CARIBBEAN EXAMINATIONS COUNCIL

CAPE® Applied Mathematics

SYLLABUS
SPECIMEN PAPER
MARK SCHEME
SUBJECT REPORTS

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CAPE® APPLIED MATHEMATICS FREE RESOURCES

LIST OF CONTENTS

Applied Mathematics

The main emphasis of the applied course is on developing the ability of the students to start with a problem in non-mathematical form and transform it into mathematical language. This will enable them to bring mathematical insights and skills in devising a solution, and then interpreting this solution in real-world terms.

Students accomplish this by exploring problems using symbolic, graphical, numerical, physical and verbal techniques in the context of finite or discrete real-world situations. Furthermore, students engage in mathematical thinking and modelling to examine and solve problems arising from a wide variety of disciplines including, but not limited to, economics, medicine, agriculture, marine science, law, transportation, engineering, banking, natural sciences, social sciences and computing.

The syllabus is divided into two (2) Units. Each Unit comprises three (3) Modules.

Unit 1: Statistical Analysis

- Module 1 Collecting and Describing Data
- Module 2 Managing Uncertainty
- Module 3 Analysing and Interpreting Data

Unit 2: Mathematical Applications

- Module 1 Discrete Mathematics
- Module 2 Probability and Distributions
- Module 3 Particle Mechanics

CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Advanced Proficiency Examination®

SYLLABUS

APPLIED MATHEMATICS

CXC A9/U2/22

Effective for examinations from May–June 2023

CAPE®

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Contents

NOTE TO TEACHERS AND LEARNERS

This document CXC A9/U2/*22* replaces CXC A9/U2/07 issued in *2007*.

Please note that the syllabuses have been revised and amendments are indicated by italics.

First Issued 1999
Revised 2004
Revised 2007
Revised 2022

Please check the website www.cxc.org for updates on **CXC®**'s syllabuses.

Please access relevant curated resources to support teaching and learning of the syllabus at https://learninghub.cxc.org/

For access to short courses, training opportunities and teacher orientation webinars and workshops go to our Learning Institute at https://cxclearninginstitute.org/

PLEASE NOTE

This icon is used throughout the syllabus to represent key features which teachers and learners may find useful.

Introduction

The Caribbean Advanced Proficiency Examination (**CAPE®**) is designed to provide certification of the academic, vocational and technical achievement of students in the Caribbean who, having completed a minimum of five years of secondary education, wish to further their studies. The examinations address the skills and know ledge acquired by students under a flexible and articulated system where subjects are organised in 1-Unit or 2-Unit courses with each Unit containing three Modules. Subjects examined under **CAPE®** may be studied concurrently or singly.

The Caribbean Examinations Council offers three types of certification at the **CAPE®** level. The first is the award of a certificate showing each **CAPE®** Unit completed. The second is the **CAPE®** Diploma, awarded to candidates who have satisfactorily completed at least six Units, including Caribbean Studies. The third is the **CXC®** Associate Degree, awarded for the satisfactory completion of a prescribed cluster of *ten* **CAPE®** Units including Caribbean Studies, Communication Studies *and Integrated Mathematics. Integrated Mathematics is not a requirement for candidates pursuing the **CXC®** Associate Degree in Mathematics or pursuing Pure and Applied Mathematics Unit 1 OR 2.* The complete list of Associate Degrees may be found in the **CXC®** Associate Degree Handbook.

For the **CAPE®** Diploma and the **CXC®** Associate Degree, candidates must complete the cluster of required Units within a maximum period of five years. *To be eligible for a **CXC®** Associate Degree, the educational institution presenting the candidates for the award, must select the Associate Degree of choice at the time of registration at the sitting (year) the candidates are expected to qualify for the award.* Candidates will not be awarded an Associate Degree for which they were not registered.

Applied Mathematics Syllabus

♦ RATIONALE

The Caribbean society is an integral part of an ever-changing world. The impact of globalisation on most societies encourages this diverse Caribbean region to revisit the education and career opportunities of our current and future citizens. A common denominator is for Caribbean societies to create among its citizens a plethora of quality leadership with the acumen required to make meaningful projections and innovations for further development. Learning appropriate problem-solving techniques, inherent to the study of mathematics, is vital for such leaders. Mathematics promotes intellectual development, is utilitarian and applicable to all disciplines. Additionally, its aesthetics and epistemological approaches provide solutions fit for any purpose. Therefore, Mathematics is the essential tool to empower people with the knowledge, competencies and attitudes that are required for academia as well as quality leadership for sustainability in this dynamic world.

The main emphasis of the Applied Mathematics course of study is on developing the ability of the students to start with a problem in non-mathematical form and transform it into mathematical language. This will enable them to bring mathematical insights and skills in devising a solution, and then interpreting this solution in real-world terms. Students will accomplish *this through learning and assessment activities that require them to explore* problems using symbolic, graphical, numerical, physical and verbal techniques in the context of finite or discrete real-world situations. Furthermore, students will engage in mathematical thinking and modelling to examine and solve problems arising from a wide variety of disciplines including, but not limited to, economics, medicine, agriculture, marine science, law, transportation, engineering, banking, natural sciences, social sciences and computing.

This course of study incorporates the features of the Science, Technology, Engineering, and Mathematics (STEM) principles. On completion of this Syllabus, students will be able to make a smooth transition to further studies in Mathematics and other related subject areas or move on to career choices where a deeper knowledge of the general concepts of Mathematics is required. It will enable students to develop and enhance Twenty-first century skills including critical and creative thinking, problem solving, logical reasoning, modelling, collaboration, decision making, research and, information communication and technological competencies which are integral to everyday life and for life-long learning. This course thus provides insight into the exciting world of advanced mathematics, thereby equipping students with the tools necessary to approach any mathematical situation with confidence.

This Syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government. *This person will* demonstrate multiple literacies, independent and critical thinking; and *question the beliefs and practices of the past and present, bringing it to bear on the* innovative application of science and technology to problem solving. Such a person should also demonstrate a positive work *ethic* and values and display creative imagination and entrepreneurship. In keeping with the UNESCO Pillars of Learning, on completion of this course of study, students will *learn to know,* learn to do, learn to be, *learn to live together,* and learn to transform themselves and society.

♦ AIMS

The syllabus aims to enable students to:

1. *be* equip*ped* with *the skills needed* for data collection, organisation and analysis in order to make valid decisions and predictions;

2. use appropriate statistical language and form in written and oral presentations;

3. develop an awareness of the exciting applications of Mathematics;

4. develop a willingness to apply Mathematics to relevant problems that are encountered in daily activities;

5. understand certain mathematical concepts and structures, their development and their interrelationships;

6. use *relevant technology* to enhance mathematical investigations;

7. develop the skills of recognising essential aspects of real-world problems and translating these problems into mathematical forms;

8. develop the skills of defining the limitations of the model and the solution;

9. apply Mathematics across the subjects of the school curriculum;

10. acquire relevant skills and knowledge for access to advanced courses in Mathematics and/or its applications in other subject areas;

11. gain experiences that will act as a motivating tool for the use of technology;

12. *develop skills such as, critical and creative thinking, problem solving, logical reasoning, modelling, collaboration, decision making, research, information and communication and technological competencies which are integral to everyday life and for life-long learning; and,*

13. *integrate Information Communication and Technology (ICT) tools and skills in the learning process.*

♦ SKILLS AND ABILITIES TO BE ASSESSED

The assessment will test candidates' skills and abilities in relation to three cognitive levels.

1. **Conceptual knowledge** - *the ability to* ***recall****, and* ***understand*** *appropriate facts, concepts and principles in a variety of contexts.*

2. ***Algorithmic knowledge*** *- the ability to* ***manipulate*** *mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.*

3. ***Reasoning*** *- the ability to* ***select*** *appropriate strategy or select,* ***use*** *and* ***evaluate*** *mathematical models and* ***interpret*** *the results of a mathematical solution in terms of a given real-world problem and engage in problem-solving.*

♦ PREREQUISITES OF THE SYLLABUS

Any person with a **good** grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (**CSEC®**) course in Mathematics, or equivalent, should be able to undertake the course. *However, persons with a **good** grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (**CSEC®**) course in Additional Mathematics would be better prepared to pursue this course of study.* Successful participation in the course will also depend on the possession of good verbal and written communication skills.

♦ STRUCTURE OF THE SYLLABUS

The *syllabus* is organised in two *(2)* Units. A Unit comprises three Modules each requiring 50 hours. The total time for each Unit is, therefore, expected to be 150 hours. Each Unit can independently offer students a comprehensive programme of study with appropriate balance between depth and coverage to provide a basis for further study in this field.

Unit 1: Statistical Analysis

Module 1	-	Collecting and Describing Data
Module 2	-	Managing Uncertainty
Module 3	-	Analysing and Interpreting Data

Unit 2: Mathematical Applications

Module 1	-	Discrete *Mathematics*
Module 2	-	Probability and Distributions
Module 3	-	Particle Mechanics

♦ APPROACHES TO TEACHING THE SYLLABUS

The Specific Objectives indicate the scope of the content and activities that should be covered. Teachers are encouraged to utilise a learner-centered approach to teaching and learning. They are also encouraged to model the process for completing, solving, and calculating mathematical problems. It is recommended that activities to develop these skills be incorporated in every lesson through the use of collaborative, integrative and practical teaching strategies. Note as well that additional notes and the formulae sheet are included in the syllabus.

♦ RECOMMENDED 2-UNIT OPTIONS FOR CAPE® MATHEMATICS

1. Pure Mathematics Unit 1 **AND** Pure Mathematics Unit 2.

2. Applied Mathematics Unit 1 **AND** Applied Mathematics Unit 2.

3. Pure Mathematics Unit 1 **AND** Applied Mathematics Unit 2.

♦ UNIT 1: STATISTICAL ANALYSIS
MODULE 1: COLLECTING AND DESCRIBING DATA

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand the concept of *sampling* and its role in data collection;
2. *develop appropriate skills for data collection and (describing) data analysis; and,*
3. appreciate that data can be represented both graphically and numerically.

SPECIFIC OBJECTIVES	CONTENTS
1. *Data Collection*	
Students should be able to:	
1.1 *determine appropriate sources of data;*	*Advantages and disadvantages of primary and secondary sources of data.*
1.2 distinguish *between types of data;*	*Types of data:* *(a)* primary and secondary; *(b)* *quantitative and qualitative; and,* *(c)* *discrete and continuous.*
1.3 distinguish *between statistical concepts;*	*Explanation of and distinction among statistical concepts:* (a) *population and sample;* (b) *census and sample survey; and,* (c) *parameter and statistic.* *Appropriate use of the concepts.*
1.4 *determine* appropriate sampling frames for given situations;	*Definition of sampling frame.* *Selection of appropriate* sampling frame *from the population.*
1.5 *justify the need for sampling;*	*Reasons for sampling.* *Ideal characteristics of a sample.*

SPECIFIC OBJECTIVES	CONTENTS
Data Collection (cont'd)	
Students should be able to:	
1.6 use sampling methods appropriately;	*Role of randomness in statistical work.* *Random sampling: simple random, stratified random, systematic random.* *Non-random sampling: cluster, quota, convenience and snowballing sampling.* *Advantages and disadvantages of sampling methods.*
1.7 use appropriate methods to obtain a simple random sample;	*Simple Random Methods to include:* *(a) random numbers (from a table or calculator); and,* *(b) lottery techniques.*
1.8 design *data collection instruments*;	*Appropriate use of the following instruments:* (a) *q*uestionnaires; (b) *i*nterviews; and, (c) *o*bservation schedules. *Structure and components of good data collection instruments.*
2. *Describing Data*	
Students should be able to:	
2.1 construct frequency distribution *tables* from raw data;	*Frequency distributions tables for grouped or ungrouped data.*
2.2 *interpret data from frequency distribution tables;*	*Class boundaries, class width, frequency density.*

SPECIFIC OBJECTIVES		CONTENT
Describing Data (cont'd)		
Students should be able to:		
2.3	*use appropriate statistical charts and diagrams to illustrate data;*	*Use the relative advantages and disadvantages to determine appropriate use:* *Pie charts, bar charts, stem-and-leaf diagrams and box-and-whisker plots, histograms, frequency polygons in data analysis.*
2.4	construct *statistical charts and diagrams based on given data;*	*Pie charts, bar charts, stem-and-leaf diagrams (including back-to-back diagrams), box-and-whisker plots, histograms, frequency polygons, cumulative frequency curves (ogives).*
2.5	*calculate measures of central tendency;*	*Ungrouped data:* *Mean, trimmed mean (given percentage from both ends), median, mode.* *Grouped data:* *Mean, mode and median.*
2.6	*use measures of central tendency appropriately;*	Advantages and disadvantages *of various measures of central tendency.* *Impact of outliers on measures of central tendency.*
2.7	calculate *measures of variability; and,*	*Calculations from ungrouped and grouped data:* *Range, quartiles, interquartile range, semi-interquartile range, percentiles, variance, standard deviation.* *From statistical charts, and diagrams and given data.*
2.8	*interpret results from statistical calculations.*	*Using measures of variability and measures of central tendency to determine:* *(a) shape of distribution (uniformity, symmetry and skewness); and,* *(b) impact of outliers.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to participate in small-group projects that require them* to collect data *using an appropriate data collection instrument.*

2. *Students should be guided in gathering secondary data on* the advantages and disadvantages of different sampling methods. *The findings should be presented, using PowerPoint or other technologies, and discussed in class. Students can also be encouraged to reflect on their experiences with the method (s) used to collect data in (a) above.*

3. *Students should be encouraged to, and be guided in, critiquing data collection instruments.*

4. *Students should be encouraged to construct questionnaires using an appropriate programme for example survey gizmo, to obtain information about the subjects, other than Mathematics done by students in the class.*

5. *Teachers are encouraged to incorporate the use of group work in which each group is encouraged to report on the results from the number of tosses of a fair die. Different groups can be asked to carry-out a set number of tosses and report on their results.*

6. *Students should be encouraged to work together to investigate the relationship between the height of a person and the distance with which the student throws a ball. Ask students to record their findings. The findings will be analysed when looking at Unit 1, Module 3.*

7. *Students should be encouraged to work in groups to investigate the number of siblings of students in the class. They should then report on their findings using either a table or graph.*

8. *Teachers are encouraged to demonstrate how* graphical representations such as histograms, pie-charts, box-and-whisker plots should be used for preliminary analysis of data. *Students should then be asked to represent data that has been collected in an appropriate form using the methods discussed.*

9. *Students should be encouraged to use back-to-back stem and leaf diagrams comparing money spent on lunch on different days of the week.*

10. *Teachers should encourage discussions on the relative advantages and usefulness of the mean, quartiles, standard deviation of grouped and ungrouped data and on the shape of frequency distributions. Students should also be guided in utilizing the functions of the calculator to calculate the mean and standard deviation of given values.*

11. *In groups of three or four, students should be required to measure the length of 10 leaves (from the base to the tip of the leaf) from a* ***single tree*** *in the school garden. Using tally charts, these measurements can then be put into groups, and an estimate given of the mean length of the leaves. Students should also calculate the standard deviation of the length.*

UNIT 1
MODULE 1: COLLECTING AND DESCRIBING DATA (cont'd)

RESOURCES

Anderson, D.R., Sweeney, D.J., Williams, T. A., Camm, J.D. and Cochran, J.J.	*Statistics for Business and economics (13th Ed.) Ohio: South-Western college Publisher, 2016.*
Crawshaw, J. and Chambers, J.	*A Concise Course in A-Level Statistics (4th Ed).* Cheltenham: Stanley Thornes Limited, 2001.
Mahadeo, R.	*Statistical Analysis – The Caribbean Advanced Proficiency Examinations A Comprehensive Text*. Trinidad and Tobago: Caribbean Educational Publishers Limited, 2007.
Mann, P.S.	*Introductory Statistics (9th Ed). New Jersey: Wiley, 2016.*
Upton, G. and Cook, I.	*Introducing Statistics*. Oxford: Oxford University Press, 2001.

UNIT 1
MODULE 2: MANAGING UNCERTAINTY

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand the concept of probability;

2. appreciate that probability models can be used to describe real world situations and to manage uncertainty; *and,*

3. *understand how to construct diagrams to facilitate probability problem solving.*

SPECIFIC OBJECTIVES	CONTENTS
1. Probability Theory	
Students should be able to:	
1.1 determine the outcomes of a given experiment;	*Elements of a possibility space.* *Elements of an event.*
1.2 calculate the probability of event A;	*Concept of probability.* $P(A)$, as the number of outcomes of A divided by the total number of possible outcomes $$P(A) = \frac{n(A)}{n(S)}$$
1.3 use the *basic rules of probability;*	*Basic rules:* (a) probability of an event A is a real number between *0 and 1 inclusive;* $(0 \leq P(A) \leq 1)$*;* (b) the sum of all the n probabilities of points in the sample space is 1; *and,* $$\sum_{i=1}^{n} p_i = 1$$

SPECIFIC OBJECTIVES		CONTENT
Probability Theory (cont'd)		
Students should be able to:		
		(c) $P(A') = 1 - P(A)$, where P(Á) is the probability that event *A* does not occur.
1.4	calculate *the probability of the union and the intersection of two sets;*	*Addition rule of probability* *$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.*
1.5	*determine types of* events;	*Types of events:* *(a) Exhaustive events:* $P(A \cup B) = 1$ *(b) Mutually exclusive events:* $P(A \cap B) = 0$ *or* $P(A \cup B) = P(A) + P(B)$. *(c) Independent events:* $P(A \cap B) = P(A) \times P(B)$ *or* $P(A\|B) = P(A)$.
1.6	calculate *the conditional probability; and,*	*Conditional probability:* $P(A\|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0.$ $P(A\|B) = P(A)$ *for independent events.*
1.7	*use possibility space diagrams to calculate probabilities.*	*Construction of possibility space diagrams.* *Possibility space diagrams (probability sample space), tree diagrams, Venn diagrams, contingency tables.*

SPECIFIC OBJECTIVES	CONTENT
2. Random Variables	
Students should be able to:	
2.1 construct probability distribution tables for discrete random variables;	Probability distribution *tables which* assign probabilities to values *of the discrete random variable x.*
2.2 use a given probability function of a discrete random variable;	*Probabilities:* $P(X = a), P(X > a)$, $P(X < b), P(X \geq b), P(X \leq a)$, *or any combination of these, where a and b are real numbers.*
2.3 *use the properties of the probability distribution* of a *discrete* random variable X;	*Properties of discrete random variables.* *0 ≤ P(X = x) ≤ 1 for all* X $\sum_{i=1}^{n} P_i = 1$ expected value $E(X) = \sum_{i=1}^{n} x_i p_i$ variance $Var(X) = E(X^2) - (E(X))^2$ and standard deviation $= \sqrt{Var(X)}$
2.4 use the laws of expectation and variance for one variable;	*Laws of expectation:* $E(aX \pm b)$ *Laws of variance:* $Var(aX \pm b)$
2.5 construct a cumulative distribution function table from a probability distribution table *for discrete random variables*;	*Cumulative distribution function table from a probability density function and vice versa.*
2.6 use cumulative distribution function tables *for discrete random variables;*	*Calculation of probabilities.*

SPECIFIC OBJECTIVES	CONTENT
Random Variables (cont'd)	
Students should be able to:	
2.7 use the properties of a probability density function, $f(x)$ of a continuous random variable X; *and,*	*Continuous random variables.*
	Probability density function.
	$0 \leq f(x) \leq 1$ for all x
2.8 use areas under the graph of a probability density function as measures of probabilities *for continuous random variables*.	*The total area under the graph is 1.*
	Measures of dispersion and Measures of central tendency
	(integration will not be tested), $P(X = a) = 0$ for any continuous random variable X and real number a.
3. Binomial Distribution	
Students should be able to:	
3.1 state the assumptions made in modelling data by a binomial distribution;	*Conditions for discrete data to be modelled as a binomial distribution.*
3.2 use *the binomial distribution as a model of data, where appropriate;*	*Binomial distribution notation.*
	X ~ Bin (n, p), where n is the number of independent trials and p is the probability of a successful outcome in each trial.
3.3 use the mean and variance of a binomial distribution; and,	*Expected value E(X), and variance Var(X), of the binomial distribution.*
3.4 calculate *binomial* probabilities.	*Binomial probabilities*
	P(X = a), P(X > a),
	$P(X < a), P(X \geq a), P(X \leq a)$ *or any combination of these, where X ~ Bin (n, p).*
	$P(X = r) = {}^{n}C_{r}\,p^{r}(1-p)^{n-r}$

SPECIFIC OBJECTIVES		CONTENT
4.	**Normal Distribution**	
Students should be able to:		
4.1	describe the main features of the normal distribution;	*Properties of the normal distribution.*
4.2	use the normal distribution as a model of data *representation*, as appropriate;	*Normal distribution notation,* $X \sim N\ (\mu, \sigma^2)$, *where μ is the population mean and* σ^2 *is the population variance.*
4.3	*standardize the normal distribution;*	*Standardization of normal distribution.* *Standard normal distribution ,* $Z(0,1)$.
4.4	determine probabilities from tabulated values of the standard normal distribution, *Z* ~ N (0, 1);	The standard normal distribution and the use of standard normal distribution tables. $P(Z < a)$
4.5	*de-standardize the variable of the normal distribution;*	*Standard normal distribution ,* $Z(0,1)$.
4.6	*solve problems involving probabilities* z-scores; and,	*Probabilities involving the normal distribution.*
4.7	use the normal distribution as an approximation to the binomial distribution.	*Continuity correction in the context of a normal distribution approximation to a binomial distribution.* *Normal approximation to the binomial distribution (np > 5 and nq > 5), applying an appropriate continuity correction* (± 0.5).

