candidates used degrees instead of radians in their solutions. They were, therefore, unable to use the cancellation to assist in finding the answer. The given value of $y=\frac{15\sqrt{2}\pi}{32}$ was also not interpreted correctly by some candidates resulting in incorrect constants of integration.

In Part (b), most candidates appeared to know the appropriate form of the complementary function given the auxiliary equation. However, several candidates did not use the quadratic formula correctly to evaluate and many were unable to evaluate $\sqrt{-16}$. Most candidates who attempted Part (b) (iii) were able to find the particular solution correctly although some provided the general solution. Candidates were awarded the marks for either of these solutions.

Solutions

(a) (i) $y=x^2\cos x+C\cos x$ (ii) $C=\frac{7\pi^2}{8}$

(b) (i) (a) $\lambda=-1\pm 2i$ (b) $u_p(t)=C_1e^{-t}\sin 2t+C_2e^{-t}\cos 2t$

(iii) $y(t) = 0.981e^{-t}\sin 2t + 0.255e^{-t}\cos 2t - \frac{16}{17}\cos 2t + \frac{4}{17}\sin 2t$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Objectives: (a) 4–6, 9; (b) 3, 4, 8; (c) 4, 5, 7, 8, 10

In Part (a), it was evident that candidates were unable to interpret the partial derivatives required. Similarly, in Part (b), most candidates were unable to choose appropriate expressions for the integration by parts. Consequently, they were unable to solve the problem. Those who were able to begin the integration by parts ended up with expressions containing incorrect signs. More exposure to the reduction formula is recommended.

In Part (c), most candidates were able to generate the simultaneous equations needed to solve for the square root. However, they experienced difficulty recognizing that the resulting equation was a quadratic equation in x^2.

Solutions

(a) 0.16% approximately

(c) $z^{1/2} = 1.453 + 0.344i$ and $z^{1/2} = -1.453 - 0.344i$

Section B

Module 2: Sequences, Series and Approximation

Question 2

Specific Objective(s): (b) 1, 2, 4, 6, 8; (c) 4

This question examined the use of binomial theorem to approximate a decimal number; Maclaurin and geometric series.

In Part (a) (i), a significant number of candidates did not express $\frac{1}{2}x$ in brackets. As a result, they raised only x to the various powers instead of the entire expression $\frac{1}{2}x$. In Part (a) (ii), some candidates substituted $x = 1.377$ into their expansion rather than stating that $1 + \frac{1}{2}x = 1.377$ to find x as 0.754.

In Part (b), quite a few candidates simply copied the Maclaurin expansion from the formula booklet instead of deriving it as the question required and were unable to find the derivatives of the logarithmetic function given. Of the few who recognized the general term, some did not use the sigma notation and others could not derive the appropriate sign for the terms of the sum.

In Part (c) (ii), most candidates were unable to show that $S_2 < 4$. Some found it difficult to work with the algebraic expressions as the terms of the series while others could not manage the reasoning required to complete the proof.

Solutions

(a) (ii) 3.60

(b) (i) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3$ (ii) $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^k}{k}$

(c) (i) $-2 < x < 2$

Section C

Module 3: Counting, Matrices and Differential Equations

Question 3

Objectives: (a) 2, 3, 6; (b) 2, 7, 8

In general, candidates knew how to find the determinant of the matrix in Part (a) and demonstrated the ability to multiply matrices. However, many did not write out the 3×3 zero matrix. Instead, they simply wrote 0.

Most candidates did not recognize the link between the equation in Part (a) (ii) and the inverse of the matrix and instead attempted to use row reduction to find the inverse. Similarly, row reduction was used to solve the simultaneous equations. This approach required much more work and many computational errors were made.

In Part (b), the majority of candidates were unable to recognize that the problem involved permutations and used the $\binom{n}{r}$ format, instead of ${}^6P_3$. Further, the concept of probability was not understood.

Solutions

(a) (i) 18 (ii) b) $A^{-1} = \frac{1}{18}C^T = \frac{1}{18}\begin{bmatrix} 8 & 4 & 2 \\ 7 & -1 & -5 \\ -3 & 3 & -3 \end{bmatrix}$

(ii) (c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

(b) (i) 120 (ii) 0.8

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2014

PURE MATHEMATICS

GENERAL COMMENTS

In 2014, approximately 5 312 and 2 909 candidates wrote the Unit 1 and 2 examinations respectively. Overall, the performance of candidates in both units was consistent with performance in 2013. In Unit 1, 70 per cent of the candidates achieved acceptable grades compared with 72 per cent in 2013; while in Unit 2, 85 per cent of the candidates achieved acceptable grades compared with 81 per cent in 2013. Candidates continue to experience challenges with algebraic manipulation, reasoning skills and analytic approaches to problem solving.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 6 to a maximum of 90. The mean mark for the paper was 55.46 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 149 out of 150. The mean score was 50.52.

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (a) (A) 1, 2, (B) 1, 2, 6, (C) 2

The topics tested in this question included the use of truth tables, binary operations, proof by mathematical induction and the factor theorem. Overall, candidates demonstrated competence in this question with approximately 98 per cent of them attempting it and obtaining a range of 9 to 12 marks. A few candidates were also able to obtain the maximum score.

In answering Part (a), candidates used a variety of styles to represent the inputs and outputs such as *1 and 0* and *T and F*. A number of candidates used only four propositions for *p* and *q* with no illustration of the proposition *r* in constructing the truth table. Generally, the results of *implication* were not well done. However, based on the candidates’ responses, marks were awarded for the *conjunction*.

Part (b) (i) required candidates to give a reason for determining whether a binary operation is commutative. Generally, candidates stated a correct or incorrect result without giving a reason. Responses varied with candidates explaining the properties of commutative law, reasoning that since $y \oplus x = x \oplus y$ then $\oplus$ is commutative or that since $-5y - 5x \neq -5x - 5y$ then $\oplus$ is not commutative. No response of $-5(y + x) = -5(x + y)$ was given. A few candidates substituted real numbers for *x* and *y* to state a result.