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to use the* probability calculations and the properties of probability based on data obtained from activities carried out under *Description of Data*, Module 1.

2. *Students should be asked to draw a tree diagram to show the total outcomes of tossing a coin three times. They could then be asked to determine the number of outcomes when a coin is tossed four or five times.*

3. *Students should be encouraged to work in groups. Each group should be given a frequency table(s), requiring them to use the results to calculate simple probabilities. Selected groups can share how they arrived at their answers.*

4. *Students should be engaged in discussions* on the concepts of mutually exclusive and independent events using real world concepts. A variety of ways to represent mutually exclusive and independent events should be utilised.

5. *Students should be encouraged to complete activities similar to the following: let the random variable X represent the times that horses take to complete a race on a given race day, and Y represent the times that cars take to complete a race on a race day.*

   (a) *Are X and Y independent events?*

   (b) *Are X and Y mutually exclusive events?*

6. *Students should be encouraged to respond to the scenario 'a die is loaded in such a way that each even number is twice as likely to occur as each odd number'. They should then be asked to calculate P(H), where H is the event that a number greater than 3 occurs in a single roll of the die.*

7. *Students should be encouraged to participate in t class discussions* and activities to clarify the concepts of discrete and continuous random variables.

8. *Students should be encouraged to use their knowledge of Binomial Distribution to do activities similar to the following:*

   (a) *The probability that a student will get exactly 8 correct answers in a multiple choice test which has 15 items and each item has 4 equally likely correct choices.*

   (b) *A shipment of 10 sewing machines includes 3 that are defective. What is the probability that of the 5 machines purchased by a school, exactly 2 are defective?*

9. *Teachers are encouraged to incorporate the use of video presentations explaining when to use normal distribution. Examples should be included which show that the distribution is symmetric (the mean, mode and median are approximately equal).*

10. *Students should be encouraged to sketch a normal curve and locate the mean of the distribution. Students should also be encouraged to shade the area under the curve showing the given or required probability. Allow students to convert from Normal distribution to standard normal distribution.*

    Examples should include:

    (a) *A manufacturer makes two sizes of chocolate bars. The smaller bar has a mean weight of 110 grams with a standard deviation of 2 grams. The weights are normally distributed. Calculate the proportion of bars that are likely to have a weight (a) less than 106 grams;(b) between 108 and 112 grams.*

    (b) *The bigger size bars have a mean weight of 115 grams with the same standard deviation. These weights are also normally distributed. What proportion of the larger bars will have a weight less than the mean of the smaller bars?*

    (c) *It has been determined that the probability that a bank will reject a loan application is 0.20. Calculate the probability that of the 225 loan applications made last week, the bank will reject at most 40.*

    (d) *Using the measurement of the leaves collected in Module one, calculate the median length and the modal length of the leaves collected by the entire class. Discuss the skewness of the distribution of the length of the leaves of the tree.*

11. *Students should be guided in using the data collected from the activity in Unit 1, Module 1 (relationship between height of a student and the distance with which they can throw the ball). Students should be encouraged to do the following:*

    (a) *Draw a scatter diagram for the data.*

    (b) *Determine the independent variable.*

    (c) *Estimate the distance that a person of a given height will throw a ball.*

UNIT 1
MODULE 2: MANAGING UNCERTAINTY (cont'd)

RESOURCES

Anderson, D.R., Sweeney, D.J., Williams, T.A., Camm, J.D. and Cochran, J.J.	*Statistics for Business and economics (13th Ed.). Ohio: South-Western college Publisher, 2016.*
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Mann, P.	*Introductory Statistics (9th Ed). Wiley Global Education, 2016.*
Upton, G. and Cook, I.	*Introducing Statistics. Oxford: Oxford University Press, 2001.*

UNIT 1
MODULE 3: ANALYSING AND INTERPRETING DATA

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand *the uses of* the sampling distribution and confidence intervals *in providing* information about a population;

2. understand the *relevance* of tests of hypotheses regarding statements about a population parameter;

3. *appreciate the use of statistical information to make inferences;*

4. appreciate that finding possible associations between variables and measuring their strengths are key ideas of statistical inference; and,

5. *understand the use of scatter diagrams to illustrate bivariate data.*

SPECIFIC OBJECTIVES	**CONTENT**
1. Sampling Distribution and Estimation	
Students should be able to:	
1.1 *formulate sampling distributions;*	*Distribution of sample mean,* $\bar{X}$ $E(\bar{X}) = \mu$ and Var $(\bar{X}) = \frac{\sigma^2}{n}$ where μ the population mean, σ^2 is the population variance and n is the sample size; *Distribution of sample proportion,* P $E(P) = p_s$ and $Var(P) = \frac{p_s(1-p_s)}{n}$ where p_s is the *sample proportion*, and n is the sample size;

SPECIFIC OBJECTIVES	CONTENT
Sampling Distribution and Estimation (cont'd)	
Students should be able to:	
1.2 *apply the Central Limit Theorem where appropriate;*	*Central Limit Theorem (no proof required).* *If the random variable X has a normal distribution, then the sample mean $\overline{X}$ will have a normal distribution.* *If the random variable X does not have a normal distribution, then the sample size n must be large in order to use the CLT.*
1.3 calculate *point* estimates;	*Unbiased estimators for the population mean, proportion and variance.*
1.4 calculate confidence intervals; *and,*	*Confidence intervals for a population mean or proportion using a large sample drawn from a population of known or unknown variance with the condition that $n \geq 30$.*
1.5 *interpret the confidence intervals.*	*Concept of confidence intervals for a population mean and proportion.* *Probability that the interval contains the mean.*
***2.* Hypothesis Testing**	
Students should be able to:	
2.1 formulate hypothesis *tests*;	Null hypothesis, H_0. *A*lternative hypotheses, H_1.
2.2 *determine* whether a one-tailed test or a two-tailed test is appropriate;	One-tailed (*upper tail or lower tail)* and two-tailed tests.
2.3 *describe the type of errors associated with hypothesis tests;*	*Type I and Type II errors.*

SPECIFIC OBJECTIVES	CONTENT
Hypothesis Testing (cont'd)	
Students should be able to:	
2.4 *determine* the critical values from tables for a given test;	Critical values *(given the level of significance).*
2.5 *identify* the critical *or rejection region* for a given test;	*Use of critical values to identify the critical region or the rejection region/s (given the level of significance).*
2.6 evaluate from sample data the test statistic for testing a population mean or proportion;	*Test statistic for the population mean,* μ $$Z_{test}=\frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$$ *Test statistic for the population proportion* p $$Z_{test}=\frac{p_s-p}{\sqrt{\frac{p(1-p)}{n}}}$$
2.7 apply a z-test; *and,*	Hypothesis test for: (a) a population mean when a sample is drawn from a normal distribution of known variance*;* (b) *a population mean where a large sample* $(n \geq 30)$ *is drawn from a non-normal population using the Central Limit Theorem; and,* (c) *test for proportion when a large sample (n ≥ 30) is drawn from a binomial distribution,* with the appropriate *continuity correction.*
2.8 *interpret the results of the* z*-test.*	*Identification of the critical values and the rejection region for the test.* *Valid conclusion for the test (consider the level of significance).*

SPECIFIC OBJECTIVES	CONTENT
Hypothesis Testing (cont'd)	
3. t-test	
Students should be able to:	
3.1 determine if a t-test is appropriate for a given situation;	*t distribution.* $T \sim t(n-1)$ $n < 30$, *and the population variance is unknown.*
3.2 evaluate the t-test statistic;	t-test statistic $t_{test} = \frac{\overline{X}-\mu}{\frac{\hat{s}}{\sqrt{n-1}}}$
3.3 determine the appropriate number of degrees of freedom for a given data set;	Degrees of freedom *in the context of a t-test.*
3.4 *read* probabilities from *t*-distribution tables;	*Use of t-distribution tables.*
3.5 apply a t*-test* for a *population* mean*; and,*	Hypothesis test for a population mean using a small sample *(n < 30)* drawn from a normal population of unknown variance.
3.6 *interpret the results of the t-test.*	*Identification of the critical values and the rejection region for the test.* *Valid conclusion for the test.*

SPECIFIC OBJECTIVES	CONTENT
4. *Chi-Squared* (χ^2-) Test	
Students should be able to:	
4.1 *formulate a χ^2-test for independence;*	*Null and Alternative Hypotheses for chi-squared tests.* *Hypothesis test for independence. (2×2 contingency tables and cells with expected frequency of less than 5* ***not*** *included).*
4.2 *evaluate the χ^2-test critical value;*	*Degrees of freedom, in context of the Chi-squared test, for a contingency table.* *Reading and interpreting probabilities from χ^2-tables.*
4.3 *evaluate expected frequency for each cell;*	$E_{ij} = \left(\frac{R_i \times C_j}{G}\right),$ *where R_i is the total of the i^{th} row, C_j is the total of the j^{th} column and G is the grand total.*
4.4 *evaluate* the *χ^2-test* statistic; and,	*Chi-squared test for independence* $\chi^2_{test} = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ *Where O_i is the observed frequency, E_i is the expected frequency and N is the total frequency.*
4.5 *interpret the results of the χ^2-test.*	*Identification of the critical values and the rejection region for the test.* *Valid conclusion for the test.*

SPECIFIC OBJECTIVES	CONTENT
5. Correlation and Linear Regression Bivariate Data	
Students should be able to:	
5.1 distinguish between dependent and independent variables;	*Bivariate data.* *Dependent and independent variables.*
5.2 draw scatter diagrams to represent bivariate data;	*Scatter diagrams.*
5.3 *interpret* scatter diagrams;	
5.4 interpret the value of r, as related to the data;	*Calculation of the value of (r), the Product moment correlation coefficient.* *Calculation of S_{xy}, S_{xx}, S_{yy}* *Strength of relationship based on the value of r ($-1 \leq r \leq 1$)* *Correlation Coefficient:* *0 = no correlation* $0 < \|r\| \leq 0.2$ *(very weak)* $0.2 < \|r\| \leq 0.4$ *(weak)* $0.4 < \|r\| \leq 0.6$ *(fair/moderate)* $0.6 < \|r\| \leq 0.8$ *(strong)* $0.8 < \|r\| \leq 1$ *(very strong)* *1 = Perfect correlation*
5.5 *draw* the regression line of y on x or x on y passing through $(\bar{x}, \bar{y})$ on a scatter diagram;	*Calculation of the regression coefficients for the line y on x or x on y using the values of* S_{xy}, S_{xx}, S_{yy} *Estimating the regression line in the form $\hat{y} = a + bx$ where a and b are regression coefficients.*
5.6 *interpret* the regression coefficients;	*Type of relationship.* *Using the regression coefficients in problem solving through practical examples.* *Interpretation of regression coefficients.*
5.7 make estimations using the appropriate *values in the* regression line; and,	*Estimation from regression lines.*

SPECIFIC OBJECTIVES | **CONTENT**

Correlation and Linear Regression Bivariate Data (cont'd)

Students should be able to:

5.8 outline the limitations of simple correlation and regression analyses.

Limitations of simple correlation and regression analyses.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to work in groups to analyse data that has been collected. Students should then be required, using the data, to identify the population and the sample size. Teachers are also encouraged to guide students in discussing how the sample size was determined.*

2. *Students should be engaged in activities that require them to recognise* and use the sample mean, $\bar{X}$, as a random variable.

3. *Students should be encouraged* to obtain the means from samples of size 3 and construct a histogram of the sample means illustrating sampling distribution. *They should then be guided* in repeat*ing* the exercise with increasing sample sizes to illustrate the Central Limit Theorem.

4. *Students should be encouraged to work in groups of two or three, where they will be required to use the results of activities like the leaf collecting activity in Unit 1, Module 1, to d*etermine the sample mean and sample standard deviation from random samples of size 5 chosen from a population of size 50. For each sample, *they should be asked to* calculate a 95 *per cent* confidence interval and determine the number of confidence intervals which contain the population mean. *Groups can be selected to show how they arrived at their answer. Encourage students to use Excel or any other statistical programme or software.*

5. *Students should be guided in using regression and correlation to* test whether there is a relationship, *the type of relationship, and the strength of the relationship* between student performance in *English and student performance in* Mathematics.

UNIT 1
MODULE 3: ANALYSING AND INTERPRETING DATA (cont'd)

RESOURCES

Anderson, D.R., Sweeney, D.J., Williams, T.A., Camm, J.D. and Cochran, J.J.	*Statistics for Business and economics (13th Ed.) Ohio: South-Western college Publisher, 2016.*
Crawshaw, J. and Chambers, J.	*A Concise Course in A-Level Statistics (4th ed.)*. Cheltenham: *Stanley Thornes Limited*, *2001*.
Mahadeo, R.	*Statistical Analysis – The Caribbean Advanced Proficiency Examinations A Comprehensive Text*. San Fernando: Caribbean Educational Publishers Limited, 2007.
Mann, P.	*Introductory Statistics (9th Ed). Wiley Global Education, 2016.*
Upton, G. and Cook, J.	*Introducing Statistics. Oxford: Oxford University Press, 2001.*

UNIT 2: MATHEMATICAL APPLICATIONS
MODULE 1: DISCRETE MATHEMATICS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand the concept of linear programming to formulate models in a real-world context;
2. understand graph theory;
3. understand basic network concepts;
4. understand basic concepts and applications of Boolean Algebra;
5. have the ability to *construct* truth tables to establish the validity of statements; and,
6. appreciate the application of discrete methods in efficiently addressing real-world situations.

SPECIFIC OBJECTIVES		CONTENT
1.	**Linear Programming**	
Students should be able to:		
1.1	*design linear programming models from real-world data;*	*The relationship between variables, and constraints.* *Maximization and minimization of the objective functions.*
1.2	*graph linear inequalities in two variables;*	*Graphical representation of linear inequalities in two variables.* *Shading the side of the line that satisfies the inequality.*
1.3	*determine* the solution set that satisfies a set of linear inequalities in two variables; *and,*	*Identification of feasible (common) region of the inequalities.* *Solution set for linear inequalities in two variables.*
1.4	*determine* a unique optimal solution of a linear programming problem.	*Analysis of the vertices to determine the optimal solution.*

SPECIFIC OBJECTIVES	CONTENT
2. Assignment Models	
Students should be able to:	
2.1 model a weighted assignment (or allocation) problem;	*Models of assignment problems using an m x n matrix where m is the number of rows and n is the number of columns.*
2.2 convert non-square matrix models to square matrix models;	*Dummy variables.*
2.3 convert a maximisation assignment problem into a minimisation; and,	*Maximum and minimum assignment problems (by changing the sign of each entry).*
2.4 solve a maximisation or a minimisation assignment problem *using* the Hungarian algorithm.	*Hungarian algorithm.* *Complexity of 5 x 5 or less.* *The convention of reducing rows before columns will be followed.*
3. Graph Theory, Critical Path Analysis and *Shortest Path*	
Students should be able to:	
3.1 determine *the components of a graph;*	*Graph theory terminology: vertex, edge, path, loop, degree (of a vertex).*
3.2 determine the degree of a vertex;	*Distinguishing among walk, trail and path.*
3.3 *construct activity networks;*	*Networks as models of real-world situations.* *The activity network algorithm in drawing a network diagram to model a real-world problem.* *(Activities will be represented by vertices and the duration of activities by edges.)*

SPECIFIC OBJECTIVES	CONTENT
Graph Theory, Critical Path Analysis and *Shortest Path (cont'd)*	
Students should be able to:	
3.4 *use* activity networks in decision making; *and,*	*Calculation of the earliest start time, latest start time, and float time.* *Identification of the critical path.*
3.5 *determine the shortest path in a network.*	*Dijkstra's algorithm.*
4. Logic and Boolean Algebra	
Students should be able to:	
4.1 formulate *propositions;*	*Propositions (in symbols or on words).* Simple propositions. The negation of simple propositions. Compound propositions that involve conjunctions, disjunctions and negations. Conditional and bi-conditional propositions.
4.2 *construct* truth *tables;*	*Truth tables.* The negation of simple propositions. Compound propositions that involve conjunctions, disjunctions and negations. Conditional and bi-conditional propositions.

SPECIFIC OBJECTIVES	**CONTENT**
Logic and Boolean Algebra (cont'd)	
Students should be able to:	
4.3 *analyse propositions using* truth tables;	*Truth tables for:* (a) tautology or a contradiction; (b) truth values of the converse, inverse and contrapositive of propositions; *and,* (c) equivalent propositions.
4.4 use the laws of Boolean algebra to simplify Boolean expressions;	*Application of algebra of propositions to mathematical logic.* *Idempotent, complement, identity, commutative, associative, distributive, absorption, de Morgan's Law.*
4.5 derive a Boolean expression from a given switching *circuit or logic gate*;	*Application of Boolean algebra to switching circuits and logic gates.*
4.6 represent a Boolean expression by a switching *circuit or logic gate*; and,	
4.7 use switching *circuits and logic gates* to model real-world situations.	

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to complete activities requiring them to design activity networks for local community projects (for example a small community center). They should then be required to establish the critical path to ensure that the building is completed as scheduled with a given budget.*

2. *Students should be guided in using the technique above (1) to map the shortest route from the airport to a local cricket field.*

3. *Teachers should incorporate activities which allows students to participate in classroom discussions. For example, discussions on* the route of a postman in a community with a crisscross of streets and houses situated on both sides of the street. *Students should then work in groups to plan* the route that best serves to save on time and avoid returning along the same street. A simple diagram may be used.

4. *Teachers are encouraged to present the following scenario students 'a furniture company produces dining tables and chairs. The production process for each is similar in that both require a certain number of hours for carpentry work and a certain number of labour hours in the finishing department. Each table takes 4 hours of carpentry and 2 hours in the finishing department. Each chair requires 3 hours of carpentry and 1 hour in the finishing department. The company has available 240 hours of carpentry time and 100 hours of finishing time'. Students should then be asked to use the Linear Method to do the following. Selected students can then be asked to share their responses.*

   (a) *Construct the necessary inequations given and a well labelled graph, clearly indicating the feasible region, to represent the information.*

   (b) *Using the fact that each table can be sold for a profit of $7 and each chair produced can be sold for a profit of $5, find the best combination of tables and chairs that the company should manufacture in order to reach the maximum profit.*

5. *Students should be encouraged to work in small groups where they will apply their knowledge of logic to guide a discussion on the scenario proposed that, 'Justin argues that it is bad to be depressed and watching the news makes him feel depressed. Therefore, it's good to avoid watching the news'. Each group will then be required to:*

   (a) *Write the statements in symbolic form.*

   (b) *Construct a truth table to show the argument.*

   (c) *Use Boolean algebra to prove his argument.*

RESOURCES

Bloomfield, I. and Stevens, J.	*Discrete & Decision.* Cheltenham: Nelson and Thornes, *2002.*
Bloomfield, I. and Stevens, J.	*Discrete & Decision.* Cheltenham: Teacher Resource File, *2002.*
Bryant, V.	*Advancing Mathematics for AQA Discrete Mathematics I. Oxford: Heinemann Educational Publishers, 2001.*
Peter, G. W. (Ed.)	*Discrete Mathematics.* Oxford: Heinemann Educational, 1992.
Ramirez, A. and Perriot, L.	*Applied Mathematics. Barbados: Caribbean Examinations Council, 2004.*

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *understand the use of* counting techniques and calculus in probability;
2. appreciate that probability models can be used to describe real-world situations;
3. *understand how to utilize* appropriate distributional approximations to data; and,
4. *appreciate* the appropriateness of distributions to data.

SPECIFIC OBJECTIVES	CONTENT
1. Probability	
Students should be able to:	
1.1 *apply counting principles to probabilities;*	*Counting principles:* *Permutations* – number of ordered arrangements of *n* objects taken *r* at a time, with or without restrictions. *Combinations* – number of selections of *n* objects taken *r* at a time, with or without restrictions.
1.2 calculate probabilities of events (which may be combined by unions or intersections) using appropriate counting techniques;	*Probability – union and intersection of events.*
1.3 calculate *probabilities associated with conditional, independent or mutually exclusive events; and,*	*Probabilities associated with conditional, independent or mutually exclusive events.*
1.4 *use the results from probabilities to make decisions.*	*Data driven decision making from probability analysis.*

SPECIFIC OBJECTIVES	CONTENT
2. Discrete Random Variables	
Students should be able to:	
2.1 use a *given* probability function;	*f(x) = P(X = x) where f is a simple polynomial or rational function.*
2.2 use the laws of expectations and variance of a linear combination of independent random variables;	*Expectation and variance of a linear combination of independent random variables.* *Laws of expectation:* $E(aX \pm bY)$ $E(X_1 + X_2 + X_3 + \cdots + X_n)$ *Laws of variance:* $Var(aX \pm bY)$ $Var(X_1 + X_2 + X_3 + \cdots + X_n)$
2.3 model practical situations in which the discrete *distributions* are suitable;	*Discrete distributions: uniform, binomial, geometric and Poisson.*
2.4 *calculate probabilities for discrete random variables;*	*uniform:* X *is* $r(\frac{1}{b-a})$ $P(X = x) = \frac{1}{n}$, *where* $x = x_1, x_2, ..x_n$; *Binomial:* X *is* $Bin(n, p)$ *P (X = x) =* ${}^nC_x\, p^x (1-p)^{n-x}$ *x = 0, 1,2, …n;* *Geometric:* X *is* $Geo(p)$ $P(X = x) = q^{x-1}p$, *where x = 1, 2, 3.;* *Poisson:* X *is* $Po(\lambda)$ *P (X = x)=*$\frac{\lambda^x e^{-\lambda}}{x!}$, *x = 0, 1, 2, 3..;*

SPECIFIC OBJECTIVES	CONTENT
Discrete Random Variables (cont'd)	
2.5 *use* the formulae for *E(X) and Var(X) of discrete;* and,	*Expectation and variance where X follows:* *Uniform* $E(X) = \frac{\sum x}{n}$ *OR* $E(X) = \sum x P(X = x)$ $Var(X) = E(X^2) - (E(X))^2$ *OR* $E(X) = \frac{n+1}{2}$ $Var(X) = \frac{n^2 - 1}{12}$ *Where x= 1,2,3,…n* *Binomial* $E(X) = np$ $Var(X) = n(1 - p)$ *Geometric* $E(X) = \frac{1}{p}$ $Var(X) \frac{q}{p^2}$ *Poisson* $E(X) = \lambda$ $Var(X) = \lambda$
2.6 *use* the Poisson distribution as an approximation to the binomial distribution, where appropriate ($n > 50$ and $np < 5$).	*Poisson approximation to binomial distribution.*

SPECIFIC OBJECTIVES	CONTENT
3. Continuous Random Variables	
3.1 *calculate probabilities* of a continuous random variable X;	*Application of the properties of the probability density function.* $0 \leq f(x) \leq 1$ $\int_{-\infty}^{+\infty} f(x)\,dx = 1$ Where f is a probability density function (f will be restricted to simple polynomials)
3.2 *determine the* cumulative distribution function;	*Cumulative distribution function.* $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$
3.3 *calculate probabilities in a continuous distribution;*	*Probability.* $P(a \leq X \leq b) = \int_a^b f(x)dx = F(b) - F(a)$
3.4 *calculate measures of central tendency for a continuous distribution;*	*Expected value and medians using integration.*
3.5 *calculate measures of dispersion of a continuous distribution;*	*Variance, standard deviation, quartiles and percentiles using integration.*
3.6 *interpret various statistics related to continuous distributions; and,*	*Measures of central tendency.* *Measures of dispersion.*
3.7 use the normal distribution, *as an approximation to* the Poisson distribution.	*Normal approximation to the Poisson distribution* $(\lambda > 15)$ with a continuity correction as appropriate.

SPECIFIC OBJECTIVES	CONTENT
4. ***Chi-squared*** *(*χ^{2}*)* **test**	
Students should be able to:	
4.1 *formulate a* χ^2 *goodness-of-fit test;*	*Goodness–fit–test with appropriate number of degrees of freedom.* *Hypotheses test statistic.*
4.2 *evaluate expected frequency;*	*Expected values for situations modelled by a given ratio, discrete uniform, binomial, geometric, Poisson or normal distribution will be tested.* *Classes should be combined in cases where the expected frequency is less than 5.*
4.3 *evaluate the* χ^2 *critical value;*	*Degrees of freedom, critical values, and rejection region in context of the Chi-squared test.* *Reading and interpreting the* χ^2*-table.*
4.4 *evaluate the* χ^2 *test statistic; and,*	*Chi-squared test for goodness-fit* $$\chi^2{}_{test} = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$ *Where* O_i *is the observed frequency,* E_i *is the expected frequency and n is the number of groups.*
4.5 *use the results of the* χ^2 *test in problem solving.*	*Identification of the critical values and the rejection region for the test.* *Valid conclusion for the test.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to work in groups of three or four to complete the following:*

   Five friends, three girls and two boys, are waiting in a line to purchase lunch from a fast-food restaurant. However, the COVID-19 protocols restrict them from going to the cashier at the same time. As a result only one can go at a time.

   (a) Determine the number of different lines that they can make to go to the cashier.

   (b) If Mario wants to be third to order his lunch, in how many ways can they make their orders.

   (c) If Leroy insists of being first in line and Renee agrees to the last, how many lines can they make.

   (d) Calculate the probability that the three girls get their lunch first.

2. *Teacher should guide a class discussion on whether absenteeism from school is determined by the day of the week. Students should then be encouraged to work in groups to use a survey to gather data on the number of students absent by year groups on different days of the week. Using the data collected students should then determine whether the same number of students absent each day. Students should be guided to use their knowledge of Goodness-of-fit tests to:*

   (a) Clearly state their hypothesis.

   (b) Determine which test method they would use.

   (c) Carry out the testing procedures.

   (d) Make an appropriate conclusion of the test.

   Each group should be required to use a PowerPoint presentation or any other presentation method to present their findings to the class.

3. *Teachers are encouraged to model situations in which discrete distributions are appropriate. They can utilise PowerPoint Presentations and YouTube videos to concretize processes to solve problems. Activities like the following should be given to students and responses discussed. For example, students should be asked to name the distribution, giving the value(s) of its parameter(s) which may be used to model each of the following random variables.*

(a) *A is the number of persons in a group of 25 persons who are right-handed, assuming that 70% of all persons are right-handed.*

(b) *B is the number of flaws in a 15 metre length of sheet metal, given that there is an average of 2 flaws in every 10 metres of the sheet metal.*

(c) *C is the number of driving tests that an applicant takes before finally passing the test on the fourth try, if the probability of passing the test is 0.75.*

4. *Students should be encouraged to perform calculations using the Laws of Expectation and Variance.*

RESOURCES

Anderson, D.R., Sweeney, D.J., Williams, T.A., Camm, J.D. and Cochran, J.J.	*Statistics for Business and economics (13th Ed.). Ohio: South-Western college Publisher, 2016.*
Chambers, J., and Crawshaw, J.	A Concise Course in Advanced Level Statistics *(4th Ed.)*. Cheltenham: Stanley Thornes Limited, 2001.
Mann, P.S.	*Introductory Statistics (9th Ed.). New Jersey: Wiley, 2016.*

UNIT 2
MODULE 3: PARTICLE MECHANICS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand forces and their applications;

2. understand the concepts of work, energy and power; and,

3. appreciate the application of mathematical models to the motion of a particle.

SPECIFIC OBJECTIVES		CONTENT
1.	**Coplanar Forces and Equilibrium**	
Students should be able to:		
1.1	*Illustrate how* forces act on a body in a given situation;	*Forces on a body to include but not limited to force normal to the plane, frictional, weight, gravitational, tractive.*
1.2	use vector notation to represent forces;	*Vectors.* *Forces as Vectors (including gravitational forces).*
1.3	*illustrate* the contact force between two surfaces;	*Normal and frictional component.*
1.4	*determine the components* of forces;	*Resolution of forces on particles, in mutually perpendicular directions (including those on inclined planes).*
1.5	calculate the resultant of two or more coplanar forces;	*Two or more coplanar forces.* *Resolving forces - particle in equilibrium* principle that *when a* particle *is* in equilibrium, the vector sum of its forces is zero, (or equivalently the sum of its components in any direction is zero.
1.6	*use Lami's Theorem with* concurrent forces;	*Lami's Theorem.* *Unknown forces and angles.* *Three forces acting at a point in equilibrium.*

SPECIFIC OBJECTIVES	CONTENT
Coplanar Forces and Equilibrium (cont'd)	
Students should be able to:	
1.7 *calculate friction; and,*	*Friction.* The appropriate relationship $F \leq \mu R$ for two *particles* in limiting equilibrium.
1.8 *interpret the results of mathematical solutions.*	*Valid conclusions from calculation (resultants, friction, system in equilibrium).*
2. Kinematics and Dynamics	
Students should be able to:	
2.1 distinguish between *Kinematics of motion in a straight*;	*Kinematics of motion:* *(a) distance and displacement; and,* *(b) speed and velocity.*
2.2 *construct graphs of motion;*	*Velocity-time and displacement-time graphs.*
2.3 *determine kinematic quantities;*	*Displacement, velocity and acceleration.* *Using graphs of motion.* *Using equations of motion.* *Assumptions – Constant acceleration.* *Motion in a straight line.*
2.4 apply Newton's laws of motion *to real world situations;*	*Newton's laws of motion* *Action of constant force.* *Rough or smooth planes.* a constant mass moving in a straight line under the action of a constant force. a particle moving vertically or on an inclined plane with constant acceleration. a system of two connected particles.

SPECIFIC OBJECTIVES	CONTENT
Kinematics and Dynamics (cont'd)	
Students should be able to:	
2.5 *calculate* rates of change *of motion;*	*Differential relationship between displacement (x), velocity (v) and acceleration.* $v = \frac{dx}{dt} = \dot{x}$ $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \ddot{x}$ *Where* $\dot{x}, \ddot{x}$ *represent velocity (v) and acceleration (a) respectively.*
2.6 solve first order differential equations of linear motion; *and,*	*Formulating and solving first-order differential equations as models of the linear motion of a particle when the applied force is proportional to its displacement.* *(Only differential equations where the variables are separable will be required).*
2.7 apply the principle of conservation of linear *momentum.*	*Linear momentum.* *Impulse.* *Direct impact of two inelastic particles moving in the same straight line* *Problems may involve two-dimensional vectors.*
3. Projectiles	
Students should be able to:	
3.1 model the projectile of a particle moving under constant gravitational force (neglecting air resistance);	*Modelling the projectile of a particle (including the use of vectors).*

SPECIFIC OBJECTIVES	CONTENT
Projectiles (cont'd)	
Students should be able to:	
3.2 formulate the equation of the trajectory of a projectile; *and,*	*Horizontal, inclined above and below the horizontal.* (Problems may involve velocity expressed in vector notation.)
3.3 use the equations of motion *for* projectiles*.*	*Properties of a projectile.* *Motion of a projectile.* Quadratic equations. Resolving velocities. *Uniform accelerated motion.* *Time of flight* $T = \dfrac{2V\sin\theta}{g}$ *Greatest height* $H = \dfrac{V^2\sin^2\theta}{2g}$ *Horizontal range =* $R = \dfrac{V^2\sin 2\theta}{g}$ *Equation of the trajectory* $y = x\tan\theta - \dfrac{gx^2}{2V^2\cos^2\theta}$ $y = x\tan\theta - \dfrac{gx^2}{2V^2}\left(1+\tan^2\theta\right)$
4. Work, Energy and Power	
Students should be able to:	
4.1 calculate the work done;	*Work done by a constant force.* *Work done by a variable force in one dimension.*

SPECIFIC OBJECTIVES	CONTENT
Work, Energy and Power (Cont'd)	
Students should be able to:	
4.2 *calculate* kinetic energy and gravitational potential energy;	*Kinetic energy* $= \frac{1}{2}mv^2$ *Gravitational potential energy* $= mgh$ *Where* m *is mass,* v *is velocity,* g *is acceleration due to gravity and* h *is vertical height.*
4.3 *use the work-energy principle in real world situations; and,*	*Principle of conservation of energy.* *Application of the work-energy principle to calculate potential and kinetic energy* (KE) *(including change in kinetic energy).* $Work = \Delta KE$
4.4 *calculate* power.	*Definition of Power.* $P = Fv$ *Where* F *is force (driving or tractive) and* v *is velocity.* $P = \frac{work\ done}{time}$ *Measuring power.* $1\ watt\ (W) = 1\ joule\ (J)\ per\ second$ $1\ kilowatt\ (kW) = 1000\ watts\ (W)$

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. *Students should be encouraged to work in groups in which they will be required to research Newton's Laws of Motion and use examples to show how it should be applied when solving problems.*

Place students in groups and assign tasks outlined below:

2. Draw a diagram of a uniform ladder or beam resting on rough horizontal ground and leaning against a rough (smooth) vertical, with the figure of a man some way up the ladder. Show the forces acting on the ladder and on the man.

3. Draw a diagram of an inclined plane on which a body is placed and is about to be pulled up the plane by a force acting at an angle to the inclined plane. The plane may be smooth or rough. Show all the forces acting on the body. The system may also be considered as the body about to move down the plane.

4. Draw a diagram of a large smooth sphere of weight W resting inside a smooth cylinder and held in place by a small smooth sphere of weight, w. Show the forces acting on the large sphere and on the small sphere.

5. Draw diagrams showing forces acting on a block of wood which is:

   (a) sliding down a rough inclined plane at steady speed; and,

   (b) accelerating down a rough plane.

6. Draw a diagram showing the forces acting on a car which is driven up an incline at steady speed.

7. Draw a diagram of a car towing a caravan on level road and show the forces acting on the car and on the caravan.

8. Show the forces acting if air resistance is present when a stone is thrown through the air.

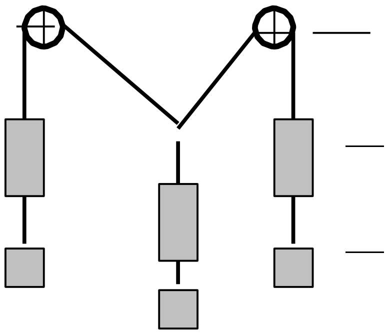
9. A man is standing alone in a moving lift. Draw a diagram to show the forces acting on:

   (a) the man;
   (b) the lift, when it is accelerating upwards;
   (c) the lift, when it is travelling at steady speed; and,
   (d) the lift, when it is accelerating downwards.

10. A railway engine is pulling a train up an incline against frictional resistances. If the combined engine and train are experiencing a retardation, draw diagrams to show the forces acting on the engine and forces acting on the train.

11. Students should be allowed to experiment with a system of three spring balances as illustrated in the figure below to investigate the resultant, resolution and equilibrium of forces.

Resource material: 2 pulleys, string, weights, a sheet of paper, 3 spring balances.



Students should consider:

(a) ***body** is an object to which a force can be applied; and,*

(b) ***particle** is a body whose dimensions, except mass, are negligible.*

RESOURCES

Bostock, L. and Chandler, S.	*Mechanics for A-Level,* Cheltenham. London: Stanley Thornes (Publishers) Limited, *1996*.
Graham, T.	*Mechanics (Collins Advanced Mathematics).* Oxford: *Harper* Collins Educational, *2011*.
Hebborn, J., Littlewood J. and Norton, F.	*Heinemann Modular Mathematics for London AS and A-Level Mechanics 1 and 2.* Oxford: Heinemann Educational Publishers, 1994.
Jefferson, B., and Beadsworth, T.	*Introducing Mechanics,* Oxford: University Press, *2000*.
Price, N. (Editor)	*AEB Mechanics for AS and A-Level*. Oxford: Heinemann Publishers, 1997.
Sadler, A.J., and Thorning, D.W.S.	*Understanding Mechanics.* Oxford: Oxford University Press, 1996.

♦ OUTLINE OF ASSESSMENT

A candidate's performance is reported as an overall grade and a grade on each Module. The assessment comprises two components, one external and one internal.

EXTERNAL ASSESSMENT		(80 %)
Paper 01 ***(1 hour 30 minutes)***	The Paper will consist of forty-five (45) multiple-choice items, fifteen (15) items on each Module. Each item is worth 1 mark *(weighted up to 2).*	30%
Paper 02 (2 hours 30 minutes)	The paper consists of six compulsory extended response questions, *two from each Module. Each question is worth 25 marks.*	50 %

SCHOOL-BASED ASSESSMENT		(20 %)
Paper 031	This paper is intended for candidates registered through schools or *other approved institutions.* **UNIT 1** The paper consists of a project designed and internally assessed by the teacher and externally moderated by **CXC®**. *The project is written work based on personal research or investigation involving collection, analysis, and evaluation of data.* ***UNIT 2*** *The paper consists of a portfolio designed and internally assessed by the teacher and externally moderated by **CXC®**. The portfolio is written work which shows how at least THREE mathematical concepts (ONE from EACH module) can be modules to solve real world problems.* See pages *54-70* for the details on the School Based Assessment (SBA)	
Paper 032 ***(2 hours)***	This paper is an alternative for Paper *031, the School-Based Assessment* and is intended for private candidates. The paper comprises three questions and *tests skills similar to those assessed in the School-Based Assessment.* The duration of the paper is 2 hours. *The case to be assessed in the papers will be given to candidates one week in advance of the examination dates.*	

MODERATION OF SCHOOL-BASED ASSSESMENT

All School-Based Assessment Record of marks must be submitted online using the SBA data capture module of the Online Registration System (ORS) *by stipulated deadlines*. Assignments will be requested by **CXC®** for moderation purposes. These assignments will be reassessed by **CXC®** Examiners who moderate the School-Based Assessment. Teachers' marks may be adjusted as a result of moderation. The Examiners' comments will be sent to schools.

Copies of the students' assignments must be retained by the school until three months after publication by **CXC®** of the examination results.

ASSESSMENT DETAILS

External Assessment by Written Papers (80% of Total Assessment)

<u>Paper 01 (1 hour *30* minutes - 30% of Total Assessment)</u>

1. Composition of papers

(a) This paper consists of forty-five multiple-choice items and is partitioned into three sections (Module 1, 2 and 3). Each section contains fifteen questions.

(b) All *items* are compulsory.

2. Syllabus Coverage

(a) Knowledge of the entire syllabus is required.

(b) The paper is designed to test candidates' knowledge across the breadth of the syllabus.

3. Question Type

Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

(a) Each item is allocated 1 mark, *which will be weighted to 2 marks.*

(b) Each Module is allocated 15 marks, *which will be weighted to 30 marks.*

(c) The total number of marks available for this paper is *45, which will be weighted to 90 marks.*

(d) This paper contributes *30* per cent towards the total assessment.

5. Award of Marks

Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

6. **Use of Calculators**

(a) Each candidate is required to have a silent non-programmable calculator and is responsible for its functioning.

(b) The use of calculators with graphical displays will not be permitted.

(c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.

(d) Calculators must not be shared during the examination.

Paper 02 (*2 hours* 30 minutes – *50 per cent* of Total Assessment)

1. **Composition of Paper**

(a) This paper consists of *six* questions, two questions from each Module.

(b) All questions are compulsory. A question may require knowledge of several topics in a Module. However, all topics in a Module may not be given equal emphasis.

2. **Syllabus Coverage**

(a) Each question may require knowledge from more than one topic in the Module from which the question is taken and will require sustained reasoning.

(b) Each question may address a single theme or unconnected themes.

3. **Question Type**

(a) Questions may require an extended response.

(b) Questions may be presented using words, symbols, diagrams, tables or combinations of these.

4. **Mark Allocation**

(a) Each question is worth *25* marks.

(b) The number of marks allocated to each sub-question will appear in brackets on the examination paper.

(c) Each M*odule* is allocated *50* marks.

(d) The total marks available for this paper is *150.*

(e) The paper contributes *50%* towards the final assessment.

5. **Award of Marks**

(a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

(b) Full marks are awarded for **correct** answers and the presence of **appropriate working**.

(c) It may be possible to earn partial credit for a correct method where the answer is incorrect.

(d) If an incorrect answer in an earlier question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.

(e) A correct answer given with no indication of the method used (in the form of written work) will receive no marks. Candidates are, therefore, advised to show all relevant working.

6. **Use of Calculators**

(a) Each candidate is required to have a silent non-programmable calculator and is responsible for its functioning.

(b) The use of calculators with graphical displays will not be permitted.

(c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.

(d) Calculators must not be shared during the examination.

*7. **Use of Mathematical Tables***

A booklet of mathematical formulae and tables will be provided.

SCHOOL-BASED ASSESSMENT

School-Based Assessment is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills, and attitudes that are associated with the subject. The activities for the School-Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus. *Group work should be encouraged.*

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their S*chool-Based Assessment* assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of *School-Based Assessment*.

The guidelines provided for the assessment of these assignments are intended to assist teachers in awarding marks that are reliable estimates of the achievement of students in the *School-Based Assessmen*t component of the course. In order to ensure that the scores awarded by teachers are not out of line with the **CXC®** standards, the Council undertakes the moderation of a sample of the *School-Based Assessment* assignments marked by each teacher.

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the student at various stages of the experience. This helps to build the self- confidence of students as they proceed with their studies. *School-Based Assessment* also facilitates the development of the critical skills and abilities emphasised by this **CAPE®** subject and enhance the validity of the examination on which candidate performance is reported. *School-Based Assessment*, therefore, makes a significant and unique contribution to both the development of relevant skills and the testing and rewarding of students for the development of those skills.

The Caribbean Examinations Council seeks to ensure that the *School-Based Assessment* scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

CRITERIA FOR THE SCHOOL-BASED ASSESSMENT (SBA) (*Paper 031)*

Unit1

This paper is compulsory and consists of a project. *Candidates have the option to work in small groups (maximum of 5 members in a group) to complete their SBA's*

1. **The aims of the project are to:**

   (a) *develop candidates' ability to work collaboratively;*

   (b) develop candidates' insights into the nature of statistical analysis;

   (c) develop candidates' abilities to formulate their own questions about statistics;

   (d) encourage candidates to initiate and sustain a statistical investigation;

   (e) provide opportunities for all candidates to show, with confidence, that they have mastered the syllabus; and,

   (f) enable candidates to use the methods and procedures of statistical analysis to describe or explain real-life phenomena.

2. **Requirements**

   (a) The project is **written work** based on personal research or investigation involving **collection**, **analysis** and **evaluation** of data.

   (b) Each project should include:

      (i) a statement of the task;

      (ii) description of method of data collection;

      (iii) presentation of data;

      (iv) analysis of data or measures; and,

      (v) discussion of findings.

   (c) The project may *utilise* mathematical modelling, statistical applications or surveys.

   (d) Teachers are expected to guide candidates in choosing appropriate projects that relate to their interests and mathematical expertise.

   (e) Candidates should make use of mathematical and statistical skills from *ALL* Modules.

3. **Integration of Project into the Course**

(a) The activities related to project work should be integrated into the course so as to enable candidates to learn and practise the skills of undertaking a successful project.

(b) Class time should be allocated for general discussion of project work. For example, discussion of how data should be collected, how data should be analysed and how data should be presented.

(c) Class time should also be allocated for discussion between teacher and student, and student and student.