Part (b) (ii) a) was well done by most candidates. A few candidates used the operation as $2 \oplus x = 2 \oplus 1$. However, poor algebraic substitution resulted in obtaining an incorrect cubic equation. Either by long division, or otherwise, many candidates were unable to obtain the other two linear factors correctly. Some candidates left their answers in the form $f(x) = (x - 1)Q(x)$ where $Q(x)$ is a quadratic in *x*.

Mathematical induction, as tested in Part (c), continues to be challenging for many candidates although the first phase of proving the statement P_n for $n = 1$ and $n = 2$ was generally well done. Many candidates do not state the assumption for the statement P_k following the proof for $n = 1$ and $n = 2$. The inductive steps required to show the algebraic expression for the $(k + 1)^{th}$ term, thus establishing the proof for P_k and subsequently for P_n, was generally poorly done.

Solutions

(a) (i) and (ii)

p	**q**	**r**	**p → q**	**r → q**	**(p → q) ∧ (r → q)**
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

(b) (i) $\left(y^3 + x^3\right) + a\left(y^2 + x^2\right) - 5\left(y + x\right) + 16 = \left(x^3 + y^3\right) + a\left(x^2 + y^2\right) - 5\left(x + y\right) + 16$

Since addition is commutative for *x*, *y*, $\oplus$ is commutative

(ii) a) $a = -2$

b) $f(x) = (x - 1)(x - 3)(x + 2)$

Recommendations

Teachers are advised to use truth tables extensively for more than two propositions. The correct number of rows, 2^n, (where n is the number of propositions) should be understood. The correct layout of the rows of the truth tables is important for subsequent connectives and conclusion of truth values. Rules on propositional calculus should be understood.

Students must understand the concepts of *identity, closure, inverse, commutativity, associativity, distributivity* and other simple binary operations. Reasons for conclusions must be based on these concepts. Tabular presentations of binary operations and algebraic expressions must be given equal attention in teaching this topic.

The steps required for proof by Mathematical Induction are not followed with the sophistry required. The concepts of *the inductive process* are not fully understood by most students. Rigid teaching of these processes must be employed. The algebra required to simplify the $P_{(k+1)}$ form of the statement $P_{(k)}$ is generally quoted and not shown by algebraic manipulation. Students must be able to demonstrate their abilities to simplify the algebra that results in the required form. Please see Example 2: Page 14 of the syllabus.

Question 2

Specific Objectives: (B) 3, (D) 6, 7, (E) 1, 4

This question tested the concepts of *composite functions*, the use of the law of logarithms to simplify algebraic expressions, the solution of equations involving $\log_e$ and $\log_a$ and the simplification of surds. The question was attempted by approximately 90 per cent of the candidates with varied levels of responses.

Part (a) (i) a) required determining $f^2(x)$ where f is a polynomial in *x*. A significant number of candidates interpreted $f^2(x)$ as $[f(x)]^2$ and $f(x^2)$ instead of $f[f(x)]$.

In Part (a) (i) b), some candidates had difficulties simplifying $2\left[\sqrt{\left(\frac{x-1}{2}\right)}\right]^2 + 1 = x$. In a few cases candidates found gf (x). In stating the relationship between f (x) and g (x), candidates used descriptions such as injective, surjective, bijective and directly or inversely proportional.

Candidates' responses to Part (b) was generally poor. In some cases candidates attempted to use the law of logs to simplify the right hand side and found it challenging to express the left hand side in terms of logarithms. Conversely, those candidates who attempted to show the expression given found the algebra beyond their capacity. It was clear that most of the candidates were unable to use the expression given as an aid to simplifying the problem and to show the required solution.

Approximately 99 per cent of candidates attempted Part (c) (i). Common errors included incorrectly factorizing the quadratic equation in terms of the variable substituted for e^x and not stating the value of $x = 0$ but instead $x = \ln 1$.

Part (c) (ii) was challenging for many candidates. Lack of knowledge of the laws of logarithms was evident. Errors included:

$$\log_2 (x+1) - \log_2 (3x+1) = 2 \Rightarrow \frac{x-1}{3x+1} = 2,$$

$$\log_2 x + \log_2 1 - \log_2 3x - \log_2 1 = 2$$

and other variations.

Approximately 98 per cent of candidates attempted Part (d). Generally, candidates demonstrated knowledge of the concept of *rationalizing surds*. Some candidates were unable to show the required answer due to poor multiplication of terms including surds.

Solutions

(a) (i) a) $\quad \mathrm{f}^2(x) = \mathrm{f}\left[\mathrm{f}(x)\right] = 2\left(2x^2+1\right)+1 = 8x^4 + 8x^2 + 3$

b) $\quad \mathrm{f}\left[g(x)\right] = 2\left[\sqrt{\left(\frac{x-1}{2}\right)}\right]^2 + 1 = x$

(ii) $\quad$ f and g are inverse functions

(b)

$$3\log\left(\frac{a+b}{2}\right) = \log\left(\frac{a+b}{2}\right)^3 = \log\left(\frac{a^3+3a^2b+3ab^2+b^3}{8}\right) = \log\left(ab^2\right) = \log a + 2\log b$$

(c) (i) $\quad e^x + \frac{1}{e^x} - 2 = 0 \Rightarrow e^{2x} - 2e^2 + 1 = 0 \Rightarrow \left(e^x - 1\right)^2 = 0 \Rightarrow x = 0$

(ii) $\quad \log_2\left(\frac{x+1}{3x+1}\right) = 2\log_2 2 \Rightarrow \frac{x+1}{3x+1} = 4 \Rightarrow x = -\frac{3}{11}$

Recommendations

It is recommended that teachers use different notations for composite functions. These may include $f^2(x) = ff(x) = f\left[f(x)\right] = f \circ f = f(x) \circ f(x)$. Further, it is important to emphasize that it is not necessarily true that *fg(x)= gf(x).*

More application of the laws of logarithms should be done. Properties of f (x) = e^x and ln (x) must be fully understood, particularly, if given f (x) = e^x then $f^{-1}(x)$ = ln (x).