4. **Management of Project**

(a) **Planning**

An early start to planning project work is highly recommended and the schedule of the dates for submission should be developed by teachers and candidates.

(b) **Length**

The project must not exceed 1500 words. The word count does not include: Tables, References, Table of contents, Appendices and Figures. TEN percent of candidates' earned marks will be deducted for exceeding the word limit by 1000 words.

(c) **Guidance**

Each candidate should know the requirements of the project and its assessment process.

Although candidates may consult with resource persons besides the teacher the candidates' submission should be their own work.

Candidates are not expected to work on their own. The teacher is expected to give appropriate guidance at all stages of project work, for example, chapters to read, alternative procedures to follow and other sources of information.

(d) **Authenticity**

Teachers are required to ensure that all projects are the candidates' work.

The recommended procedures are to:

(i) engage candidates in discussion;

(ii) ask candidates to describe procedures used and summarise findings either orally or written; and,

(iii) ask candidates to explain specific aspects of the analysis.

ASSESSMENT CRITERIA FOR THE PROJECT

General

It is recommended that candidates be provided with assessment criteria before commencing the project.

1. For each component, the aim is to find the level of achievement reached by the candidates.

2. For each component, only whole numbers should be awarded.

3. It is recommended that the assessment criteria be available to candidates at all times.

ASSESSING THE PROJECT

The project will be marked out of a total of 60 marks. Marks for the Project will be allocated across Modules in the ratio 1:1:1. The marks earned by a student are assigned to each Module. For example, if a student earns 50 out of 60 for his School-Based Assessment, 50 marks will be assigned to Module 1, 50 marks to Module 2 and 50 marks to Module 3. The total score will be 50+50+50= 150 out of 180. The marks for the portfolio are allocated to each task as outlined below:

Project Descriptors

1.	***Project Title***	***2 marks***
	(a) *Title is clear and concise.* ***(1 mark)*** *(b)* *Title relates to the project.* ***(1 mark)***	
2.	***Purpose of Project***	***6 marks***
	(a) *Background describes the context of the problem.* ***(1 mark)*** *(b)* *Purpose Statement* - *The purpose of the project is relevant to the background.* ***(1 mark)*** - *Method of data analysis (hypothesis testing and/or correlation and regression analysis) is relevant to the purpose of the project.* ***(1 mark)*** - *Method of managing uncertainty is relevant to the purpose of the project.* ***(1 mark)*** *(c)* *Dependent variable(s) accurately identified.* ***(1 mark)*** *(d)* *Independent and/or random variable(s) accurately identified.* ***(1 mark)***	
3.	***Method of Data Collection***	***12 marks***
	(a) *Sampling (selection of a sample):* - *Population is clearly identified.* ***(1 mark)*** - *Clear justification of the use of a sample is presented.* ***(1 mark)*** - *A relevant sampling method is identified.* ***(1 mark)*** - *Justification of the selected sampling method is clearly stated.* ***(1 mark)*** - *Sampling Procedure* ***(4 marks)*** • *Award* ***1 mark*** *for a sequential procedure clearly describing how the sample method was used to select the sample.* • *Award* ***1 mark*** *for a procedure which aligns with the sampling method selected.* • *Award* ***1 mark*** *for stating the sample size.* • *Award* ***1 mark*** *for a sample size that is representative of the population.*	

	(b) *Instruments* ***(4 marks)*** - *Appropriate Data Collection Instruments identified.* ***(1 mark)*** - *Data Collection instruments described.* ***(2 marks)*** • *Award* ***1 mark*** *for description.* • *Award* ***1 mark*** *for evidence of the instrument used. (appendix)* - *Administration of the instrument is clearly described.* ***(1 mark)***	
4.	***Presentation of Data***	***12 marks***
	(a) ***Tables*** - *At least one table used.* ***(1 mark)*** - *An appropriate title is presented.* ***(1 mark)*** - *Table is clearly written (unambiguous and systematic).* ***(1 mark)*** - *Appropriate headers (columns and rows).* ***(1 mark)*** *(b)* ***Graph/Chart (as mentioned in the syllabus)*** - *At least one graph/chart used.* ***(1 mark)*** - *An appropriate title is presented.* ***(1 mark)*** - *Scale/Key.* ***(1 mark)*** - *Correct Labels (Axis/Sectors).* ***(1 mark)*** - *At least 4 correct values used.* ***(4 marks)*** • *Award* ***1 mark*** *for each correct value (candidate table).*	
5.	***Analysis of Data***	***18 marks***
	Probability Analysis ***Candidates must use the statistical technique stated in the purpose. No mark will be awarded for the calculations if the technique stated is not used.*** *(a)* *At least TWO probability calculations attempted.* ***(1 mark)*** *(b)* *Award* ***4 marks*** *for EACH probability calculation attempted as follows.* ***(8 marks)*** - *Accurate Formula/Equation shown.* ***(1 mark)***	

	- *Accurate utilization of the data presented in candidate's tables/charts/graphs.* ***(1 mark)*** - *Accurate steps are shown.* ***Candidates who do not show working will NOT be awarded marks for this section. (2 marks)*** • *Award* ***2 marks*** *if there are no errors.* • *Award* ***1 marks*** *if there are errors.* • *Award* ***zero*** *for completely inaccurate calculations.* ***Hypothesis (Z-test/t-test/Chi-squared) and/or Correlation and Regression Analysis.*** ***Candidates must use the statistical technique stated in the purpose. No mark will be awarded for the calculations if the technique stated is not used.*** *(a) A hypotheses OR correlation and regression analysis attempted.* ***(1 mark)*** *(b) Accurate Formula/Equation stated.* ***(1 mark)*** *(c) Accurate utilization of the data presented.* ***(1 mark)*** *(d) ALL steps of the analysis are shown.* ***(1 mark)*** *(e) Accurate steps are shown.* ***Candidates who do not show working will NOT be awarded marks for this section. (3 marks)*** - *Award* ***3 marks*** *if there are no errors.* - *Award* ***2 marks*** *if there are 1-2 errors.* - *Award* ***1 mark*** *if there are more than 2 errors.* - *Award* ***zero*** *for completely inaccurate calculations.* *(f) Candidate's Correct Answer.* ***(1 mark)***	
6.	***Discussions of Findings/Conclusion***	***7 marks***
	(a) Limitations - *Insights into the nature of the problem encountered in the tasks.* ***(1 mark)*** - *Insights into the resolution of problems encountered in the task.* ***(1 mark)*** *(b) Statement of most findings is clearly identified.* ***(1 mark)*** *(c) Statements of findings follow logically from the data gathered.* ***(1 mark)*** *(d) Conclusion is based on analysis of data.* ***(1 mark)*** *(e) Conclusion is related to the purpose of project.* ***(1 mark)*** *(f) At least ONE recommendation to improve the project is presented.* ***(1 mark)***	

7.	***Communication of Information***	***2 marks***
	- ***Award 2 marks*** *if information is communicated in a logical way using correct grammar, statistical jargon, and symbols most of the time.* - ***Award 1 mark*** *if there are more than two areas requiring improvement.*	
8.	***List of References***	***1 mark***
	References relevant, up to date, written using a consistent convention	
	TOTAL 60 MARKS	
For exceeding the word limit of 1,500 words by 1000 words, deduct 10 percent of the candidate's score.		

Unit 2

This paper is compulsory and consists of a *portfolio*. Candidates are encouraged to work in small groups *(maximum of 5 members in a group)* to complete their SBA's.

1. The aims of the assignment are to:

(a) *develop candidates' ability to work collaboratively;*

(b) enable the student to explore *research* possibilities *in Applied Mathematics*; and,

(c) develop mathematical ideas and communicate using mathematical tools, language and symbols.

2. Requirements

(a) *Candidates are required to create a portfolio which shows how at* ***least*** *THREE mathematical concepts (ONE from EACH module) can be utilised modules to solve real world problems;*

(b) At each stage of the task, the candidates must:

(i) describe and explain clearly their actions and thinking;

(ii) present all data (preferably in a table or chart or diagram or as a set of symbols, equations and inequalities);

(iii) process or analyse data using mathematical skills and available technology;

(iv) state assumptions and expected limitations of the selected process; and,

(v) discuss their expected findings.

(c) The *portfolio may utilise mathematical modelling, demonstrations, and investigations;*

(d) Teachers are expected to guide candidates in choosing appropriate topics that relate to candidates' interests and mathematical expertise. *During the identification-of-the-topic stage, candidates should be required to:*

(i) *list most of the mathematics they expect to use in engaging the topic; and,*

(ii) *ensure that there is substantial mathematical content in their work.*

(e) Candidates should make use of mathematical and statistical skills contained in **ALL THREE** Modules.

3. Integration of portfolio into the course

(a) The activities related to the *Portfolio* should be integrated into the course so as to enable students to learn and practise the skills of undertaking and completing a successful *Portfolio*.

(b) Class time should be allocated for general discussion of *Portfolio* work.

(c) Class time should also be allocated for discussion between teacher and students, and among students.

4. Management of *Portfolio*

(a) Planning

An early start to planning *Portfolio* work is highly recommended and the schedule of the dates for submission should be developed by teachers and candidates.

(b) Length

The Portfolio must not exceed 1500 words. The word count does not include: Tables, References, Table of contents, Appendices and Figures. TEN percent of the candidates' earned marks will be deducted for exceeding the word limit by 1000 words.

(c) *Guidance*

Each candidate should be provided with the requirements of the assignment and its assessment process.

Although candidates may consult with resource persons besides the teacher the candidates' submission should be their own work.

Candidates are not expected to work on their own. The teacher is expected to give appropriate guidance at all stages of project work, for example, chapters to read, alternative procedures to follow and other sources of information.

(d) Authenticity

Teachers are required to ensure that all assignments are the candidates' work. The recommended procedures are to:

(i) *engage candidates in discussion;*

(ii) *require candidates to describe procedures used and summarise findings either orally or written; and,*

(iii) *require candidates to replicate parts of the analysis.*

ASSESSMENT CRITERIA FOR THE *PORTFOLIO*

General

It is recommended that candidates be provided with an assessment criterion before commencing the Assignment.

1. For each component, the aim is to ascertain the level of achievement reached by the candidates.

2. For each component, fractional marks should not be awarded.

3. It is recommended that the assessment criteria be available to candidates at all times.

ASSESSING THE *PORTFOLIO*

The portfolio will be marked out of a total of 60 marks. Marks for the portfolio will be allocated across Modules in the ratio 1:1:1. The marks earned by a student are assigned to each Module. For example, if a student earns 50 out of 60 for his School-Based Assessment, 50 marks will be assigned to Module 1, 50 marks to Module 2 and 50 marks to Module 3. The total score will be 50+50+50= 150 out of 180. The marks for the portfolio are allocated to each task as outlined below:

1.	*Sections A – Module 1* *Section B – Module 2* *Section C – Module 3* ***Each section will be marked out of 19 using the outline below:***	***(57 marks)***
	Statement of Task (4 marks)	
	(a) Background detailing a real-world problem is presented. ***(1 mark)*** *(b) Purpose Statement.* ***(2 marks)*** - *Award* ***1 mark*** *if the purpose stated is relevant to the background.* - *Award* ***1 mark if*** *the purpose statement includes a relevant method of analysis.* *(c) The relevant variable/s to be used is/are identified.* ***(1 mark)***	
	Methodology (15 marks) *Description of the plan for carrying out task (and mathematics involved).*	

	(a) Method of ***solving details*** *how mathematical processes will be carried out correctly in analysing the data.* ***<u>Candidates may use provided data, data from conducting their own investigations or simulated data.</u>***
	
	- *Justification of the appropriateness of the method of solving.* ***(2 marks)***
	
	• *Award* ***1 mark*** *if* ***ALL*** *assumptions/conditions of the method of analysis are presented and accurate. (**1 mark)***
	• *Award* ***1 mark*** *if the justification provided links* ***ALL*** *assumptions/conditions of the method of analysis to the problem described.* ***(1 mark)***
	
	- *Data is presented using appropriate mathematical tools. (**4 marks)***
	
	• *An appropriate title is presented.* ***(1 mark)***
	• *The data presentation is clear and systematic.* ***(1 mark)***
	• *The mathematical tool is appropriately labelled (headings/scale/keys/sectors). (**1 mark)***
	• *The data presentation is without flaws.* ***(1 mark)***
	
	- *Data Analysis* ***(6 marks)***
	
	• *accurate Algorithm/Formula/Equation stated.* ***(1 mark)***
	• *accurate utilization of the data presented.* ***(1 mark)***
	• *ALL steps of the analysis are shown.* ***(1 mark)***
	• *The steps shown are accurate.* ***Candidates who do not show working will NOT be awarded marks for this section. (3 marks)***
	  - *Award* ***3 marks*** *if there are no errors*
	  - *Award* ***2 marks*** *if there are 1-2 errors*
	  - *Award* ***1 mark*** *if there are more than 2 errors*	

	(b) Method of interpreting ***(3 marks)*** - *At least ONE relevant expected limitation/challenge is clearly stated.* ***(1 mark)*** - *At least ONE relevant conclusion based on candidates' analysis is stated.* ***(1 mark)*** - *At least ONE recommendation to improve the project is presented.* ***(1 mark)***	
2.	***Communication of Information (Overall Portfolio)***	***(2 marks)***
	Award 2 marks *if information is communicated in a logical way using correct grammar, statistical jargon, and symbols most of the time.* ***Award 1 mark*** *if there are more than two areas requiring improvement.*	
3.	***List of References (Overall Portfolio)***	***(1 mark)***
	At least ONE relevant reference for EACH section is provided and is written using a consistent convention.	*1*
	TOTAL 60 MARKS	
For exceeding the word limit of 1,500 words by 1000 words, deduct 10 percent of the candidate's score.		

GENERAL GUIDELINES FOR TEACHERS

1. *All School-Based Assessment Record of marks must be submitted online using the SBA data capture module of the Online Registration System (ORS).*

2. **CXC®** requires *that ALL* assignments *are submitted* for external moderation.

3. Teachers should note that the reliability of marks awarded is a significant factor in School-Based Assessment, and has far-reaching implications for the candidate's final grade.

4. Candidates who do not fulfill the requirements of the School-Based Assessment will be considered absent from the whole examination.

5. Teachers are asked to note the following:

   (a) the relationship between the marks for the assignments and those submitted to **CXC®** on the School-Based Assessment form should be clearly shown; *and,*

   (b) the standard of marking should be consistent.

♦ REGULATIONS FOR PRIVATE CANDIDATES

Candidates who are registered privately will be required to sit Paper 01, Paper 02 and Paper 032. Paper 032 will be 2 hours' duration and will contribute 20 per cent of the total assessment of a candidate's performance on that Unit.

The paper comprises THREE questions and tests skills similar to those assessed in the School-Based Assessment. ***This case to be assessed in the papers will be given to candidates ONE week in advance of the examination dates***

♦ REGULATIONS FOR RESIT CANDIDATES

CAPE® candidates may reuse any moderated SBA score within a two-year period. In order to assist candidates in making decisions about whether or not to reuse a moderated SBA score, the Council will continue to indicate on the preliminary results if a candidate's moderated SBA score is less than 50% in a particular Unit. Candidates reusing SBA scores should register as "Resit candidates" and must provide the previous candidate number when registering. Resit candidates must complete Papers 01 and 02 of the examination for the year in which they register.

♦ ASSESSMENT GRID

The Assessment Grid for this Unit contains marks assigned to papers and to Modules, and percentage contributions of each paper to total scores.

Papers	Module 1	Module 2	Module 3	Total	(%)
External Assessment **Paper 01** (1 hour 30 minutes) Multiple Choice	30 *(15 raw)*	*30* *(15 raw)*	*30* *(15 raw)*	*90* *(45 raw)*	(30)
Paper 02 (2 hours 30 minutes) Extended Response	*50*	*50*	*50*	*150*	(50)
Paper 03 ***Paper 031*** **- School Based Assessment (SBA)** **Papers 032** *(2 hours)*	*20* *(60 raw)*	*20* *(60 raw)*	*20* *(60 raw)*	*60* *(180 raw)*	(20)
Total (Weighted)	100	100	100	300	(100)

♦ APPLIED MATHEMATICS NOTATION

The following list summarises the notation used in Applied Mathematics papers of the Caribbean Advanced Proficiency Examinations.

Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x: \ldots\}$	the set of all *x* such that …
n(*A*)	the number of elements in set *A*
$\emptyset$	the empty set
U	the universal set
A′	the complement of the set *A*
W	the set of whole numbers {0, 1, 2, 3, …}
N	the set of natural numbers {1, 2, 3, …}
Z	the set of integers
Q	the set of rational numbers
$\overline{Q}$	the set of irrational numbers
R	the set of real numbers
C	the set of complex numbers
$\subset$	is a subset of
$\not\subset$	is not a subset of
$\cup$	union
$\cap$	intersection
[a, b]	the closed interval $\{x \, \varepsilon \, \mathbf{R} : a \leq x \leq b\}$
(a, b)	the closed interval $\{x \, \varepsilon \, \mathbf{R} : a < x < b\}$
$[a, b)$	the interval $\{x \, \varepsilon \, \mathbf{R} : a \leq x < b\}$
$(a, b]$	the interval $\{x \, \varepsilon \, \mathbf{R} : a < x \leq b\}$

♦ MISCELLANEOUS SYMBOLS

$\equiv$	is identical to
$\approx$	is approximately equal to
$\propto$	is proportional to
∞	infinity

Operations

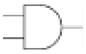
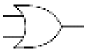
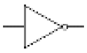
$\sum_{i=1}^{n} x_i$	$x_1 \; x_2 \; \ldots \; x_n$
$\angle \vert x \angle \vert$	the modulus of the real number x
$n!$	n factorial, $1\times 2\times 3\times \ldots\times n$, for $n \varepsilon \mathbf{N}$ $(0! = 1)$
${}^{n}C_{r}$	the binomial coefficient, $\frac{n!}{(n-r)!\,r!}$, for $n \varepsilon \mathbf{R}$, $0 \le r \le n$
${}^{n}P_{r}$	$\frac{n!}{(n-r)!}$

Functions

$\Delta x, \delta x$	an increment of x
$\frac{dy}{dx}, y'$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}, y^{(n)}$	the n^{th} derivative of y with respect to x
$f'(x), f''(x), \cdots, f^{(n)}(x)$	the first, second, …, n^{th} derivatives of $f(x)$ with respect to x
$\dot{x}, \ddot{x}$	the first and second derivatives of x with respect to time
e	the exponential constant
$\ln x$	the natural logarithm of x (to base e)
$\lg x$	the logarithm of x to base 10

Logic

p, q, r	propositions
$\wedge$	conjunction
$\vee$	(inclusive) disjunction
$\sim$	negation
$\rightarrow$	conditionality
$\leftrightarrow$	bi-conditionality
•	implication
$\Leftrightarrow$	equivalence

	AND gate
	OR gate
	NOT gate
T, 1	true
F,0	false

Probability and Statistics

S	the possibility space
A, B, …	the events *A*, *B*, …
$P(A)$	the probability of the event A occurring
P(A′)	the probability of the event not occurring
P(A∠| B)	the conditional probability of the event A given the event B
X, Y, R …	random variables
x, y, r …	values of the random variable X, Y, R …
x_1, x_2, …	observations
f_1, f_2, …	the frequencies with which the observations x_1, x_2, … occur
f(x)	the probability density function of the random variable X
F(x)	the value of the cumulative distribution function of the random variable X
E(X)	the expectation of the random variable X
Var(X)	the variance of the random variable X
μ	the population mean
$\bar{x}$	the sample mean
σ^2	the population variance
s^2	the sample variance
$\hat{\sigma}^2$	an unbiased estimate of the population variance
r	the linear product-moment correlation coefficient
Bin(n, p)	the binomial distribution, parameters n and p
Po(λ)	the Poisson distribution, mean and variance λ
N(μ, σ^2)	the normal distribution, mean μ and variance σ2
N(μ, σ^2)	the normal distribution, mean μ and variance σ2 read σ^2
Z	standard normal random variable
Φ	cumulative distribution function of the standard normal distribution N(0, 1)
χ^2_v	the chi-squared distribution with ν degrees of
t_v	freedom the t-distribution with ν degrees of freedom

Vectors

$\underset{\sim}{a}$, **a**, $\overrightarrow{AB}$	vectors
$\hat{a}$	a unit vector in the direction of **a**
∠|a∠|	the magnitude of **a**
a . b	the scalar product of **a** and **b**
i, **j**, **k**	unit vectors in the direction of the cartesian coordinate axes
$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	$x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

Mechanics

x	displacement
v, $\dot{x}$	velocity
a, $\dot{v}$ $\ddot{x}$	acceleration
μ	the coefficient of
F	force
R	normal reaction
T	tension
m	mass
g	the gravitational
W	work
P	power
I	impulse

♦ GLOSSARY OF EXAMINATION TERMS

WORD	DEFINITION	NOTES
Analyse	examine in detail	
Annotate	add a brief note to a label	Simple phrase or a few words only.
Apply	use knowledge/principles to solve problems	Make inferences/conclusions.
Assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
Calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
Classify	divide into groups according to observable characteristics	
Comment	state opinion or view with supporting reasons	
Compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
Construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.
Deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	
Define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
Demonstrate	show; direct attention to...	

WORD	DEFINITION	NOTES
Derive	to deduce, determine or extract from data by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
Describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
Determine	find the value of a physical quantity	
Design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analysed and presented.
Develop	expand or elaborate an idea or argument with supporting reasons	
Diagram	simplified representation showing the relationship between components	
Differentiate/Distinguish (between/among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
Discuss	present reasoned argument; consider points both for and against; explain the relative merits of a case	
Draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.
Estimate	make an approximate quantitative judgement	
Evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.

WORD	DEFINITION	NOTES
Explain	give reasons based on recall; account for	
Find	locate a feature or obtain as from a graph	
Formulate	devise a hypothesis	
Identify	name or point out specific components or features	
Illustrate	show clearly by using appropriate examples or diagrams, sketches	
Interpret	explain the meaning of	
Investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
Justify	explain the correctness of	
Label	add names to identify structures or parts indicated by pointers	
List	itemise without detail	
Measure	take accurate quantitative readings using appropriate instruments	
Name	give only the name of	No additional information is required.
Note	write down observations	
Observe	pay attention to details which characterise a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
Outline	give basic steps only	
Plan	prepare to conduct an investigation	
Predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	
Record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs,

WORD	DEFINITION	NOTES
		histograms or tables.
Relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
Sketch	make a simple freehand diagram showing relevant proportions and any important details	
State	provide factual information in concise terms outlining explanations	
Suggest	offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalisation which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
Use	apply knowledge/principles to solve problems	Make inferences/conclusions.

♦ GLOSSARY OF MATHEMATICAL TERMS

WORDS	MEANING
Absolute Value	The absolute value of a real number x, denoted by $\|x\|$, is defined by $\|x\| = x$ if $x > 0$ and $\|x\| = -x$ if $x < 0$. For example, $\|-4\| = 4$.
Algorithm	A process consisting of a specific sequence of operations to solve a certain types of problems. See **Heuristic**.
Argand Diagram	An Argand diagram is a rectangular coordinate system where the complex number $x + iy$ is represented by the point whose coordinates are x and y. The x-axis is called the real axis and the y-axis is called the imaginary axis.
Argument of a Complex Number	The angle, $\theta = tan^{-1}\left(\frac{y}{x}\right)$, is called the argument of a complex number $z = x + iy$.
Arithmetic Mean	The average of a set of values found by dividing the sum of the values by the amount of values.
Arithmetic Progression	An arithmetic progression is a sequence of elements, $a1, a2, a3$,….., such that there is a common difference of successive terms. For example, the sequence {2, 5, 8, 11, 14, …} has common difference, $d = 3$.
Asymptotes	A straight line is said to be an asymptote of a curve if the curve has the property of becoming and staying arbitrarily close to the line as the distance from the origin increases to infinity.
Augmented Matrix	If a system of linear equations is written in matrix form $Ax = b$, then the matrix $[A\|b]$ is called the augmented matrix.
Average	The average of a set of values is the number which represents the usual or typical value in that set. Average is synonymous with measures of central tendency. These include the mean, mode and median.
Axis of symmetry	A line that passes through a figure such that the portion of the figure on one side of the line is the mirror image of the portion on the other side of the line.
Bar Chart	A bar chart is a diagram which is used to represent the frequency of each category of a set of data in such a way that the height of each bar if proportionate to the frequency of the category it represents. Equal space should be left between consecutive bars to indicate it is not a histogram.

WORDS	MEANING
Base	In the equation $y = log_a x$, the quantity a is called the base. The base of a polygon is one of its sides; for example, a side of a triangle. The base of a solid is one of its faces; for example, the flat face of a cylinder. The base of a number system is the number of digits it contains; for example, the base of the binary system is two.
Bias	Bias is systematically misestimating the characteristics of a population (parameters) with the corresponding characteristics of the sample (statistics).
Biased Sample	A biased sample is a sample produced by methods which ensures that the statistics is systematically different from the corresponding parameters.
Bijective	A function is bijective if it is both injective and surjective; that is, both one-to-one and unto.
Bimodal	Bimodal refers to a set of data with two equally common modes.
Binomial	An algebraic expression consisting of the sum or difference of two terms. For example, $(ax + b)$ is a binomial.
Binomial Coefficients	The coefficients of the expansion $(x + y)^n$ are called binomial coefficients. For example, the coefficients of $(x + y)^3$ are 1, 3, 3 and 1.
Box-and-whiskers Plot	A box-and-whiskers plot is a diagram which displays the distribution of a set of data using the five number summary. Lines perpendicular to the axis are used to represent the five number summary. Single lines parallel to the axis are used to connect the lowest and highest values to the first and third quartiles respectively and double lines parallel to the axis form a box with the inner three values.
Categorical Variable	A categorical variable is a variable measured in terms possession of quality and not in terms of quantity.
Class Intervals	Non-overlapping intervals, which together contain every piece of data in a survey.
Closed Interval	A closed interval is an interval that contains its end points; it is denoted with square brackets $[a, b]$. For example, the interval $[-1,2]$ contains -1 and 2. For contrast see **open interval**.

WORDS	MEANING
Composite Function	A function consisting of two or more functions such that the output of one function is the input of the other function. For example, in the composite function $f(g(x))$ the input of f is g.
Compound Interest	A system of calculating interest on the sum of the initial amount invested together with the interest previously awarded; if A is the initial sum invested in an account and r is the rate of interest per period invested, then the total after n periods is $A(1+r)^n$.
Combinations	The term combinations refers to the number of possible ways of selecting r objects chosen from a total sample of size n if you don't care about the order in which the objects are arranged. Combinations is calculated using the formula $nCr = \binom{n}{r} = C_r^n = \frac{n!}{r!(n-r)!}$. See **factorial**.
Complex Numbers	A complex number is formed by adding a pure imaginary number to a real number. The general form of a complex number is $z = x + iy$, where x and y are both real numbers and i is the imaginary unit: $i^2 = -1$. The number x is called the real part of the complex number, while the number y is called the imaginary part of the complex number.
Conditional Probability	The conditional probability is the probability of the occurrence of one event affecting another event. The conditional probability of event A occurring given that even B has occurred is denoted $P(A\|B)$ (read "probability of A given B"). The formula for conditional probability is $P(A\|B) = \frac{P(A \text{ and } B)}{P(B)}$.
Conjugate of a Complex Number	The conjugate of a complex number $z = x + iy$ is the complex number $\bar{z} = x - iy$, found by changing the sign of the imaginary part. For example, if $z = 3 - 4i$, then $\bar{z} = 3 + 4i$.
Continuous	The graph of $y = f(x)$ is continuous at a point a if: 1. $f(a)$ exists, 2. $\lim_{x \to a} f(x)$ exists, and 3. $\lim_{x \to a} f(x) = f(a)$. A function is said to be continuous in an interval if it is continuous at each point in the interval.
Continuous Random Variable	A continuous random variable is a random variable that can take on any real number value within a specified range. For contrast, see **Discrete Random Variable.**
Coterminal	Two angles are said to be coterminal if they have the same initial and terminal arms. For example, $\theta = 30^o$ is coterminal with $\alpha = 390^o$.

WORDS	MEANING
Critical Point	A critical point of a function $f(x)$ is the point $P(x, y)$ where the first derivative, $f'(x)$ is zero. See also **stationary points.**
Data	Data (plural of datum) are the facts about something. For example, the height of a building.
Degree	1. The degree is a unit of measure for angles. One degree is $\frac{1}{360}$ of a complete rotation. See also **Radian.** 2. The degree of a polynomial is the highest power of the variable that appears in the polynomial. For example, the polynomial $p(x) = 2 + 3x - x^2 + 7x^3$ has degree 3.
Delta	The Greek capital letter delta, which has the shape of a triangle: Δ, is used to represent "change in". For example Δx represents "change in x".
Dependent Events	In Statistics, two events A and B are said to be dependent if the occurrence of one event affects the probability of the occurrence of the other event. For contrast, see **Independent Events.**
Derivative	The derivative of a function $y = f(x)$ is the rate of change of that function. The notations used for derivative include: $y' = f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$.
Descriptive Statistics	Descriptive statistics refers to a variety of techniques that allows for general description of the characteristics of the data collected. It also refers to the study of ways to describe data. For example, the mean, median, variance and standard deviation are descriptive statistics. For contrast, see **Inferential Statistics.**
Determinant	The determinant of a matrix is a number that is useful for describing the characteristics of the matrix. For example if the determinant is zero then the matrix has no inverse.
Differentiable	A continuous function is said to be differentiable over an interval if its derivative exists for every point in that interval. That means that the graph of the function is smooth with no kinks, cusps or breaks.
Differential Equation	A differential equation is an equation involving the derivatives of a function of one or more variables. For example, the equation $\frac{dy}{dx} - y = 0$ is a differential equation.
Differentiation	Differentiation is the process of finding the derivative.
Discrete	A set of values are said to be discrete if they are all distinct and separated from each other. For example the set of shoe sizes where the elements of this set can only take on a limited and distinct set of

WORDS	MEANING
	values. See **Discrete Random Variables.**
Discrete Random Variable	A discrete random variable is a random variable that can only take on values from a discrete list. For contrast, see **Continuous Random Variables.**
Estimate	The best guess for an unknown quantity arrived at after considering all the information given in a problem.
Even Function	A function $y = f(x)$ is said to be even if it satisfies the property that $f(x) = f(-x)$. For example, $f(x) = \cos x$ and $g(x) = x^2$ are even functions. For contrast, see Odd Function.
Event	In probability, an event is a set of outcomes of an experiment. For example, the even A may be defined as obtaining two heads from tossing a coin twice.
Expected Value	The average amount that is predicted if an experiment is repeated many times. The expected value of a random variable X is denoted by $E[X]$. The expected value of a discrete random variable is found by taking the sum of the product of each outcome and its associated probability. In short, $$E[X] = \sum_{i=1}^{n} x_i p(x_i).$$
Experimental Probability	Experimental probability is the chances of something happening, based on repeated testing and observing results. It is the ratio of the number of times an event occurred to the number of times tested. For example, to find the experimental probability of winning a game, one must play the game many times, then divide by the number of games won by the total number of games played.
Exponent	An exponent is a symbol or a number written above and to the right of another number. It indicates the operation of repeated multiplication.
Exponential Function	A function that has the form $y = a^x$, where a is any real number and is called the base.
Extrapolation	An extrapolation is a predicted value that is outside the range of previously observed values. For contrast, see **Interpolation**.
Factor	A factor is one of two or more expressions which are multiplied together. A prime factor is an indecomposable factor. For example, the factors of $(x^2 - 4)(x + 3)$ include $(x^2 - 4)$ and $(x + 3)$, where $(x + 3)$ is prime but $(x^2 - 4)$ is not prime as it can be further decomposed into $(x - 2)(x + 2)$.
Factorial	The factorial of a positive integer n is the product of all the integers

WORDS	MEANING
	from 1 up to n and is denoted by $n!$, where $1! = 0! = 1$. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
Function	A correspondence in which each member of one set is mapped unto a member of another set.
Geometric Progression	A geometric progression is a sequence of terms obtained by multiplying the previous term by a fixed number which is called the common ratio. A geometric progression is of the form $a, ar, ar^2, ar^3, \ldots$.
Graph	A visual representation of data that displays the relationship among variables, usually cast along x and y axes.
Grouped Data	Grouped data refers to a range of values which are combined together so as to make trends in the data more apparent.
Heterogeneity	Heterogeneity is the state of being of incomparable magnitudes. For contrast, see **Homogeneity.**
Heuristic	A heuristic method of solving problems involve intelligent trial and error. For contrast, see **Algorithm.**
Histogram	A histogram is a bar graph with no spaces between the bars where the area of the bars are proportionate to the corresponding frequencies. If the bars have the same width then the heights are proportionate to the frequencies.
Homogeneity	Homogeneity is the state of being of comparable magnitudes. For contrast, see **Heterogeneity.**
Identity	1. An equation that is true for every possible value of the variables. For example $x^2 - 1 \equiv (x-1)(x+1)$ is an identity while $x^2 - 1 = 3$ is not, as it is only true for the values $x = \pm 2$. 2. The **identity element** of an operation is a number such that when operated on with any other number results in the other number. For example, the identity element under addition of real numbers is zero; the identity element under multiplication of 2×2 matrices is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
Independent Events	In Statistics, two events are said to be independent if they do not affect each other. That is, the occurrence of one event does not depend on whether or not the other event occurred.
Inferential Statistics	Inferential Statistics is the branch of mathematics which deals with the generalisations of samples to the population of values.

WORDS	MEANING
Infinity	The symbol ∞ indicating a limitless quantity. For example, the result of a nonzero number divided by zero is infinity.
Integration	Integration is the process of finding the integral which is the antiderivative of a function.
Interpolation	An interpolation is an estimate of an unknown value which is within the range of previously observed values. For contrast, see **Extrapolation.**
Interval	An interval on a number line is a continuum of points bounded by two limits (end points). An **Open Interval** refers to an interval that excludes the end points and is denoted (a, b). For example, $(0,1)$. A **Closed Interval** in an interval which includes the end points and is denoted $[a, b]$. For example $[-1,3]$. A **Half-Open Interval** is an interval which includes one end point and excludes the other. For example, $[0, \infty)$.
Interval Scale	Interval scale refers to data where the difference between values can be quantified in absolute terms and any zero value is arbitrary. Finding a ratio of data values on this scale gives meaningless results. For example, temperature is measured on the interval scale: the difference between $19^o C$ and $38^o C$ is $19^o C$, however, $38^o C$ is not twice as warm as $19^o C$ and a temperature of $0^o C$ does not mean there is no temperature. See also Nominal, Ordinal and Ratio scales.
Inverse	1. **The inverse of an element under an operation** is another element which when operated on with the first element results in the identity. For example, the inverse of a real number under addition is the negative of that number. 2. **The inverse of a function** $f(x)$ is another function denoted $f^{-1}(x)$, which is such that $f[f^{-1}(x)] = f^{-1}[f(x)] = x$.
Irrational Number	A number that cannot be represented as an exact ratio of two integers. For example, π or the square root of 2.
Limit	The limit of a function is the value which the dependent variable approaches as the independent variable approaches some fixed value.
Line of Best Fit	The line of best fit is the line that minimises the sum of the squares of the deviations between each point and the line.
Linear Expression	An expression of the form $ax + b$ where x is a variable and a and b are constants, or in more variables, an expression of the form $ax + by + c, ax + by + cz + d$ where a, b, c and d are constants.

WORDS	MEANING
Logarithm	A logarithm is the power of another number called the base that is required to make its value a third number. For example 3 is the logarithm which carries 2 to 8. In general, if y is the logarithm which carries a to x, then it is written as $y = \log_a x$ where a is called the base. There are two popular bases: base 10 and base e. 1. **The Common Logarithm (Log)**: the equation $y = \log x$ is the shortened form for $y = \log_{10} x$. 2. **The Natural Logarithm (Ln)**: The equation $y = \ln x$ is the shortened form for $y = \log_e x$
Matrix	A rectangular arrangement of numbers in rows and columns.
Method	In Statistics, the research methods are the tools, techniques or processes that we use in our research. These might be, for example, surveys, interviews, or participant observation. Methods and how they are used are shaped by methodology.
Methodology	Methodology is the study of how research is done, how we find out about things, and how knowledge is gained. In other words, methodology is about the principles that guide our research practices. Methodology therefore explains why we're using certain methods or tools in our research.
Modulus	The modulus of a complex number $z = x + iy$ is the real number $\|z\| = \sqrt{x^2 + y^2}$. For example, the modulus of $z = -7 + 24i$ is $\|z\| = \sqrt{(-7)^2 + 24^2} = 25$
Mutually Exclusive Events	Two events are said to be mutually exclusive if they cannot occur simultaneously, in other words, if they have nothing in common. For example, the event "Head" is mutually exclusive to the event "Tail" when a coin is tossed.
Mutually Exhaustive Events	Two events are said to be mutually exhaustive if their union represents the sample space.
Nominal Scale	Nominal scale refers to data which names of the outcome of an experiment. For example, the country of origin of the members of the West Indies cricket team. See also Ordinal, Interval and Ratio scales.
Normal	The normal to a curve is a line which is perpendicular to the tangent to the curve at the point of contact.
Odd Function	A function is an odd function if it satisfies the property that $f(-x) = -f(x)$. For example, $f(x) = \sin x$ and $g(x) = x^3$ are odd functions. For contrast, see **Even Function**.

WORDS	MEANING
Ordinal Scale	Data is said be in the ordinal scale if they are names of outcomes where sequential values are assigned to each name. For example, if Daniel is ranked number 3 on the most prolific goal scorer at the Football World Cup, then it indicates that two other players scored more goals than Daniel. However, the difference between the 3rd ranked and the 10th ranked is not necessarily the same as the difference between the 23rd and 30th ranked players. See also **Nominal, Interval and Ratio scales**.
Outlier	An outlier is an observed value that is significantly different from the other observed values.
Parameter	In statistics, a parameter is a value that characterises a population.
Partial Derivative	The partial derivative of $y = f(x_1, x_2, x_3, \ldots, x_n)$ with respect to x_i is the derivative of y with respect to x, while all other independent variables are treated as constants. The patrial derivative is denoted by $\frac{\partial f}{\partial x}$. For example, if $f(x, y, z) = 2xy + x^2z - \frac{3x^3y}{z}$, then $\frac{\partial f}{\partial x} = 2y + 2xz - \frac{9x^2y}{z}$
Pascal Triangle	The Pascal triangle is a triangular array of numbers such that each number is the sum of the two numbers above it (one left and one right). The numbers in the n^{th} row of the triangle are the coefficients of the binomial expansion $(x + y)^n$.
Percentile	The *p*th percentile of in a list of numbers is the smallest value such that *p*% of the numbers in the list is below that value. See also **Quartiles.**
Permutations	Permutations refers to the number of different ways of selecting a group of r objects from a set of n object when the order of the elements in the group is of importance and the items are not replaced. If $r = n$ then the permutations is $n!$; if $r < n$ then the number of permutation is $P_r^n = \frac{n!}{(n-r)!}$.
Piecewise Continuous Function	A function is said to be piecewise continuous if it can be broken into different segments where each segment is continuous.
Polynomial	A polynomial is an algebraic expression involving a sum of algebraic terms with nonnegative integer powers. For example, $2x^3 + 3x^2 - x + 6$ is a polynomial in one variable.
Population	In statistics, a population is the set of all items under consideration.
Principal Root	The principal root of a number is the positive root. For example, the principal square root of 36 is 6 (not -6).
Principal Value	The principal value of the arcsin and arctan functions lies on the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. The principal value of the arcos function lies on

WORDS	MEANING
	the interval $0 \leq x \leq \pi$.
Probability	1. The probability of an event is a measure of how likely it is for an event to occur. The probability of an event is always a number between zero and 1. 2. Probability is the study of chance occurrences.
Probability Distribution	A probability distribution is a table or function that gives all the possible values of a random variable together with their respective probabilities.
Probability Space	The probability space is the set of all outcomes of a probability experiment.
Proportion	1. A relationship between two ratios in which the first ratio is always equal to the second. Usually of the form $\frac{a}{b} = \frac{c}{d}$. 2. The fraction of a part and the whole. If two parts of a whole are in the ratio 2:7, then the corresponding proportions are $\frac{2}{9}$ and $\frac{7}{9}$ respectively.
Pythagorean Triple	A Pythagorean triple refers to three numbers, a, b & c, satisfying the property that $a^2 + b^2 = c^2$.
Quadrant	The four parts of the coordinate plane divided by the x and y axes are called quadrants. Each of these quadrants has a number designation. First quadrant – contains all the points with positive x and positive y coordinates. Second quadrant – contains all the points with negative x and positive y coordinates. The third quadrant contains all the points with both coordinates negative. Fourth quadrant – contains all the points with positive x and negative y coordinates.
Quadrantal Angles	Quadrantal Angles are the angles measuring $0^{o}, 90^{o}, 180^{o}$ & 270^{o} and all angles coterminal with these. See **Coterminal.**
Quartic	A quartic equation is a polynomial of degree 4.
Quartiles	Consider a set of numbers arranged in ascending or descending order. The quartiles are the three numbers which divide the set into four parts of equal amount of numbers. The first quartile in a list of numbers is the number such that a quarter of the numbers is below it. The second quartile is the median. The third quartile is the number such that three quarters of the numbers are below it. See also **Percentile.**
Quintic	A quintic equation is a polynomial of degree 5.