Section B

Module 2: Trigonometry, Geometry and Vectors

Question 3

Specific Objectives: (A) 1, 2, 5, 7, 8

This question tested trigonometric identities of reciprocal angles; the compound angle formulae; the solution of trigonometric equations involving reciprocal and compound angles; expressing $a \cos 2\theta + b \sin 2\theta$ in the form $r \sin (2\theta + \alpha)$; and determining maximum and minimum values of trigonometric expressions.

In Part (a) (i), approximately 80 per cent of candidates recognized the need to express cot x and cot y in terms of $\frac{\cos x}{\sin x}$ and $\frac{\cos y}{\sin y}$ respectively. However, a large number of candidates failed to expand $\frac{\sin(x - y)}{\sin(x + y)}$ and could not readily link or simplify the two sides of the equation, thus proving the identity. Part (a) (ii) was generally poorly done. The majority of candidates who attempted this part of the question merely stated $\frac{\sin(x - y)}{\sin(x + y)} = 1$ and could not follow through with the simplification and subsequent solution. Many of those candidates who followed through beyond this step did not make use of the given condition $\sin x = \frac{1}{2}$. In some cases candidates expressed $\sin(x \pm y)$ as $\sin x \pm \sin y$.

The majority of candidates attempted Part (b) (i) with a few making the error of calculating $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ instead of $\tan^{-1}\left(\frac{4}{3}\right)$. Further, some candidates gave α as degrees and not radians. Part (b) (ii) a) was poorly done. Approximately half of the candidates who attempted this part of the question knew that $-1 \le \sin\theta \le 1$ but could not deduce that

$-r \le r\sin(2\theta + \alpha) \le r$. Many candidates failed to use the range given for θ and this resulted in their value of θ being incorrect.

Candidates who attempted Part (b) (ii) b) found the values $\frac{1}{12}$ and $\frac{1}{2}$ but could not distinguish the minimum and maximum values.

Solutions

(a) (ii) $y = 0, \pi, 2\pi$

(b) (i) $r\,\sin(2\theta + \alpha) = 5\,\sin(2\theta + 0.927^{c})$

(b) (ii) a) $\theta = 1.89^{c}$

b) minimum $= \frac{1}{12}$ and maximum $= \frac{1}{2}$

Recommendations

Proofs of trigonometric identities should involve expressing reciprocal ratios in terms of sine and cosine. Generally, either the left hand side (LHS) or the right hand side (RHS) of an equation is simplified but in some cases both sides require simplification. Candidates must be able to choose the most suitable formulae or ratios to simplify expressions. This is best achieved by repeated practice.

The concepts of *minimum and maximum of reciprocal functions* are related to inequalities. Candidates will better understand these concepts using the properties of reciprocals.

Question 4

Specific Objectives: Content (B) 1, (B) 4, 5, 6, (C) 1, 6, 7

This question tested geometry of the circle; the locus of a point satisfying given properties; the Cartesian equation of a curve given its parametric representation; three-dimensional vectors; the use of the modulus and the scalar product of three-dimensional vectors.

Part (a) (i) was generally well done. Some candidates substituted the coordinates given to show that the required answer is correct. In Part (a) (ii), approximately half of the candidates used the formula for the midpoint of a line between two points to deduce the correct coordinates. Some candidates determined the equation of the circle and found the point of intersection of the circle with L_1. This entailed much more work and time.

A significant number of candidates recognized that the locus of *p* in Part (a) (iii) is a circle since the distance from the given point is fixed. This part of the question was well done.

Part (b) was poorly done. Most candidates who expressed *t* in terms of *x* were able to substitute correctly for *t* in *y*. However, poor algebraic manipulation by the majority of candidates resulted in challenges simplifying the expression of *y* in terms of *x*.

Part (c) (i) was generally well done. A small number of candidates calculated $\overrightarrow{PQ} = \vec{P} + \vec{Q}$ and $\vec{P} - \vec{Q}$.

Most of the candidates who attempted Part (c) (ii) used the scalar product since the property of perpendicular vectors was given in the problem. Some candidates used the Pythagorean method for a right-angled triangle.

Solutions

(a) (ii) B (3, 4) (iii) $(x-2)^2 + (y-3)^2 = 2$ circle: centre (2, 3) radius = $\sqrt{2}$ units.

(b) $x^2 + 2xy - x - y = 0$

(c) (i) $\overrightarrow{PQ} = -4\mathbf{i} + (\lambda + 2)\mathbf{j} + 4\mathbf{k}$ $\overrightarrow{QR} = 3\mathbf{i} + (1 - \lambda)\mathbf{j} - 9\mathbf{k}$ $\overrightarrow{RP} = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$

(ii) $\lambda = 15$

Recommendations

Expressing a curve given by parametric equations into Cartesian form and vice versa requires good algebraic skills and knowledge of trigonometric identities. Eliminating the given parameter will require proper algebraic substitution and simplification. Teachers are advised to use a wide variety of expressions in *x* and *y* given by parametric equations and which can be expressed in Cartesian form using algebraic manipulation. The same applies for determining the parametric equations for curves expressed in Cartesian form.

The concepts of an *angle between vectors*, *modulus of vectors* and *applications of vectors in geometry* are basic fundamentals for use in problem solving. These concepts must be thoroughly understood by students. Teachers are advised to ensure that these topics are covered comprehensively.

Section C

Module 3: Calculus I

<u>Question 5</u>

Specific Objectives: (A) 4, 8, 10, (B) 2 (b), 4 (a), 5 (b),

This question tested the use of simple limits theorems; identification of a point for which a function is continuous; differentiation using first principles, the quotient rule and parametric differentiation.

The majority of candidates attempted the question with fairly satisfactory responses. A small number of candidates obtained full marks and simple arithmetical errors were responsible for a few candidates not obtaining full marks.

Part (a) (ii) was generally understood by most candidates. However, poor substitution and algebraic skills continue to pose difficulties for a number of candidates.

Responses to Part (b) (i) were generally poor. Candidates demonstrated understanding of the concepts of *differentiation from first principles* and used the correct identity to simplify the expression $\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$. However, poor algebraic skills particularly rationalizing surds, hindered further work in this regard. In other cases, the steps required were poorly set out giving rise to very untidy presentations. Candidates used either the quotient rule or the product rule for completing Part (b) (ii). The concepts were fully understood but poor algebraic skills resulted in incorrect answers. Some candidates did not simplify the answer fully.

The major challenge seen in Part (c) was candidates' inability to apply the chain rule for parametric differentiation. Some candidates obtained the Cartesian equation of the curve but could not use it effectively to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Solutions

(a) (i) $a = \frac{1}{3}$ (ii) $b = 7$

(b) (i) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2x\sqrt{x}}$ (ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x+2}{2\sqrt{(x+1)^3}}$

(c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\cot\theta$

Recommendations

Candidates must fully understand the concepts of *left-handed limits, right-handed limits* and *limit at a point*. Further, the concepts of *continuity* and *discontinuity* are very important when determining limits. It must be emphasized that a derivative at a point is the limit of the rate of change as the change approaches that point. Teachers are advised to allow students to determine the limits at a point using an intuitive approach and to confirm the results using a graphical method or the calculator.

Students should be exposed to differentiation of simple expressions from first principles noting that the derivative of f (x) = $\mathrm{f}'(x) = \lim\limits_{\delta x \to 0} \dfrac{\mathrm{f}\left(x+\delta x\right) - \mathrm{f}\,(x)}{\left(x+\delta x\right) - x}$. The use of the chain rule must be applied when differentiating composite functions and parametric equations.

<u>Question 6</u>

Specific Objectives: (B) 8, 11, 12, (C) 1, 2, 4, 5 (b), 6, 7 (c), 8 (c)

This question tested definite integration given boundary conditions; the theorem of the integral of sums being equal to the sum of integrals; the minimum and maximum stationary points of a cubic curve; definite integration using substitution; and the volume of a solid generated by rotating part of a polynomial curve about the x-axis.

Part (a) (i) a) required candidates to find the equation of a curve given boundary conditions. Most candidates performed satisfactorily although a few made arithmetical errors in the calculation of the constant of integration. In Part (a) (i) b), common errors included using the wrong substitution to find the coordinates of y and incorrectly distinguishing between minimum and maximum stationary points. The use of the second derivative to distinguish the nature of stationary points was amply demonstrated. Part (a) (ii) was generally well done. Minor errors included not showing clearly the coordinates of the stationary points and the intercepts of the coordinate axes.

Many candidates demonstrated a good understanding of integration using substitution as required to answer Part (b) (i). Poor algebraic manipulation resulted in some candidates not obtaining the correct function to complete the integration. A few candidates did not effect a change of the limits given for x to the limits for the variable taken for substitution.

Part (b) (ii) required candidates to find the volume of the solid formed when the curve defined was rotated completely about the x-axis. A significant number of candidates quoted an incorrect formula for the volume thus being unable to integrate the function easily.

Solutions

(a) (i) a) $y = x^3 - 2x^2 + x$

b) $\left(\frac{1}{3}, \frac{4}{27}\right)_{\max}$ $(1, 0)_{\min}$

(b) (i) $\frac{2}{3}\left(\sqrt{10^3} - 1\right)$

(ii) Vol = $\frac{544\pi}{15}$ units^3

Recommendations

Teachers must emphasize the importance of including the constant of integration when completing indefinite integration. Integration by substitution requires extensive reinforcement to ensure that students fully understand the processes involved. Instances are seen where students merely substitute for the variable without expressing dx in terms of du if u is the substitution given for x. The resulting function to be integrated becomes a mix of the variable x and the variable u. In addition, no changes are made to the limits given in terms of x.

Students also require practice in finding areas and volumes of regions enclosed by curves and the coordinate axes, curves enclosed by $x = a$ and $x = b$ and two curves.

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (B) 2, F 3, G

This question required candidates to use a given table of a binary operation to determine whether the operation is commutative; name the identity element of a binary operation and determine the inverse of two elements of a set under a binary operation.

Generally, most candidates did not successfully answer this question. They appeared to be unfamiliar with binary operations, as well as the concepts of the *identity* and the *inverse*.

Candidates knew the concepts of *the sum of roots, the sum of products of roots pairwise* and *the product of roots of a cubic equation*. However, the level of algebra required to complete Parts (b) (i) a) and b) was beyond their capacity. Consequently, they were unable to complete Part (b) (ii) successfully.

In Part (c) (i), candidates experienced difficulties sketching a modulus graph given a rational function in *x*. In Part (c) (ii), failure to sketch the correct graphs in (i) resulted in many candidates being unable to solve the equation. The ‘otherwise’ approach seemed out of their scope.

Solutions

(a) (i) * is not commutative

(ii) identity element is e

(iii) a) inverse of *d* is *f*

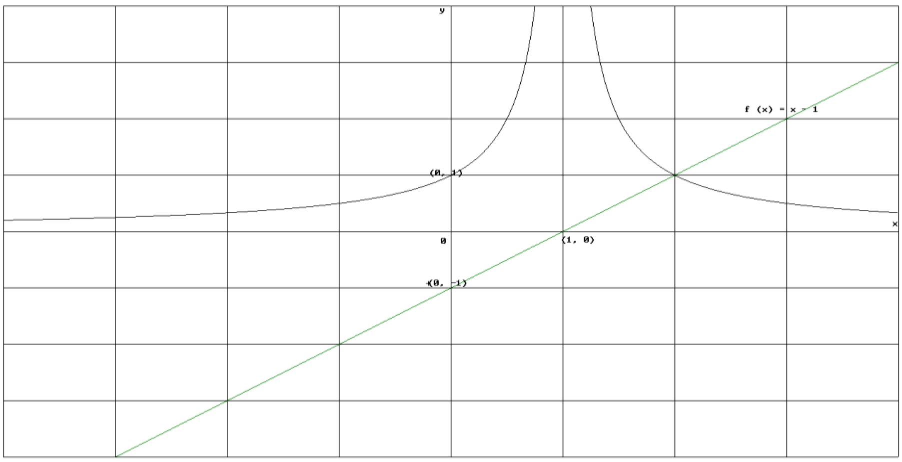
b) inverse of *c* is *c*

(b) (i) a) 17

b) 4

(ii) $x^3 - 17x^2 + 4x - 4 = 0$

(c) (i)



(ii) $x = 2$

Section B

Module 2: Trigonometry, Geometry and Vectors

Question 2

Specific Objectives: (A) 1, (C) 3, 4, 5, 6, 7

This question tested the derivation of displacement vectors, the modulus of vectors, the angle between two vectors and the use of the compound angle to verify the exact value of a trigonometric ratio.