WORDS	MEANING
Radian	The radian is a unit of measure for angles, where one radian is $\frac{1}{2\pi}$ of a complete rotation. One radian is the angle in a circle subtended by an arc of length equal to that of the radius of the circle. See also **Degrees**.
Radical	The radical symbol ($\sqrt{}$) is used to indicate the taking of a root of a number. $\sqrt[q]{x}$ means the q^{th} root of x; if $q = 2$ then it is usually written as $\sqrt{x}$. For example $\sqrt[5]{243} = 3$, $\sqrt[4]{16} = 2$. The radical always means to take the positive value. For example, both 5 and -5 satisfy the equation $\mathrm{x}^2 = 25$, but $\sqrt{25} = 5$.
Random Variable	A random variable is a variable that takes on a particular value when a random event occurs.
Ratio Scale	Data are said to be on the ratio scale if they can be ranked, the distance between two values can be measured and the zero is absolute, that is, zero means "absence of". See also Nominal, Ordinal and Interval Scales.
Regression	Regression is a statistical technique used for determining the relationship between two quantities.
Residual	In linear regression, the residual refers to the difference between the actual point and the point predicted by the regression line. That is the vertical distance between the two points.
Root	1. The root of an equation is the same as the solution of that equation. For example, if $y = f(x)$, then the roots are the values of x for which $y = 0$. Graphically, the roots are the x-intercepts of the graph. 2. The n^{th} root of a real number x is a number which, when multiplied by itself n times, gives x. If n is odd then there is one root for every value of x; if n is even then there are two roots (one positive and one negative) for positive values of x and no real roots for negative values of x. The positive root is called the **Principal root** and is represented by the radical sign ($\sqrt{}$). For example, the principal square root of 9 is written as $\sqrt{9} = 3$ but the square roots of 9 are $\pm\sqrt{9} = \pm 3$.
Sample	A group of items chosen from a population.
Sample Space	The set of all possible outcomes of a probability experiment. Also called probability space.
Sampling Frame	In statistics, the sampling frame refers to the list of cases from which a sample is to be taken.
Scientific Notation	A shorthand way of writing very large or very small numbers. A

WORDS	MEANING
	number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (for example, $7000 = 7x10^3$ or $0.0000019 = 1.9x10^{-6}$).
Series	A series is an indicated sum of a sequence.
Sigma	1. The Greek capital letter sigma, Σ, denotes the summation of a set of values. 2. The corresponding lowercase letter sigma, σ, denotes the standard deviation.
Significant Digits	The amount of digits required for calculations or measurements to be close enough to the actual value. Some rules in determining the number of digits considered significant in a number: - The leftmost non-zero digit is the first significant digit. - Zeros between two non-zero digits are significant. - Trailing zeros to the right of the decimal point are considered significant.
Simple Event	A non-decomposable outcome of a probability experiment.
Skew	Skewness is a measure of the asymmetry of a distribution of data.
Square Matrix	A matrix with equal number of rows and columns.
Square Root	The square root of a positive real number n is the number m such that $m^2 = n$. For example, the square roots of 16 are 4 and -4.
Standard Deviation	The standard deviation of a set of numbers is a measure of the average deviation of the set of numbers from their mean.
Stationary Point	The stationary point of a function $f(x)$ is the point $P(x_o, y_o)$ where $f'(x) = 0$. There are three type of stationary points, these are: 1. Maximum point is the stationary point such that $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} \leq 0$; 2. Minimum point is the stationary point such that $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} \geq 0$; 3. Point of Inflexion is the stationary point where $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 0$ and the point is neither a maximum nor a minimum point.
Statistic	A statistic is a quantity calculated from among the set of items in a sample.
Statistical Inference	The process of estimating unobservable characteristics of a population by using information obtained from a sample.
Symmetry	Two points A and B are symmetric with respect to a line if the line is a perpendicular bisector of the segment AB.

WORDS	MEANING
Tangent	A line is a tangent to a curve at a point A if it just touches the curve at A in such a way that it remains on one side of the curve at A. A tangent to a circle intersects the circle only once.
Theoretical Probability	The chances of events happening as determined by calculating results that would occur under ideal circumstances. For example, the theoretical probability of rolling a 4 on a fair four-sided die is ¼ or 25%, because there is one chance in four to roll a 4, and under ideal circumstances one out of every four rolls would be a 4.
Trigonometry	The study of triangles. Three trigonometric functions defined for either acute angles in the right-angled triangle are: Sine of the angle x is the ratio of the side opposite the angle and the hypotenuse. In short, $\sin x = \frac{O}{H}$; Cosine of the angle x is the ratio of the short side adjacent to the angle and the hypotenuse. In short, $\cos x = \frac{A}{H}$; Tangent of the angle x is the ratio of the side opposite the angle and the short side adjacent to the angle. In short $\tan x = \frac{O}{A}$.
Z-Score	The z-score of a value x is the number of standard deviations it is away from the mean of the set of all values. $z - score = \frac{x-\bar{x}}{\sigma}$.

♦ *ADDITIONAL NOTES FOR TEACHING AND LEARNING*

UNIT 1

MODULE 1: COLLECTING AND DESCRIBING DATA

Data Collection

It is critical that students collect data *that can* be used in subsequent Modules. The data collected should also be used as stimulus material for hypothesis testing or linear regression and correlation, by investigating relationships between variables.

Description of Data

Calculators or statistical software should be used whenever possible to display and analyse the collected data. Strengths and weaknesses of the different forms of data representation should be emphasised.

UNIT 1

MODULE 2: MANAGING UNCERTAINTY

Most concepts in this Module are best understood by linking them to the data collected and concepts learnt in Module 1. Only simple arithmetic, algebraic and geometric operations are required for this Module.

Probability Theory

The main emphasis is on understanding the nature of probability as applied to modelling and data interpretation such as the waiting times for taxis and heights of students. Many problems are often best solved with the aid of a Venn diagram, tree diagram or possibility space diagram. Therefore, *students* should be encouraged to draw diagrams *(Venn, tree or possibility space)* as aids or explanations to the solution of problems.

Concepts of possibility spaces and events may be motivated through the practical activities undertaken in *the* Data *Collection section* of *Unit 1,* Module 1.

Random Variables

Clarify the concepts of discrete and continuous random variables.

Examples: Examples of discrete random variables include the number of televisions per household and the number of people queuing at checkouts; while examples of continuous random variables include the waiting times for taxis and heights of students.

It should be emphasised that, for continuous random variables, the area under the graph of a probability density function is a measure of probability and note the important fact that $P(X = a) = 0$.

Normal Distribution

Concept of continuous random variables with particular reference to the normal distribution *should be discussed.* Students should be made aware that a normal distribution N (μ, σ^2) is uniquely defined by its mean, μ, and variance, σ^2. The shape of the normal distribution for varying values of μ, and σ^2 could then be explored.

It can be demonstrated that the binomial distribution may be approximated by the normal distribution.

Example:

Use a graphical calculator to study the graph of $(p + q)^n$ where
$p = 0.25$, $q = 0.75$ and $n = 3, 10, 25, 50, 100$.

Repeat the above activity with different values of $(p + q)^n$ where
$p < 0.25$ and $q > 0.75$ or $(p > 0.25)$ and $(q < 0.75)$ and $n = 3, 10, 25, 50, 100$.

Arithmetic, Algebraic and Geometric Operations required for this Module.

The concepts below showed be discussed.

1. Use of the operations $+, -, \times, \div$ on integers, decimals and fractions.

Arithmetic

2. Knowledge of *real* numbers.

3. Simple applications of ratio, percentage and proportion.

4. Absolute value, $|a|$.

Algebra

1. Language of sets.

2. Operations on sets: union, intersection, complement.

3. Venn diagram and set notation.

4. Basic manipulation of simple algebraic expressions including factorisation and expansion.

5. Solutions of linear equations and inequalities in one variable.

6. Solutions of simultaneous linear equations in two variables.

7. Solutions of quadratic equations.

8. Ordered relations $<, >, \leq, \geq$ and their properties.

Geometry

1. Elementary geometric ideas of the plane.

2. Concepts of a point, line and plane.

3. Simple two-dimensional shapes and their properties.

4. Areas of polygons and simple closed curves.

UNIT 1

MODULE 3: ANALYSING AND INTERPRETING DATA

Allow students to use the data collected in Modules 1 and 2 to facilitate the attainment of the objectives of this Module. Classroom discussions and oral presentations of work done by students, individually or in groups, should be stressed at all times.

Sampling Distributions *and Estimation*

Shoppers often sample a plum to determine its sweetness before purchasing any. They decide from one plum what the larger bunch or lot will taste like. A chemist does the same thing when he takes a sample of rum from a curing vat. He determines *if* it is 90 *per cent* proof and infers that all the rum in the vat is 90 *per cent* proof. If the chemist tests all of the rum, and the shopper tastes all the plums, *then* there may be none to sell. Testing all the product often destroys it and is unnecessary. To determine the characteristics of the whole we have to sample a portion'. *This is the same when sampling the population.*

Time is also a factor when managers need information quickly in order to adjust an operation or to change policy. Take an automatic machine that sorts thousands of pieces of mail daily. Why wait for an entire day's output to check whether the machine is working accurately? Instead, samples can be taken at specific intervals, and if necessary, the machine can be adjusted right away.

Sampling distributions of data collected by students working in groups should be presented in tables and graphs. The emerging patterns should be discussed and used to explore the concepts and principles.

Hypothesis Testing

Suppose a manager of a large shopping mall tells us that the average work efficiency of the employees is 90 per cent. How can we test the validity of that manager's claim or hypothesis? Using a sampling method discussed, the efficiency of a sample could be calculated. If the sample statistic came out to 93 per cent, would the manager's statement be readily accepted? If the sample statistic were 43 per cent, we may reject the claim as untrue. Using common sense the claim can either be accepted or rejected based on the results of the sample. Suppose the sample statistic revealed an efficiency of 83 per cent. This is relatively close to 90 per cent. Is it close enough to 90 per cent for us to accept or reject the manager's claim or hypothesis?

Whether we accept or reject the claim we cannot be absolutely certain that our decision is correct. Decisions on acceptance or rejection of a hypothesis cannot be made on intuition. One needs to learn how to decide objectively on the basis of sample information, whether to accept or reject a hypothesis.

Correlation and Linear Regression – Bivariate Data

Information collected in Module 1, from the section Data Analysis can be applied to the concepts of linear regression and correlation.

Students should become proficient in the use of computer *software such as SPSS, Excel, Minitab* or scientific calculators *(non-programmable)* to perform statistical calculations, as in obtaining regression estimates and correlation coefficients.

UNIT 2

MODULE 1: DISCRETE MATHEMATICS

Critical Path Analysis

The critical path in an activity network has proven to be very useful to plan, schedule and control a wide variety of activities and projects in real-world situations. These projects include construction of plants, buildings, roads, the design and installation of new systems, finding the shortest route in a connected set of roads, organising a wedding, and organising a regional cricket competition.

Staying on the critical path in an activity network designed for the construction of a building, for example, ensures that the building is completed as scheduled.

UNIT 2

MODULE 2: PROBABILITY AND DISTRIBUTION

Probability

While teaching counting principles, *introduce* the concepts of independence and mutually exclusive events.

Discrete Random Variables

Computation of expected values and variances will not entail lengthy calculations or the summation of series.

The difference between discrete and continuous random variables could be illustrated by using real life situations.

Continuous Random Variables

Students may need to be introduced to the integration of simple polynomials.

UNIT 2

MODULE 3: PARTICLE MECHANICS

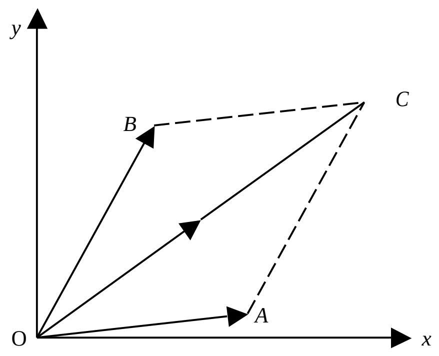
COPLANAR FORCES *AND EQUILIBRIUM*

Vectors

Be advised that students should have *practice* in dealing with vectors (see Module 2 Unit 1 of the Pure Mathematics syllabus) in order to represent a force as a vector.

Resolution of Forces

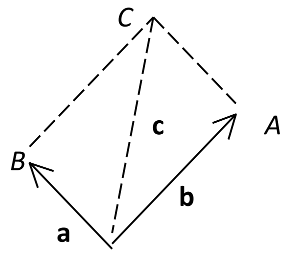
Have students consider two vectors **a**, **b** which have the same initial point O as in the figure below. Ask students to complete the parallelogram *OACB* as shown by the dotted lines, draw the diagonal *OC* and denote the vector $\overrightarrow{OC}$ by **c**.



*The vector c represents the resultant of the vectors **a** and **b**. Conversely, the vectors **a, b** can be regarded as the components of **c**. In other words, starting with the parallelogram OACB, the vector $\overrightarrow{OC}$ is said to be resolved into vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.*

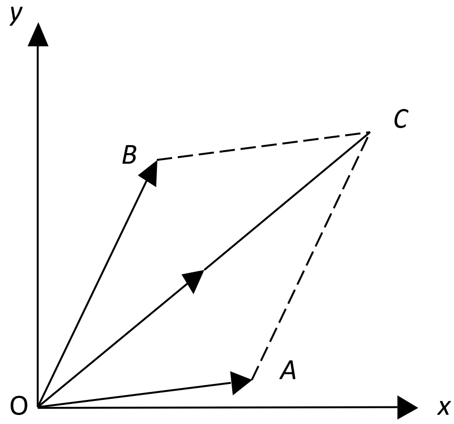
Let $\overrightarrow{OA} = \mathbf{a}$ *and* $\overrightarrow{OB} = \mathbf{b}$ *and* $\overrightarrow{OC} = \mathbf{c}$

By the parallelogram $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} = \mathbf{a} + \mathbf{b}$
Forces acting on a particle in equilibrium are equivalent to a single force acting at a common point.



*Diagrams are not in sequence to examples, the fixed point **O** is not shown in parallelogram **OACB** and points **B** and **A** are not correctly labelled.*

See example below.



Let $\overrightarrow{\boldsymbol{OA}} = \boldsymbol{a}$ *and* $\overrightarrow{\boldsymbol{OB}} = \boldsymbol{b}$ *and* $\overrightarrow{\boldsymbol{OC}} = \boldsymbol{c}$

By the parallelogram law $\overrightarrow{\boldsymbol{OC}} = \overrightarrow{\boldsymbol{OA}} + \overrightarrow{\boldsymbol{OB}} = \boldsymbol{a} + \boldsymbol{b}$

Forces

Students should be made aware of the definitions used in Mechanics.

Examples include:

(a) ***body*** *is any object to which a force can be applied;*

(b) ***particle*** *is a body whose dimensions, except mass, are negligible;*

(c) ***weight*** *is the force with which the earth attracts the body. It acts at the body's centre of gravity and is always vertically downwards;*

(d) ***a light body*** *is considered to be weightless;*

(e) ***pull and push*** *(P) are forces which act on a body at the point(s) where they are applied; and,*

(f) *normal reaction (R) is a force which acts on a body in contact with a surface. It acts in a direction at right angles to the surfaces in contact.*

Drawing Force Diagrams

Students should know that drawing a clear **force diagram** is an essential first step in the solution of any problem in mechanics which is concerned with the action of forces on a body.

***Students should be aware of the* important points *as listed below* to remember when drawing force diagrams:**

1. make the diagram large enough to show clearly all the forces acting on the body and to enable any necessary geometry and trigonometry to be done;

2. show only forces which are acting on the body being considered;

3. weight always acts on the body unless the body is described as light;

4. contact with another object or surface gives rise to a normal reaction and sometimes friction;

5. attachment to another object (by string, spring, hinge) gives rise to a force on the body at the point of attachment;

6. forces acting on a particle act at the same point; and,

7. check that no forces have been omitted or included more than once.

KINEMATICS AND DYNAMICS

Definitions

Distance *is how much ground is covered by an object despite its starting or ending point.*

Displacement is the position of a point relative to a fixed origin O. It is a vector. The SI Unit is the metre (m). Other metric units are centimeter (cm), kilometer (km).

Velocity is the rate of change of displacement with respect to time. It is a **vector**. The SI Unit is **metre per second** ($m\ s^{-1}$). Other metric units are $cm\ s^{-1}$, kmh^{-1}.

Speed is the magnitude of the velocity and is a scalar quantity.

Uniform velocity is the constant speed in a fixed direction.

Average velocity – $\dfrac{\text{change in displacement}}{\text{time taken}}$

Average speed – $\dfrac{\text{total distance travelled}}{\text{time taken}}$

Acceleration is the rate of change of velocity with respect to time. It is a **vector**. The SI Unit is **metre per second square** ($m\ s^{-2}$). Other metric units are $cm\ s^{-2}$, $km\ h^{-2}$.

Negative acceleration is also referred to as retardation.

Uniform acceleration is the constant acceleration in a fixed direction.

Motion in one dimension – When a particle moves in **one dimension**, that is, along a straight line, it has only two possible directions in which to move. Positive and negative signs are used to identify the two directions.

Vertical motion under gravity – this is a special case of uniform acceleration in a straight line. The body is thrown **vertically upward**, or falling **freely downward**. This uniform acceleration is due to **gravity** and acts vertically downwards towards the centre of the earth. It is denoted by **g** and may be approximated by **9.8 m s^{-2}** or **10 m s^{-2}.**

Graphs in Kinematics

A **displacement-time** graph for a body moving in a straight line shows its displacement x from a fixed point on the line plotted against time, t. The **velocity** v of the body at time, t is given by the **gradient** of the graph since

$$\frac{dx}{dt} = v.$$

The **displacement-time** graph for a body moving with **constant velocity** is a **straight line**. The velocity, v of the body is given by the gradient of the line.

The **displacement-time** graph for a body moving with **variable velocity** is a **curve**.
The velocity at any time, t may be estimated from the gradient of the tangent to the curve at that time. The average velocity between two times may be estimated from the gradient of the chord joining them.

Velocity-time graph for a body moving in a straight line shows its velocity v plotted against time, t. The **acceleration,** a of the body at time, t is given by the **gradient** of the graph at t, since $a = \frac{dv}{dt}$.

The **displacement** in a time interval is given by the **area** under the **velocity-time** graph for that time interval

Since $x = \int_{t_1}^{t_2} v\,dt$.

The **velocity-time** graph for a body moving with **uniform acceleration** is a **straight line**. The acceleration of the body is given by the gradient of the line.

Particle Dynamics

Force is necessary to cause a body to **accelerate**. More than one force may act on a body. If the forces on a body are in **equilibrium,** then the body may be at rest or moving in a straight line at constant speed.

If forces are acting on a body, then the body will accelerate in the direction of the resultant force. Force is a **vector**; that is, it has magnitude and direction. The SI Unit is the **newton** (N). One newton is the force needed to give a body a mass of 1 kg an acceleration of 1 ms $^{-2}$.

Mass and Weight are different. The **mass** of a body is a measure of the matter contained in the body. A massive body will need a large force to change its motion. The mass of a body may be considered to be uniform, whatever the position of the body, provided that no part of the body is destroyed or changed.

Mass is a **scalar** quantity; that is, it has magnitude only. The SI Unit of mass is the **kilogram** (kg). However, for heavy objects it is sometimes more convenient to give mass in tonnes, where 1 tonne = 1000 kg.

The **weight** of a body is the force with which the earth attracts that body. It is dependent upon the body's distance from the centre of the earth, so a body weighs less at the top of Mount Everest than it does at sea level.

Weight is a **vector** since it is a force. The SI Unit of weight is the **newton** (N).
The weight, W, in newtons, and mass, m, in kilograms, of a body are connected by the relation $W = mg$, where g is the acceleration due to gravity, in m s^{-2}.

Newton's three laws of motion are the basis of the study of mechanics at this level.

1st Law: A body will remain at rest or continue to move in a straight line at constant speed unless an external force acts on it.

(a) If a body has an acceleration, then there must be a force acting on it.

(b) If a body has no acceleration, then the forces acting on it must be in equilibrium.

2nd Law: The rate of change of momentum of a moving body is proportional to the external forces acting on it and takes place in the direction of that force. When an external force acts on a body of uniform mass, the force produces an acceleration which is directly proportional to the force.

(a) The basic equation of motion for constant mass is

Force = mass × acceleration
(in N) (in kg) (in m s^{-2})

(b) The force and acceleration of the body are both in the same direction.

(c) A constant force on a constant mass gives a constant acceleration.

3rd Law: If a body, A exerts a force on a body, B, then B exerts an equal and opposite force on A. These forces between bodies are often called reactions. In a rigid body the internal forces occur as equal and opposite pairs and the net effect is zero. So only external forces need be considered.

The following are important points to remember when solving problems using Newton's laws of motion:

(a) Draw a clear force diagram.

(b) If there is no acceleration, that is, the body is either at rest or moving with uniform velocity, then the forces balance in each direction.

(c) If there is an acceleration:

(i) use the symbol ⟶▶▶ to represent it on the diagram;

(ii) write, if possible, an expression for the resultant force; and,

(iii) use Newton's 2nd law, that is, write the equation of motion: F = ma.

Connected Particles

Two particles connected by a light inextensible string which passes over a fixed light smooth (frictionless) pulley are called **connected particles**. The tension in the string is the same throughout its length, so each particle is acted upon by the same tension.

Problems concerned with connected particles usually involve finding the acceleration of the system and the tension in the string.

To solve problems of this type:

(a) draw a clear diagram showing the forces on each particle and the common acceleration;

(b) write the equation of motion, that is, F = ma for each particle separately; and,

(c) solve the two equations to find the common acceleration, a, and possibly the tension, T, in the string.

Systems may include:

(a) one particle resting on a **smooth** or **rough** horizontal table with a light inextensible string attached and passing over a fixed small smooth pulley at the edge of the table and with its other end attached to another particle which is allowed to hang freely;

(b) as in (i), two light inextensible strings may be attached to opposite ends of a particle resting on a **smooth** or **rough** horizontal table and passing over fixed small smooth pulleys at either edge of the table and with their other ends attached to particles of different masses which are allowed to hang freely; and,

(c) one particle resting on a **smooth** or **rough** inclined plane and attached to a light inextensible string which passes over a fixed small smooth pulley at the top of the incline and with its other end attached to another mass which is allowed to hang freely.

WORK, ENERGY AND POWER

Work may be done either by or against a force (often gravity). It is a **scalar**. When a constant force F moves its point of application along a straight line through a distance s, the **work done** by F is **F.s.** The SI Unit of work is the **joule** (J). One joule is the work done by a force of one newton in moving its point of application one metre in the direction of the force.

Energy is the capacity to do work and is a scalar. The SI Unit of energy is the **joule** (the same as work).

A body possessing energy can do work and lose energy. Work can be done on a body and increase its energy, that is, work done = change in energy.

Kinetic and Potential Energy – are types of **mechanical energy**.

(a) Kinetic energy (K.E.) is due to a body's motion. The K.E. of a body of mass, m, moving with velocity, v, is $\frac{1}{2}mv^2$.