Parts (a) (i) and (ii) were generally well done. However, candidates appeared unsure how to apply the previous results to find the area of the triangle. They experienced challenges determining the height to use in the formula A = $\frac{1}{2}bh$ or could not apply the formula

A = $\frac{1}{2}ab\sin C$.

In Part (b), despite being given the compound angle to use for the required answer, many candidates were not able to reason and use the compound formula for tan (A – B).

Solutions

(a) (i) $\overrightarrow{\mathrm{PQ}} = -2\mathbf{i} - \mathbf{j} + \mathbf{k}$ $\quad \overrightarrow{\mathrm{PR}} = 3\mathbf{k}$

(ii) a) $\left|\overrightarrow{\mathrm{PQ}}\right| = \sqrt{6}$ $\quad \left|\overrightarrow{\mathrm{PR}}\right|$ 3

b) $\cos\theta = \frac{1}{\sqrt{6}}$

c) area = $\frac{3}{2}\sqrt{5}$ units^2

Section C

Module 3: Calculus I

Question 3

Specific Objectives: (A) 4, 5, 6, (B) 1, 14, (C) 8 (b)

This question tested the evaluation of a limit using simple limit theorems; the gradient of a curve at a given point; the equation of a normal to a curve at a given point and the area of a finite region enclosed by two curves.

Part (a) required candidates to use simple limit theorems and the fact that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ to find $\lim_{x \to 0} \frac{\sin 8x}{2x}$. Very few candidates gave any evidence of the requisite knowledge of this topic and performance was generally unsatisfactory. Except for minor algebraic and arithmetic errors, Parts (b) (i) and (ii) were done satisfactorily. The majority of candidates was unable to respond to Part (c).

Solutions

(a) 4

(b) (i) $x = \frac{1}{3}$ $y = \frac{58}{27}$ and $x = -1$ $y = 2$

(ii) $x + y - 1 = 0$

(c) $\frac{1}{3}$ units^2

UNIT 2

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 10 to a maximum of 90. The mean mark for the paper was 61.48 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 145 out of 150. The mean score was 51.33.

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Specific Objectives: (A) 1, 2, 3, 12, 13, (B) 2, 3, 6

This question tested differentiation of ln f (x) and inverse trigonometric functions, using parametric differentiation where the parameter was given as a trigonometric ratio to find a tangent to the curve; the existence of complex roots of a quadratic equation; the use of de Moivre's theorem and the exponential form of a complex number.

Overall, performance on this question was unsatisfactory with approximately two-thirds of candidates earning less than 10 of the 25 marks available and a significant number scoring no marks. However, Part (a) (i) was generally well done by almost all candidates. The use of the chain rule for differentiation and subsequent simplification was successfully done by a small number of candidates. Some candidates found it difficult to work with the coordinates (x, y) to find the equation of the tangent. Limited skills in performing algebraic manipulations continue to be a challenge for candidates.

Almost all candidates seemed well prepared for Part (b) (i) and this part was generally well done. Most candidates successfully used the discriminant to determine the nature of the roots but a few failed to state the nature of the roots.

Part (b) (ii) was well done by the majority of candidates. Some candidates had difficulties stating the correct argument within the required range. In Part (b) (iii), less than half of the candidates demonstrated their ability to use de Moivre's theorem. Those candidates who got the wrong values for the sum required made arithmetic errors. A number of candidates opted to use the expansion of $(\alpha + \beta)^3$ in terms of $\alpha + \beta$ and $\alpha\beta$ with $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. This was done with varying success. In Part (b) (iv), the majority of candidates was familiar with the procedures involved and their responses were satisfactory.

Solutions

(a) (i) $\dfrac{dy}{dx} = -\tan^{-1}\left(\dfrac{x}{2}\right)$

(b) (i) Roots are complex

(ii) $\alpha = 3e^{i(2\pi/3)}$ $\beta = 3e^{i(-2\pi/3)}$

(iii) 54

(iv) $x^2 - 54x + 729 = 0$

Recommendations

More emphasis must be placed on the differentiation of composite functions using the chain rule. Teachers are advised to demonstrate as many applications of de Moivre's theorem as possible. These applications should include simplifying $(a + b\,i)^n$ where $a + bi$ is a complex number and n is real and proof of trigonometric identities.

Question 2

Specific Objectives: (C) 1 (d), 3, 5, 7, 10

This question tested the decomposition into partial fractions of a rational function with a denominator of repeated quadratic factors; definite integration of the rational function and the derivation and use of a reduction formulae to obtain a definite integral.

The formula derived in Part (a) (i) was effectively used by some of the candidates who cascaded the formula to find $F_{(3)}$ (2). However, a number of arithmetical errors were made by candidates who attempted to use the derived formula linearly. In many cases, the succeeding terms were incorrectly factored and arithmetically calculated. A few candidates attempted integration by parts after failing to obtain the reduction formula and not being able to use it effectively. However, the successive integration involved was beyond their capacity.

Part (b) (i) required candidates to decompose into partial fractions a rational function whose denominator included repeated quadratic factors. Approximately 50 per cent of candidates obtained full marks. Generally, candidates appeared to know the concept of *decomposition*. However, poor algebraic manipulation hindered their efforts to find the correct values of the coefficients and constants in the numerators. Incorrect forms of the numerators were some of the errors made.

Candidates followed through with Part (b) (ii) using the partial fractions given in Part (b) (i). The majority of the candidates successfully integrated $\int \frac{1}{y^2+1}\,\mathrm{d}y$ but found $\int \frac{2y}{\left(y^2+1\right)^2}\,\mathrm{d}y$ challenging. In addition, candidates were unable to use the substitution $u = y^2 + 1$.