(b) Gravitational Potential Energy (G.P.E.) is dependent on height. The P.E. of a body of mass m at a height h:

  (i) above an initial level is given by mgh; and,

  (ii) below an initial level is given by – mgh.

  The P.E. at the initial level is zero (any level can be chosen as the initial level).

Mechanical Energy – (M.E.) of a particle (or body) = P.E. + K.E. of the particle (or body).

M.E. is lost (as heat energy or sound energy) when we have: - resistances (friction) or impulses (collisions or strings becoming taut).

Conservation of Mechanical Energy – The total mechanical energy of a body (or system) will be **conserved** if:

(a) no external force (other than gravity) causes work to be done; and,

(b) none of the M.E. is converted to other forms. Given these conditions:

  P.E. + K.E. = constant
  or loss in P.E. = gain in K.E. or loss in K.E. = gain in P.E.

Power – is the rate at which a force does work. It is a **scalar**. The SI Unit of power is the **watt** (W). One watt

(W) = one joule per second (J s^{-1}). The **kilowatt** (kW), 1 kW = 1000 W is used for large quantities.

When a body is moving in a straight line with velocity v m s^{-1} under a tractive force F newtons, the power of the force is P = Fv.

Moving vehicles – The power of a moving vehicle is supplied by its engine. The **tractive force** of an engine is the pushing force it exerts.

To solve problems involving moving vehicles:

1. draw a clear force diagram, (non-gravitational resistance means frictional force);

2. resolve forces perpendicular to the direction of motion;

3. if the velocity is:

   (a) constant (vehicle moving with steady speed), then resolve forces parallel to the direction of motion; and,

   (b) not constant (vehicle accelerating), then find the resultant force acting and write the equation of motion in the direction of motion.

4. Use power = tractive force × speed. Common situations that may arise are:

   (a) vehicles on the level moving with steady speed, v;

   (b) vehicles moving on the level with acceleration, a, and instantaneous speed, v;

   (c) vehicles on a slope of angle, α moving with steady speed, v, either up or down the slope; and,

   (d) vehicles on a slope moving with acceleration, a and instantaneous speed, v, up or down the slope.

Impulse and Momentum

The **impulse** of a force F, constant or variable, is equal to the **change in momentum** it produces. If a force, F acts for a time, t, on a body of mass, m, changing its velocity from u to v then Impulse = mv – mu.

Impulse is the time effect of a force. It is a **vector** and for a constant force F acting for time, t; impulse = Ft.

For a variable force, F acting for time, t, impulse = $\int_{t_1}^{t_2} F\,dt$

The SI Unit of impulse is the **newton second** (Ns).

The **momentum** of a moving body is the product of its mass m and velocity v that is, mv. It is a **vector** whose direction is that of the velocity and the SI Unit of momentum is the **newton second** (Ns).

Conservation of Momentum: The principle of **conservation of momentum** states that the total momentum of a system is constant in any direction provided no external force acts in that direction. Initial momentum = final momentum. In this context a system is usually two bodies.

Problem solving

Problems concerning impulse and momentum usually involve finding the impulse acting or the velocity on the mass of a body of a system.

To find an impulse for such a system write the impulse equation on each body.
To find a velocity or mass for such a system write the equation of the conservation of momentum.

Direct Impact – takes place when **two spheres of equal radii are** moving along the same straight line and collide.

Direct Impact with a Wall – When a smooth sphere collides **directly with a smooth vertical wall**, the sphere's direction of motion is perpendicular to the wall. The sphere receives an impulse perpendicular to the wall.

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

UNIT 1 – Paper 01

STATISTICAL ANALYSIS

1 minutes 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 1 hour and 30 minutes to answer them.

2. In addition to this test booklet, you should have an answer sheet.

3. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.

4. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.

5. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   The mean of 5, 7, 9, 11 and 13 is

   (A) 5
   (B) 7
   (C) 8
   (D) 9

   Sample Answer

   (A) (B) (C) ●

   The best answer to this item is "9 ", so (D) has been shaded.

6. If you want to change your answer, erase it completely before you fill in your new choice.

7. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, go on to the next one. You may return to that item later.

8. You may do any rough work in this booklet.

9. The use of silent, non-programmable scientific calculators is allowed.

   Examination Materials

   A list of mathematical formulae and tables. **(Revised 2019)**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Which of the following is an advantage of using secondary data in research?

(A) Reliability is guaranteed.
(B) Information is up to date.
(C) Data is controlled by the researcher.
(D) Information is specific to the purpose of the research.

2. Which of the following sets of data can be classified as discrete data?

(A) The weather condition yesterday
(B) The number of rainy days in the last month
(C) The weather prediction for tomorrow
(D) The amount of rainfall (in cm) recorded last month

3. In which of the following charts or diagrams will all the original data values be displayed?

(A) Pie chart
(B) Histogram
(C) Stem-and-leaf diagram
(D) Box-and-whisker diagram

4. Which of the following relationships is true for positively skewed distributions?

(A) mean > median > mode
(B) mean > mode > median
(C) median > mean > mode
(D) mode > median > mean

Item 5 refers to the following information

A small snack bar near our school was opened to students for 50 days during the summer program of 2019. The table below summarizes the daily number of students who visited the snack bar.

Number of students	22	23	24	25	26	27	28
Number of days	14	10	8	7	6	3	2

5. The mean number of students who visited the snack bar was

(A) 7.10
(B) 23.96
(C) 22
(D) 25

Items 6–8 refer to the following stem-and-leaf diagram which illustrates the heights, in cm, of a sample of 25 seedlings.

3	**4 6 7 8 9 9**
4	**0 2 2 3 4 6 8 9**
5	**0 1 3 5 8**
6	**2 4 5**
7	**4 6**
8	**1**

Key: 6 | 5 represents 6.5

6. The median height of the seedling is

(A) 4.8
(B) 8
(C) 13
(D) 48

7. The interquartile range for the set of data is

(A) 2.05
(B) 2.3
(C) 3.95
(D) 6.0

8. The shape of the distribution is

(A) neutral
(B) symmetrical
(C) positively skewed
(D) negatively skewed

<u>Item **9**</u> refers to the following information.

The following table shows the number of goals scored by the top scorers in 2021.

Lewandowski	Messi	Ronaldo	Silva
41	30	29	28

9. A pie chart showing the goals scored is drawn. The angle of the sector, to the nearest degree, representing Ronaldo's goals is

(A) 29
(B) 79
(C) 82
(D) 90

<u>Items **10–11**</u> refer to the table below.

Class	1–5	6–15	16–20	21–25
Frequency	7	24	5	2

10. The class boundaries of the 3rd class are

(A) 15.5, 20.5
(B) 15.5, 19.5
(C) 16, 20
(D) 16.5, 20.5

11. The class density of the interval 6–15 is

(A) 2.4
(B) 9
(C) 10
(D) 24

12. The CSEC Math grades (1–6) for the 189 students in the fifth form of a secondary school are collected and placed into a frequency table. The diagram that will BEST illustrate this information is a

(A) pie chart
(B) histogram
(C) stem-and-leaf
(D) cumulative frequency curve

Items **13 and 14** refer to the information given below:

A teacher wishes to find out whether boys in primary school play more video games than girls. A researcher recommends the following sampling design.

Arrange the students into two groups, boys and girls. Then randomly select 10 students from each group using a table of random numbers.

13. The sampling design used by the researcher is MOST likely

(A) quota sampling
(B) cluster sampling
(C) simple random sampling
(D) stratified random sampling

14. The MAIN advantage of the sample design recommended is that

(A) the population is divided into two groups
(B) an equal number of boys and girls were selected
(C) it is the most unbiased way of choosing a sample
(D) every member of the population has an equal chance of being selected

---

Item **15** refers to the following information.

In a questionnaire circulated to heads of households in a community, the sanitation authority asked

How do you rate the garbage collection service in your community?

☐ **Average** ☐ **Above average**

15. Which of the following can be identified as defects with this question?

I. Not enough options given
II. The range of options is not specific
III. Not all households will generate garbage

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II and III

16. Which of the following is TRUE for mutually exclusive events?

(A) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(B) $P(A \cap B) = P(A) \times P(B)$
(C) $P(A \cup B) = P(A) + P(B)$
(D) $P(A|B) = P(A)$

---

17. An unbiased die with faces marked 1, 2, 3, 4, 5, 6 is tossed 7 times and the number facing up is noted each time. A member of the possibility space is

(A) 6, 1, 4, 2, 6, 3, 6
(B) 1, 3, 5, 7, 5, 3, 1
(C) 3, 5, 4, 2, 7, 1, 1
(D) 1, 2, 3, 4, 5, 6, 7

18. For a standard normal distribution curve, what approximate percentage of the area under the curve is covered by $\mu \pm 2\sigma$?

(A) 90%
(B) 95%
(C) 97.5%
(D) 99%

19. Which of the following statements is NOT a condition for the binomial model of probability?

(A) There are only two possible outcomes for each trail.
(B) The probability of success is the same for every trail.
(C) There are an infinite number of trails for the experiment.
(D) The trails of the experiment are independent of each other.

20. The probability that it will rain on any day in the month of March is 0.3. What is the probability that it will rain on 4 days in a given week (7 days)?

(A) 0.9028
(B) 0.3000
(C) 0.2269
(D) 0.0972

21. Events C and D are independent events. $P(C) = 0.48$ and $P(D) = 0.45$. $P(C \cup D)$ is

(A) 0.216
(B) 0.714
(C) 0.784
(D) 0.930

<u>Item **22**</u> refers to the following formation.

The discrete random variable Q, where Q takes on only the values 2, 4, 6, and 8, has a cumulative distribution function, F(q), as shown in the table below.

q	2	4	6	8
F(q)	0.35	0.61	0.84	1

22. The value of $P(Q = 4)$ is

(A) 0.23
(B) 0.26
(C) 0.39
(D) 0.61

23. The continuous random variable, X, has a probability density function, *f*, given by

$$f(x) = \begin{cases} 0.25, & \text{for } 0 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$P(X < \frac{1}{2})$ is

(A) 0.125
(B) 0.250
(C) 0.500
(D) 0.875

GO ON TO THE NEXT PAGE

Items **24 and 25** refer to the following table which shows the distribution of the faculty enrolled in by a group of 150 students at a university.

	Social Sciences	**Science and Technology**	**Humanities**
Males	26	27	15
Females	48	11	23

24. The probability that a student chosen at random is a male and is enrolled in the faculty of Science and Technology is

(A) $\frac{27}{38}$

(B) $\frac{9}{50}$

(C) $\frac{34}{75}$

(D) $\frac{19}{75}$

25. The probability that a student selected at random is enroled in the faculty of Social Sciences given that the student is a female is

(A) $\frac{8}{25}$

(B) $\frac{24}{37}$

(C) $\frac{24}{41}$

(D) $\frac{37}{75}$

26. The time taken to run the 100m dash follows a normal distribution with mean 10.7 seconds and variance 0.36 seconds. The probability, to four decimal places, that the time of a randomly chosen runner is less than 9.8 second is

(A) 0.0062
(B) 0.0668
(C) 0.9332
(D) 0.9938

27. If $X \sim N(30.25)$ and $P(X > x) = 0.025$, then the value of x is

(A) 20.2
(B) 21.8
(C) 38.2
(D) 39.8

28. If $X \sim Bin(125, 0.45)$, the BEST distribution to calculate $P(X > 95)$ is

(A) $X \sim Bin(56.25, 30.94)$

(B) $X \sim N(56.25, 30.94)$

(C) $X \sim N(125, 0.45)$

(D) $X \sim Bin(125, 0.45)$

29. A random variable, X, has the following probability distribution.

x	0	2	3	5
$P(X = x)$	0.3	0.2	0.1	0.4

The value of $E(X)$ is

(A) 1
(B) 2.5
(C) 2.7
(D) 5

30. If $X \sim Bin(5, p)$ and $Var(X) = 0.8$, one of the possible values for p could be

(A) 0.16
(B) 0.20
(C) 0.60
(D) 0.84

Items **31** and **32** refer to the following contingency table which summarizes the responses of a random sample of 300 persons to a survey on preference for three dancehall artists.

Gender	**Artist A**	**Artist B**	**Artist C**	**Total**
Male	54	61	48	163
Female	38	72	27	137
Total	92	133	75	300

A χ^2 test is carried out to determine whether there is an association between the gender of respondents and the preference of artists.

31. The expected number of male respondents who prefer Artist C is

(A) 12
(B) 26.08
(C) 40.75
(D) 48

32. The number of degrees of freedom is

(A) 2
(B) 6
(C) 9
(D) 12

33. The null hypothesis for a X^2 test for independence is always

(A) H_0: There is a relationship between the variables

(B) H_1: There is a relationship between the variables

(C) H_0: There is no relationship between the variables

(D) H_1: There is no relationship between the variables

GO ON TO THE NEXT PAGE

Item **34** refers to the following information.

A company has a fleet of similar cars of different ages. An examination of the company records shows that the cost of replacement parts for the older cars is generally greater than that for the newer cars. The cost of the parts, in $, for the cars is given by the variable y and the age of the cars, in years, is given by the variable x. The summary statistics of the records for 9 cars are given as

$$\Sigma x = 68.8 \quad \Sigma y = 180 \quad \Sigma xy = 1960 \quad \Sigma x^2 = 753.9$$

A regression line is used to show the relation between the cost and age of the cars.

34. The value of the regression coefficient, b is

(A) –2.56
(B) 2.56
(C) 2.64
(D) 7.64

Item **35** refers to the following information

The probability of becoming infected with COVID-19, (y), is believed to be impacted by the amount of physical separation between an infected person and an uninfected one. This is also know as physical distancing, (x), measured in feet.

35. Using regression analysis in the form $y = a + bx$, researchers estimate that the regression line is $y = 0.69 - 0.140x$. Which of the following is the correct interpretation of the value of b?

(A) If there were no physical distancing, the probability of being infected is expected to be 0.69.
(B) If there were no physical distancing, the probability of being infected is expected to be 0.104.
(C) For every additional foot of physical distancing, the probability of being infected is expected to decrease by 0.69.
(D) For every additional foot of physical distancing, the probability of being infected is expected to decrease by 0.104.

36. Which of the following values for the product moment correlation coefficient, r, indicates the weakest degree of linear correlation between the two variables?

(A) $r = 0.38$
(B) $r = 0.45$
(C) $r = 0.76$
(D) $r = 0.91$

37. A type II error usually occurs in hypothesis testing when the null hypothesis is

(A) rejected when the null hypothesis is true
(B) not rejected when the null hypothesis is true
(C) rejected when the alternative hypothesis is true
(D) not rejected when the alternative hypothesis is true

38. Tacks produced by a machine have a mean length of 2.3 cm. A random sample of 20 tacks has a mean length of 2.4 cm with a standard deviation of 0.09 cm. If a hypothesis test is to be done at the 5% significance level, the test statistic is

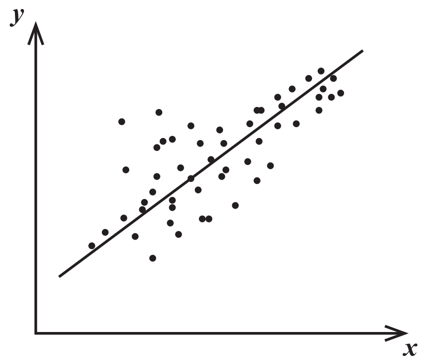
(A) 1.65
(B) 1.96
(C) 4.84
(D) 5.59

---

39. A coffee vending machine is adjusted so that, on average, it dispenses 150 ml of coffee with a standard deviation of 8.5 ml into a cup. The owner of the machine decides to check if it dispenses the correct amount. So, he takes a sample of 50 cups of coffee and performs a test. At the 5% level, what is an appropriate decision rule for this test?

(A) reject the null if the calculated $Z > 1.96$
(B) do not reject the null if the calculated Z | is < 1.96
(C) reject the null if the calculated $Z \neq 1.96$
(D) reject the null if the calculated $Z > |1.96|$

Item **40** refers to the following scatter diagram.



40. Which of the following BEST describes the above scatter diagram?

(A) There is a weak positive correlation between x and y.
(B) There is a weak negative correlation between x and y.
(C) There is a strong positive correlation between x and y.
(D) There is a strong negative correlation between x and y.

Item **41** refers to the following contingency table below which shows data collected to determine whether a person's favourite sport is dependent on gender.

GENDER	FOOTBALL	BASEBALL	BASKETBALL	GOLF	TOTAL
MALE	63	50	45	20	178
FEMALE	20	30	45	27	122
TOTAL	83	80	90	47	300

41. The expected frequency of the females who chose basketball as their favourite sport is

(A) 53.4
(B) 42.6
(C) 33.8
(D) 32.5

42. A random sample of 200 students is taken from a non-normal population with unknown variance. If the sample mean is $\bar{x}$, what is a 95% confidence interval for the population mean?

(A) $\bar{x} \pm 1.645\left(\frac{\hat{\sigma}}{\sqrt{n}}\right)$

(B) $\bar{x} \pm 1.645\left(\frac{\sigma}{\sqrt{n}}\right)$

(C) $\bar{x} \pm 1.96\left(\frac{\hat{\sigma}}{\sqrt{n}}\right)$

(D) $\bar{x} \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right)$

43. Which of the following is NOT true for a *t*-test?

(A) σ^2 is unknown
(B) n is small
(C) $v = n - 1$
(D) population is not normal

<u>Item **44**</u> refers to the following information

A test was conducted to determine whether a sample of 8 leaves with a mean length of 3.875 cm and a standard deviation of 1.45 cm came from a plant with leaves of mean 3.70 cm.

44. What is the value of the test statistic?

(A) 2645
(B) 0.3193
(C) 0.3414
(D) 0.3845

45. A *t*-test, at the 5% level of significance, was carried out on 15 volunteers to determine whether a new diet increased the speed of weight loss. What will be the critical region for this test?

(A) $t > 1.761$
(B) $t < -1.761$
(C) $t > 1.753$
(D) $t > 1.645$

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

Item	Specific Objective	Key		Item	Specific Objective	Key
1	1.1.1.1	A		26	1.2.4.6	B
2	1.1.1.2	C		27	1.2.4.5	D
3	1.1.2.3	C		28	1.2.4.8	B
4	1.1.2.8	A		29	1.2.2.4	C
5	1.1.2.5	B		30	1.2.3.3	B
6	1.1.2.5	A		31	1.3.4.4	C
7	1.1.2.7	A		32	1.3.4.2	A
8	1.1.2.8	C		33	1.3.4.1	C
9	1.1.2.4	B		34	1.3.5.5	B
10	1.1.2.2	A		35	1.3.5.6	D
11	1.1.2.2	A		36	1.3.5.4	A
12	1.1.2.3	A		37	1.3.2.3	C
13	1.1.1.6	A		38	1.3.2.6	D
14	1.1.1.6	B		39	1.3.2.8	D
15	1.1.1.8	C		40	1.3.5.3	C
16	1.2.1.5	C		41	1.3.4.3	B
17	1.2.1.9	A		42	1.3.1.6	C
18	1.2.4.1	B		43	1.3.3.1	D
19	1.2.5.1	C		44	1.3.3.2	B
20	1.2.3.1	D		45	1.3.3.5	A
21	1.2.1.5	B				
22	1.2.2.6	B				
23	1.2.2.7	A				
24	1.2.1.6	B				
25	1.2.1.6	C				

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

APPLIED MATHEMATICS

SPECIMEN

2022

TABLE OF SPECIFICATIONS

Paper 02

UNIT 1

Question	***Module***	***CK***	***AK***	***R***	***Total***
1	1	6	14	5	25
2	1	6	14	5	25
3	2	6	14	5	25
4	2	6	14	5	25
5	3	6	14	5	25
6	3	6	14	5	25
SUBTOTAL		**36**	**84**	**30**	**150**

TEST CODE **02105020**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections. Each section consists of TWO questions.
2. Answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials:

Mathematical formulae and tables **(Revised 2022)**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

“*”Barcode Area”*”
Sequential Bar Code

SECTION A

MODULE 1: COLLECTING AND DESCRIBING DATA

Answer BOTH questions.

1. (a) Differentiate between the terms ‘parameter’ and ‘statistic’.

....

....

....

....

[2 marks]

(b) A principal wants to select 16 students from among 464 students to represent the school at an event. The following sampling methods labelled A, B, C, and D are suggested as possible ways of selecting the sample of 16 students.

W: Select equal numbers of male and female students even though there are more female students than male students.

X: Group the students into year groups and select students at random from within each group in proportion to the number of students that make up that group.

Y: Assign a unique number from 001 to 464 to each student and use a random number table to select the students.

Z: First select a student randomly from among the first *r* students on the nominal roll, thereafter, selecting every *r*th student on the roll.

(i) Identify the term which BEST describes the sampling methods A–D.

W ....

X ....

Y ....

Z ....

[4 marks]

GO ON TO THE NEXT PAGE

(ii) Identify (by letter) the sampling method which would result in a sample that is MOST representative of the 464 students. Give TWO reasons for your choice.

....

....

....

....

[3 marks]

(iii) Out of the 16 students selected, there would be 4 students from the fourth form if sampling method X were used. Calculate how many of the 464 students are in fourth form.

[3 marks]

(iv) Using the Random Sampling Numbers table provided in the List of Formulae and Statistical Tables, start at the third row from the top with 720 and work across the row left to right. Select the **first** and **fourth** of the 16 students that would be selected according to sampling method Y.

....

....

[2 marks]

(c) 30 students were asked to estimate what their score would be on a recently completed test out of 60. The data is represented in the table below.

32	12	39	54	29	27	44	41	31	33
42	23	25	29	37	18	29	25	31	51
29	36	20	41	36	37	26	50	29	31

(i) Construct a stem-and-leaf diagram to illustrate the data.