Solutions

(b) (ii) $\frac{1}{4}(\pi + 2)$

Recommendation

Teachers should assist students with developing the skills of integration by substitution by integrating a variety of functions using simple polynomial, exponential and trigonometric substitutions.

Section B

Module 2: Sequences, Series and Approximation

<u>Question 3</u>

Specific Objectives: (B) 5, 6, 8,

This question tested the concepts of *mathematical induction for the sum of a series*, the *convergent sum of an infinite series* and the *use of the Maclaurin series for the expansion of a given series*.

Candidates' performance in Part (a) (i) was generally unsatisfactory. Candidates continue to demonstrate poor techniques when setting out proofs. Apart from the initial steps of proving the assumption of the statement made, the concept of the statement being true for the $(k + 1)^{\text{th}}$ term is not well understood. As a consequence, the algebra to establish the inductive process is poorly handled. In Part (a) (ii), a number of candidates simply stated that the convergent sum of the series was 2 without supporting working. Also, showing $\lim_{n \to \infty} \frac{1}{2^n} = 0$ was neglected thus denying candidates full marks.

Candidates recorded mixed success in Part (b). Those candidates who attempted to use differentiation for evaluating the coefficients of the required terms found that successive differentiation using the product rule was tedious and fraught with arithmetic and algebraic errors. The product was expressed in forms such as $\left(1 + 2x + x^2\right)\sin(x)$ and $\sin(x) + 2x\sin(x) + x^2\sin(x)$. Candidates who opted to use the formula sheet provided used the expansion $\left(x - \frac{x^3}{6}\right) + x\left(x - \frac{x^3}{6}\right) + x^2\left(x - \frac{x^3}{6}\right)$ and found the series expansion required.

Solutions

(a) (ii) 2

(b) $x + 2x^2 + \frac{5x^3}{6}$

Recommendations

Proof by induction is a sophisticated mathematical process and teachers are encouraged to emphasize the required steps and the algebra needed while teaching the concept.

Maclaurin's series expansion may involve differentiation to determine the coefficients of the polynomial or the use of derived formulae for those functions listed in the Formulae Booklet. As such, candidates should be familiar with all the differentiation skills required for determining coefficients.

<u>Question 4</u>

Specific Objectives: (C) 1, 2, 3, (D) 1, 5

This question tested the use of the binomial theorem to determine specific terms of an expansion; a partial expansion to approximate a numerical value; the use of simple properties of the ${}^{n}C_{r}$ notation; the determination of real roots in a given interval and use of the Newton-Raphson method for approximation.

Part (a) (i) was satisfactorily done by the majority of candidates who demonstrated a sound understanding of the binomial theorem in spite of some arithmetic errors. Part (a) (ii) was also well done by the majority of candidates although a significant number of candidates used an incorrect substitution for x in Part (a) (ii) b).

Part (b) required candidates to use the ${}^{n}C_{r}$ notation to simplify a sum and prove a given result. Most of the candidates exhibited very limited skills in decomposing n! and r! to simplify a common denominator.

Candidates generally experienced challenges in responding to Part (c) (i). Invariably the Intermediate Value Theorem was not quoted and, more importantly, the fact that f (x) must be continuous in the interval [a, b]. Part (c) (ii) was well done with the exception of arithmetic errors made by some candidates.

Solutions

(a) (ii) a)	$1 + 20x + 180x^2$
b)	1.1045
(c)) (ii)	2.20

Recommendations

Students must understand the definition of ${}^{n}C_{r}$. For proofs involving the ${}^{n}C_{r}$ notation the definitions and algebraic expressions should be used. Substitution of numbers for n and r are not accepted for proofs. Teachers are recommended to have extensive demonstrations of this strategy.

Section C

Module 3: Counting, Matrices and Differential Equations

<u>Question 5</u>

Objectives: (A) 2, 6, 13, (B) 1, 2, 7

This question tested the number of possible arrangements at a round table; simple probability theory; operations with conformable matrices and inversion of a 3 x 3 matrix.

In Part (a) (i), candidates were required to determine the number of possible arrangements of persons seated at a round table. From the responses it was clear that the majority of candidates had no knowledge of circular permutation. Further, determining the total number of ways of seating teams of three with the leader in the middle was challenging to most candidates. This part of the question was poorly done. However, Part (a) (ii) was generally well done.

The responses to Part (b) (i) were satisfactory. A number of candidates failed to state the range of values of *x*. In some cases an inequality was not solved hence distinct values of *x* were given. In other cases the range was stated incorrectly.

In Part (b) (ii), a few candidates demonstrated poor understanding of what is required when asked to show a result. Those candidates substituted the value of *x* asked to be shown and worked backwards. Marks were not awarded in those cases.

Part (b) (iii) was fairly well done. There were a few cases where arithmetic errors in calculating the cofactors were seen and there was little evidence of candidates applying the row reduction method to find the inverse of the matrix.

Solutions

(a) (i) $(5-1)! \times 2!^5 = 768$

(a) (ii) a) 0.1

(a) (ii) b) 3000

(b) (i) $x \neq \frac{5}{4}$ or $x < \frac{5}{4}$ and $x > \frac{5}{4}$

(b) (iii) $\frac{1}{7}\begin{pmatrix} -2 & -1 & 6 \\ 4 & 2 & -5 \\ 3 & 5 & -9 \end{pmatrix}$

Recommendations

Teachers are advised to expose students to all forms of permutations and combinations. Arrangements involving circular arrangements and beads on a circular ring are particularly important. It is also suggested that complex arrangements be illustrated using a diagrammatic approach for simplicity.

<u>Question 6</u>

Objectives: (C) 1, 2, 3 (b) (a)

Part (a) (i) was poorly done. Most candidates recognized that an integrating factor was required. However, an incorrectly calculated integrating factor was used and the required answer could not be found.

Candidates used the follow through from Part (a) (i) to complete Part (a) (ii). However, poor algebraic substitution and incorrect values for the constant C resulted in many candidates being unable to obtain full marks.

The majority of candidates successfully found the complementary function in Part (b). It was apparent that many candidates were not familiar with second order differential equations structured in this manner. For the small number of candidates who used the particular solution given, errors in differentiation and poor algebraic skills resulted in incorrect values for A and B.

Solutions

(a) (ii) $y = \sin(x) + \cos(x)$

(b) $y = C_1 + C_2 e^{5x} + \frac{1}{10}x^2 e^{5x} - \frac{1}{25} x e^{5x}$

Recommendations

Differential equations of the form $y' + f(x)y = g(x)$ must be recognized as a first order differential equation which can be solved using the integrating factor $e^{\int f(x)\,dx}$. Extensive tutorials using differentiation of polynomials and trigonometric functions must be done to prepare students for these differential equations.

In cases where the principal integral may be quoted as a general solution, it is recommended that candidates carry out first and second differentials in order to solve the unknown constants. The processes of finding the complementary function and the principal will then be combined for solution of the second order differential equation.

Paper 032 – Alternative to School-Based Assessment (SBA)

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Objectives: (B) 8, (C) 6, 7, 11

This question tested use of the trapezium rule, definite integration of an exponential function using a given substitution and first order partial derivatives.

The majority of candidates who attempted this question did not follow the instruction to use two trapezia and instead used three ordinates to calculate the width, h. In Part (a) (ii), candidates used the substitution $u = e^{-x}$ instead of $u = e^{x}$ as suggested. Further, the liimits were not changed in terms of u and in some cases candidates used $dx = e^{-x}du$ resulting in trying to evaluate $\int_{-1}^{1} \frac{1}{e^x\left(1+u^{-1}\right)}\,du$. Poor algebraic manipulation resulted in many candidates being unable to simplify the fraction in terms of u to allow for simple integration.

In Part (b) (i), many candidates were unable to determine the separate areas and the resulting total area of the box. Partial derivatives seemed unfamiliar to most of the candidates who attempted Part (b) (ii) a). Instead, a few candidates attempted to use implicit differentiation.

In Part (b) (ii) b), since there were no follow through of $\frac{\partial A}{\partial x}$ and $\frac{\partial A}{\partial y}$, candidates were not able to solve for $\frac{\partial A}{\partial x} = 0$ and $\frac{\partial A}{\partial y} = 0$. Candidates who obtained the partial derivatives correctly substituted the values of x and y to obtain the desired result.

Solutions

(a) (i) 1

(a) (ii) 1

(b) (ii) $\frac{\partial A}{\partial x} = y - \frac{1152}{x^2}$ and $\frac{\partial A}{\partial y} = x - \frac{768}{y^2}$

Section B

Module 2: Sequences, Series and Approximation

<u>Question 2</u>

Specific Objectives: (B) 2, 3, 4, 7, (D) 1, 3

This question tested the sum of a finite series using the *method of differences*; use of the partial sums of an arithmetic series to find a particular term; the existence of a root in a given interval using the Intermediate Value Theorem; and linear interpolation.

Part (a) was attempted by all candidates and was generally well done. Simple algebraic manipulation was sufficient to achieve the required result but some candidates used partial fractions to decompose the sum given to *n* terms to show the left hand side. A few candidates attempted to use mathematical induction to answer Part (a) (ii) and errors in the procedure resulted in lost marks.

Parts (b) (i) and (ii) were well done. A few arithmetic errors were evident. A few candidates attempted to use the formula for the n^{th} term in Part (b) (i) with no success.

In Part (c) (i), candidates did not make reference to the Intermediate Value Theorem and continuity of the function in the given interval in establishing the existence of a root. Too many candidates merely showed a change of sign without further information.

Many candidates seemed not to know the term *linear interpolation* as required in Part (c) (ii) but those who used it obtained the correct results. Some candidates opted to use the Newton-Raphson method.

Solutions

(b) (i) $a = \dfrac{136}{25}$ $d = \dfrac{82}{25}$

(b) (ii) $u_{15} = \dfrac{1284}{25}$

(c) (ii) 1.614

Section C

Module 3: Counting, Matrices and Differential Equations

<u>Question 3</u>

Objectives: (A) 6, 15, 17, (C) 1, 2

This question tested the use of a tree diagram to determine probabilities; the solution of a first order differential equation using an integrating factor; and determining the value of this equation given boundary conditions.

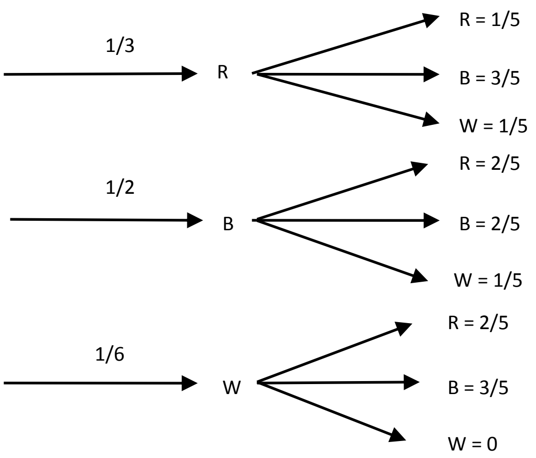
The most satisfactory responses for this question were given for Part (a). However, some candidates appeared unsure when using the tree diagram to determine the probability of the required outcome. They were divided on knowledge of the addition and multiplication rules of probability.

In Part (b) (i), most candidates were able to use the appropriate integrating factor to solve the differential equation. Common errors included incorrectly determining the constant of integration and in some cases omitting the constant of integration. Consequently, the required result could not be shown.

Most candidates did not attempt Part (b) (ii). However, those who attempted to find the limit failed to determine $\lim_{x\to\infty} \mathrm{e}^{-\frac{R}{L}t} = \lim_{x\to\infty} \frac{1}{\mathrm{e}^{-\frac{R}{L}t}} = 0$.

Solutions

(a) (i)



(a) (ii) $\frac{1}{2}$

(b) (ii) $\frac{V}{R}$

Paper 031 – School-Based Assessment (SBA)

A total of 206 Unit 1 and 157 Unit 2 SBAs were moderated this year. Overall, there were notable improvements in the quality of the SBA packages submitted and it was evident that teachers have implemented the suggestions made in the past. Notably, multiple choice tests were not part of any of the SBAs and greater care was made to ensure that the content tested was relevant to the syllabus.

However, teachers are reminded that they should:

- Submit solutions with unitary mark schemes, that is, mark schemes for questions and their subsequent parts which are not broken down to show all single mark allocations
- Submit packages in which the materials are organized according to modules
- Create tests which are neat and professionally done instead of untidy ‘cut and paste’ additions with varying font styles and sizes, scrappily written or missing information
- Write subtotals per question, test totals, instructions, dates and examination duration or time allotted on the question papers
- Include at least one mathematical modelling question in the module test
- Award marks appropriately based on the skills assessed. (Fractional marks are not allowed.)

Teachers *must* pay particular attention to the following guidelines and comments to ensure appropriate and reliable SBAs.

The SBA comprises three separate module tests which *must be administered at school, under examination conditions* with the level of difficulty similar to that of the actual CAPE examination.

The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant unit.
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level.
- Presentation of the sample (Question paper, solutions with unitary mark scheme and students’ scripts should be batched per module).
- Quality of the teachers’ solutions and mark schemes.
- Quality of teachers’ assessments — consistency of marking using the mark schemes.
- Inclusion of mathematical modelling in at least one module test for each unit.

General Comments

- Module tests must be neatly hand written or typed in at least a size 12 font.

- The stipulated time for module tests is 1 to $1\frac{1}{2}$ hours and with a range of 60 to 90 marks awarded. This must be strictly adhered to as candidates may be at an undue disadvantage when module tests are too extensive or insufficient. The following guide can be used: 1 to $1\frac{1}{2}$ minutes per single, skill mark allocation. Approximately 75 per cent of the syllabus should be tested and mathematical modelling *must* be included.

- Multiple choice questions will *not* be accepted in the module tests.

- The moderation process relies on the validity of teachers' assessments. There were instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. There were still situations where the integrity of the assessment was brought into question based on the presentation of the sample submitted. Teachers are reminded that the SBA must be administered under examination conditions at the school. *It is not to be done as a homework assignment or research project*.

- Teachers *must* present evidence of having marked each individual question on students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be written on the front of the students' scripts.

- Teachers *must* indicate any changes/omissions that were made to question papers, solutions and marking schemes and scripts. Students' names on the computer generated form must correspond to the names on the PMATH 1-3 and PMATH 2-3 forms and students' scripts.

- The maximum number of marks for each assessment should be the same for *all* students.

- The number of tests used for the SBA should be the same for *all* students.

- If a student scores zero in an SBA, the script must be sent if that student's name is in the generated sample. Teachers should also inform the examiner about the circumstances regarding missing script(s). A letter *must* be submitted with a full explanation from the school.

- If a student was absent for an assessment then an official letter explaining this absence *must* be sent with the other samples submitted.

To enhance the quality of the design of the module tests, the validity of teachers' assessments and the validity of the moderation process, the following SBA guidelines are listed below for emphasis.

Guidelines for Module Tests and Presentation of Samples

1. <u>Sample Package Considerations</u>
   - Design a separate test for each module. The module test *must* focus on objectives from that module.
   - In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.
   - One sample of *five* students will form the sample for the centre. If there are less than five students, *all* scripts will form the sample for the centre.
   - In 2014, the format of the SBA remained unchanged.

2. <u>Cover Page to Accompany Each Module Test</u>
   The following information is required on the cover of each module test.
   - Name of school and territory, Name of teacher, Centre number.
   - Unit Number and Module Number.
   - Date and duration (1 to $1\frac{1}{2}$ hours**)** of module test.
   - Clear instructions to students.
   - Total marks allotted for module test.
   - Sub-marks and total marks for each question *must* be clearly indicated.

3. <u>Coverage of the Syllabus Content</u>
   - The number of questions in each module test must be appropriate for the stipulated time of (1 to $1\frac{1}{2}$ hours).
   - *CAPE past examination papers should be used as a guide only* and should never appear in an SBA.
   - Duplication of specific objectives and questions must be avoided.
   - Specific objectives tested must be from the relevant unit of the syllabus.

4. <u>Mark Scheme</u>
   - Unitary mark schemes *must* be done on the detailed worked solution, that is, *one mark should be allocated per skill assessed and not 2, 3, 4 etc. marks per skill*.
   - *Fractional / decimal marks must not be awarded, that is, do not allocate* $\frac{1}{2}$ *marks on the mark schemes or students' scripts*.

- The total marks for module tests *must* be clearly stated on teachers' solution sheets.
- A student's mark, that is, the final mark out of 20 *must* be entered on the front page of the student's script.
- Hand-written mark schemes *must be neat and legible*. The *unitary* marks *must* be written on the right side of the page.
- *Diagrams must be neatly drawn with geometrical / mathematical instruments*.

5. Presentation of Sample
    - Candidates' responses *must* be written on A4 (210 x 297 mm) or letter-sized paper ($8\frac{1}{2} \times 11$) ins.
    - Question numbers *must* be written clearly in the left hand margin.
    - The total marks for *each question* on students' scripts **MUST** be clearly written in the left or right margin.
    - *Only original* students' scripts *must* be sent for moderation.
    - Photocopied scripts *will not be accepted***.**
    - Typed Module tests *must be neat and legible***.**
    - The following are required for each module test:
        - ❖ A question paper.
        - ❖ Detailed solutions with detailed unitary mark schemes.
        - ❖ The question paper, detailed solutions, unitary mark schemes and five students' samples should be batched together for each module.
    - Marks recorded on the PMath1–3 and PMath2–3 forms *must* be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded.
    - PMaths 1-4 and PMaths 2-4 forms are for official use *only* and should not be completed by the teacher. However, teachers may complete the relevant information: Centre Code, Name of Centre, Territory, Year of Examination and Name of Teacher(s).
    - The guidelines at the bottom of these forms should be observed. (See page 53 of the syllabus, no. 3, Part b).