[5 marks]

(ii) Comment on the shape of the distribution. Give ONE reason to support your comment.

....

....

....

[2 marks]

(iii) Calculate the 10% trimmed mean for this set of data.

[4 marks]

Total 25 marks

2. (a) The following table shows the distances travelled by some sixth form students to school on a given morning. Use the table to answer the following questions.

Distance Travelled (km)	**Frequency**
0–4	9
5–9	14
10–14	18
15–19	12
20–24	5
25–29	2

(i) State the upper boundary of the third class.

....

[1 mark]

(ii) Identify the modal class.

....

[1 mark]

(iii) Calculate the frequency density of the fourth class.

[2 marks]

(iv) Estimate the mean distance travelled.

[5 marks]

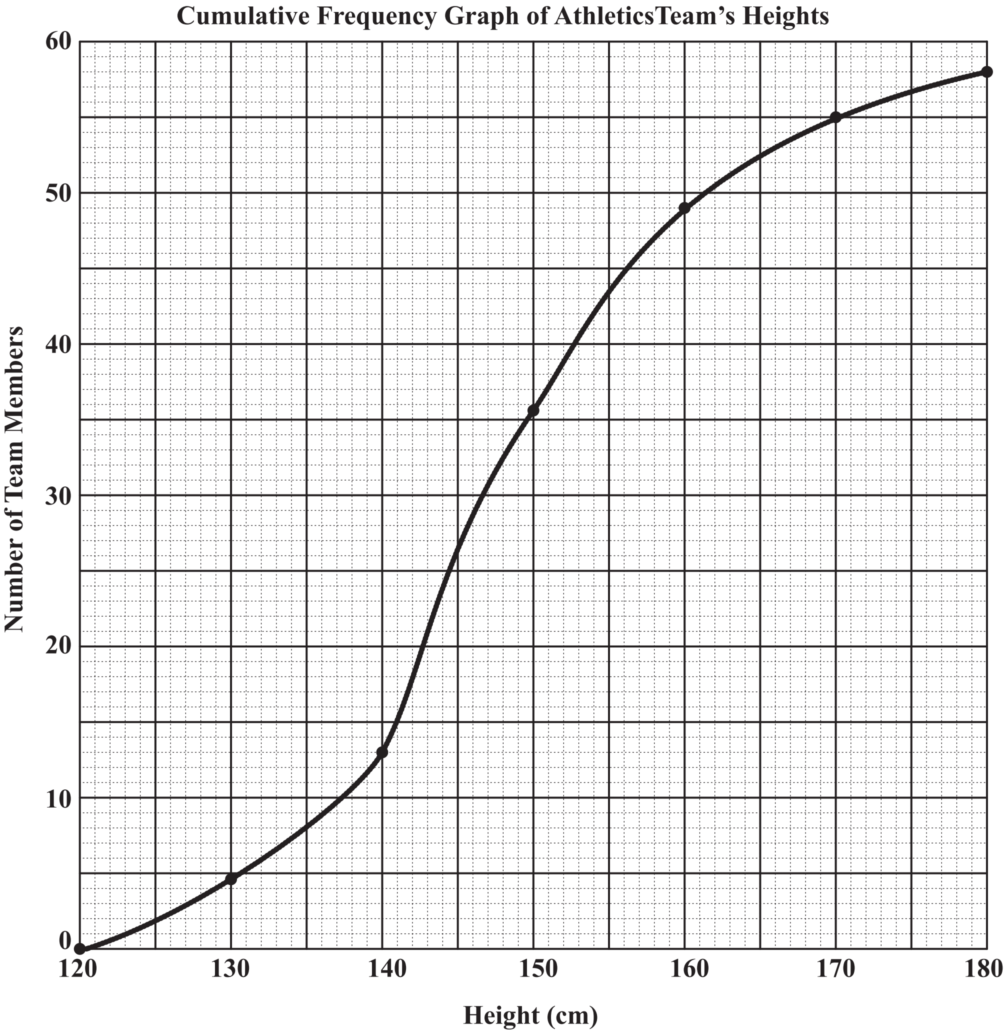
(v) Estimate the standard deviation of the distance travelled.

[5 marks]

(vi) Estimate the number of students who travelled more than 17 km.

[2 marks]

(b) The following cumulative frequency curve shows the heights of the members of the athletics team at a particular school.



GO ON TO THE NEXT PAGE

Use cumulative frequency graph, **provided on page 8**, to answer the following.

(i) Estimate the median height of the athletes.

....

....

[2 marks]

(ii) Calculate the interquartile range.

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....

....

....

[3 marks]

(iii) Estimate the number of athletes who are taller than 166 cm.

....

....

....

[2 marks]

(iv) Construct a box-and-whisker plot on the cumulative frequency graph given **on page 8** which BEST represents the data. **[2 marks]**

Total 25 marks

SECTION B

MODULE 2: MANAGING UNCERTAINTY

Answer BOTH questions.

3. (a) Sharks FC and Hurricanes FC, the two leading teams in the present football season, each have one game to play (not against each other).

If Sharks FC wins and Hurricanes FC draws or loses, Shark FC wins the championships.

If Sharks FC draws and Hurricanes FC loses, then Sharks FC wins the championship.

The following table shows the probabilities for each team winning, losing, or drawing their last game of the season.

	Win	**Draw**	**Lose**
Sharks FC	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$
Hurricanes FC	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

(i) Complete the table below to show nine possible outcomes for the last games for the **two** teams.

		Sharks FC		
		Win	**Draw**	**Lose**
Hurricanes FC	Win	$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$		
	Draw			$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
	Lose		$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$	

[6 marks]

GO ON TO THE NEXT PAGE

(ii) Determine the probability that Sharks FC wins the championship.

[4 marks]

(iii) Calculate the probability that Hurricanes FC draws given that Sharks wins the championships.

[4 marks]

(b) The probability distribution of a discrete random variable, Y, is given in the following table.

y	0	1	2	3
$P(Y = y)$	$\frac{1}{3}$	p	q	$\frac{1}{4}$

The expectation is given as $E(X) = 1\frac{1}{4}$.

(i) Calculate the value of p and q.

[6 marks]

(ii) Calculate the standard deviation of X.

[5 marks]

Total 25 marks

4. (a) At a regional hospital, five patients are tested for dengue fever. Doctors believe that the probability of a positive test is 0.3.

(i) List FOUR reasons why this situation may be modelled as a binomial distribution.

....

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....

[4 marks]

(ii) State fully the parameters of the binomial distribution.

....

....

....

[2 marks]

GO ON TO THE NEXT PAGE

(b) Determine the probability that

(i) there is exactly one positive test

[4 marks]

(ii) there are less than two positive tests.

....

....

....

....

....

....

[4 marks]

(c) Calculate the expected number of positive tests.

....

....

....

[2 marks]

(d) Calculate the variance of the number of positive tests.

....

....

....

[2 marks]

GO ON TO THE NEXT PAGE

(e) Calculate the probability that out of 100 tests, twenty-five or less will result positive. In your response, give TWO reasons for the use of the distribution selected.

[7 marks]

Total 25 marks

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SECTION C

MODULE 3: ANALYSING AND INTERPRETING DATA

Answer BOTH questions.

5. (a) The masses of mangoes in a box, can be modelled by a normal distribution with a mean of 62.2 g and a standard deviation of 3.6 g.

(i) Write a distribution for the mean mass, $\overline{X}$, of a sample of 20 mangoes.

[2 marks]

(ii) Calculate the probability that the mean mass of a random sample of 20 mangoes is less than 60 g.

[4 marks]

(iii) Calculate the probability that the mean mass of a random sample of 20 mangoes is between 63 g and 64 g.

[4 marks]

(iv) A random sample of mangoes is taken from the box. Using a 95 percent confidence interval for the mean, determine the sample size so that the sample mean will differ from the true population mean by less than 1.0 g.

[5 marks]

(v) A 90% confidence interval for the population mean, μ, is found for each sample when 60 random samples of size 20 are taken. Determine the expected number of intervals that do NOT contain μ.

[2 marks]

(b) The mean of 50 observations of a random variable X, where $X \sim \text{Bin}\,(10, \frac{1}{6})$ is $\overline{X}$.

(i) Determine the distribution modelled by $\overline{X}$, stating clearly its parameters.

[3 marks]

(ii) Calculate $P(\overline{X} > 1.9)$

[4 marks]

(iii) State the percentage of the sample that will have a mean greater than 1.9.

....

[1 mark]

Total 25 marks

GO ON TO THE NEXT PAGE

6. (a) Some fruit trees are suffering from a certain disease that causes them to wither. An agronomist wishes to investigate whether there is an association between the type of treatment for the disease and the length of survival of the affected trees.

Treatment A involved no action being taken.

Treatment B involved careful removal of the diseased branches.

Treatment C involved the careful removal of the diseased branches and then the frequent spraying of the remaining branches, with an antibiotic.

The following table shows the results of the treatment given to 200 trees.

Survival	**Treatment**		
	A	**B**	**C**
< 3 months	24	32	32
≥ 3 months	30	30	52

Use a chi-squared test at the 5% level of significance to determine whether these data provide evidence of an association between the type of treatment and the survival of the trees.

(i) State the null and alternative hypotheses.

....

....

....

....

[2 marks]

(ii) Complete the table below to show the expected frequencies.

Survival	**Treatment**		
	A	**B**	**C**
< 3 months	23.8	27.3	
≥ 3 months			47.1

[3 marks]

GO ON TO THE NEXT PAGE

(iii) For this test, state the degrees of freedom and the critical value of χ^2.

....

....

....

[2 marks]

(iv) Complete the following table to determine the χ^2 test statistic, correct to three decimal places.

Observed (O)	Expected (E)	χ^2
24	23.8	0.00168
30	30.2	
32	27.3	
30	34.7	0.63660
32	36.9	
52	47.1	0.50974

[5 marks]

(v) State a valid conclusion for the test. Justify your response.

....

....

....

[2 marks]

GO ON TO THE NEXT PAGE

(b) An Internet service provider runs a series of television advertisements at weekly intervals. A study was conducted to investigate the effectiveness of the advertisements. The company records the viewing figures in millions, v, for the programme in which the advertisement is shown, and the number of new customers, c, who sign up for its service the next day. The results are summarized as follows:

$$\bar{v} = 4.92, \quad \bar{c} = 104.4, \quad S_{vc} = 594.05, \quad S_{vv} = 85.44.$$

(i) From the situation given, identify the independent variable.

....

[1 mark]

(ii) Determine the equation of the regression line, c on v, in the form $c = a + bv$.

[5 marks]

GO ON TO THE NEXT PAGE

(iii) Give an interpretation of the constants *a* and *b* in relation to the effectiveness of the advertisements.

....

....

....

[2 marks]

(iv) Estimate the number of customers who will sign up with the company the day after an advertisement is shown during a programme watched by 3.7 million viewers.

[3 marks]

Total 25 marks

END OF TEST

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 02

KEY AND MARK SCHEME

SPECIMEN 2022

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

SECTION A

MODULE 1: COLLECTING AND DESCRIBING DATA

Question 1.

(a) A parameter is a fixed measure or characteristic of the population **(1)** while a statistic is a characteristic of a sample **(1)**.

[2 marks] [CK]

(b) (i) W – Quota sampling **(1)**
X - Stratified random sampling **(1)**
Y - Simple random sampling **(1)**
Z - Systematic random sampling **(1)** **[4 marks] [CK]**

(ii)
- Sampling method II/Stratified Random Sampling **(1)**.
- The sampling method guarantees proportional representation equal to that of the population **(1)**.
- The sampling method is random thus eliminating bias **(1)**.

[3 marks] [R]

(iii) Let x be the number of 4^{th} form students

$\frac{x}{464} \times 16 = 4$ **(1)**

$x = \frac{4}{16} \times 464$ **(1)**

$x = 116$ students **(1)** **[3 marks] [AK]**

(iv) 211, 442, 282, 398 **[2 marks] [AK]**

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 1. (continued)

(c) (i)

stem	leaf
1	2 8
2	0 3 5 5 6 7 9 9 9 9 9
3	1 1 2 3 6 6 7 7 7
4	1 1 1 2 4
5	0 1 4

Key: 3|6 = 36

Correct stem **(1)**
Key **(1)**
All rows correct **(3)**
3 - 4 rows correct **(2)**
At least 2 rows correct **(1)** **[5 marks] [AK]**

(ii) Distribution is positively skewed **(1)**.
(Right tailed is NOT accepted for the shape of the distribution)

The mode is smaller than median / most of the data lies to the left of the data / the data is right tailed/skewed towards the right **(1)**.

1 mark for comment on shape
1 mark for any ONE reason **[2 marks] [R]**

(iii) 10% from both ends of data (SOI) **(1 mark each)**

23 - 44

1/24(23 + 2(25) + 26+ 27 +5(29) + 2(31) + 32 + 33 + 2(36) + 2(37) + 39 +3(41) +42 + 44)

$\frac{\sum x}{n}$ SOI

1/24 (792) **(1)** **(dividing his sum by 24)**

= 33 **(1)** **CORRECT ANSWER ONLY** **[4 marks][AK]**

Total 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 2.

(a) (i) 14.5 **[1 mark] [CK]**

(ii) 10-14 **[1 mark] [CK]**

(iii) $\frac{12}{5} = 2.4$

division by 5 (1)

correct answer (1)

[2 marks] [AK]

(iv)

Distance Travelled (km)	**Frequency (f)**	**Midpoints (x)**	**fx**
0 - 4	9	2	18
5 - 9	14	7	98
10 - 14	18	12	216
15 - 19	12	17	204
20 - 24	5	22	110
25 - 29	2	27	54
Totals	60		700

correct fx column (1 mark) [AK]
correct midpoints (1 mark) [AK]
correct total fx column [CK]

$$mean = \frac{\sum fx}{\sum f} = \frac{700}{60} = 11.67\ km$$

correct denominator (1) [R]
candidate's correct answer (1) [AK]

[5 marks]

Question 2. (continued)

(a) (v)

Distance Travelled (km)	**Frequency (f)**	**Midpoints (x)**	**fx**	**fx^2**
0 - 4	9	2	18	36
5 - 9	14	7	98	386
10 - 14	18	12	216	2592
15 - 19	12	17	204	3468
20 - 24	5	22	110	2420
25 - 29	2	27	54	1458
Totals	60		700	10360

1 mark for correct multiplication [AK]
1 mark for correct fx^2 total [CK]

$$Variance = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$$

$= \frac{10360}{60} - (11.67)^2$ **correct substitution into formulae (1) [AK]**

$= 36.478$

$Std.Dev = \sqrt{Var} = \sqrt{36.478} = 6.04$

1 mark for correct $\sqrt{var}$ [CK]
1 mark for correct answer [AK]

Alternate method also accepted

[5 marks] [AK]

(vi) $6 + 5 + 2 = 13\ students$

1 mark for 6 (1)
addition of subsequent values (1)
[2 marks] [R]

Question 2. (continued)

(b) (i) 147 cm

1 mark for choosing the 29th value (1) [R]
1 mark for correct corresponding height (1) [AK]
[2 marks]

(ii) Q1= 141cm **(1)** (tolerance $\pm 1cm$) **[AK]**

Q3= 156cm **(1)** (tolerance $\pm 1cm$) **[AK]**

I.Q.R = Q3 – Q1 = 156 – 141 = 15 cm **subtraction of values (1) [CK]**

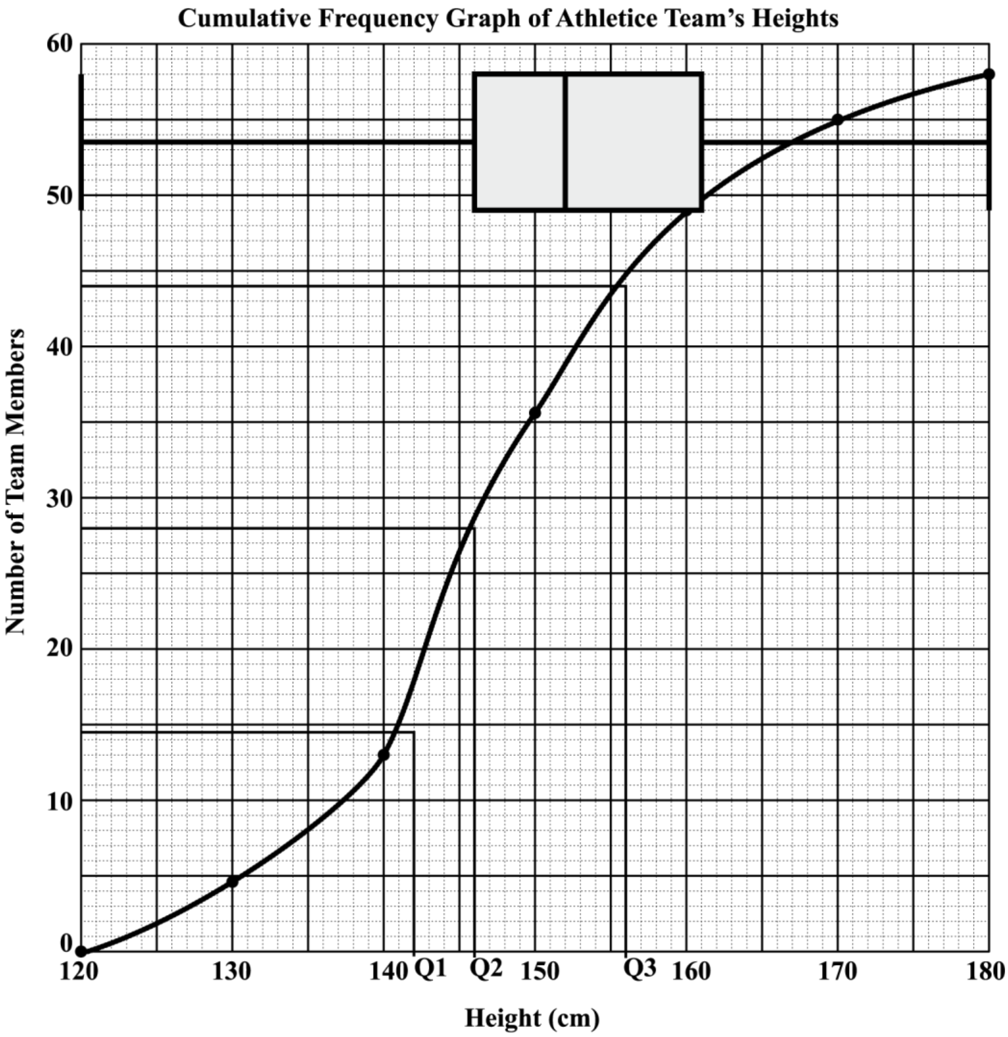
[3 marks]

(iii) 58 – 53 = 5 students

1 mark for 53 (1) [AK]
1 mark for subtraction from 58 (1) [R]
[2 marks]

Question 2. (continued)

(b) (iv)



1 mark for whiskers (1 AK)
1 mark for box (1 AK) **[2 marks]**

Total 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

SECTION B

MODULE 2: MANAGING UNCERTAINTY

Question 3.

(a) (i)

		Sharks FC		
Hurricanes FC		*Win*	*Draw*	*Lose*
	Win	$\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}$	$\mathbf{\frac{1}{6}\times\frac{2}{3}=\frac{2}{18}}$	$\mathbf{\frac{1}{3}\times\frac{2}{3}=\frac{2}{9}}$
	Draw	$\mathbf{\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}}$	$\mathbf{\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}}$	$\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}$
	Lose	$\mathbf{\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}}$	$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$	$\mathbf{\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}}$

1 for each correct probability AK
[6 marks]

(ii) P(Sharks wins championship)

$= P(SW \times HW') + P(SD \times HL)$ **(1) mark for correct interpretation [R]**

$P(HW') = 1-\frac{2}{3}=\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$

P(Sharks wins championship)

$=\left(\frac{1}{2}\times\frac{1}{3}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)$ **1 mark each for correct partial product [AK]**

$=\frac{7}{36}$ **1 mark for correct answer only[AK]**

[4 marks]

Alternate Response

or from table $\mathbf{\frac{1}{12}+\frac{1}{12}+\frac{1}{36}=\frac{7}{36}}$

Any two correct probabilities from the table, 1 mark each [AK]
Addition of their probabilities 1 mark [R]
1 mark for correct answer [AK]

Question 3. (continued)

(a) (iii) $\frac{P(HD \cap SW)}{P(SWC)}$ **(1) conditional formula stated correctly [R]**

$= \frac{\left(\frac{1}{12}\right)}{\frac{7}{36}}$ **1 mark for correct numerator [CK]**

1 mark for candidates' correct denominator [CK]

$= \frac{3}{7}$ **1 mark Correct Answer only [AK]**

[4 marks]

(b) (i) $\sum P(Y = y) = 1$ **(1) mark for correct formula [CK]**

$\frac{1}{3} + p + q + \frac{1}{4} = 1$ **(1) mark for equation {1} [R]**

$p + q = \frac{5}{12} \ldots\{1\}$

$E(Y) = \sum yP(Y = y)$ **(1) mark for expectation formula CK**

$\frac{5}{4} = 0 + p + 2q + \frac{3}{4}$

$\frac{1}{2} = p + 2q \ \ldots\{2\}$ **(1) mark for equation {2} [R]**

working with $\{2\}$, **(1) mark for solving simultaneously [R]**

$p = \frac{1}{2} - 2q \ldots\{3\}$

subtituting $\{3\}$ *in* $\{1\}$

$\left(\frac{1}{2} - 2q\right) + q = \frac{5}{12}$

$q = \frac{1}{12}$

subtitute $q = \frac{1}{12}$ *in* $\{3\}$

$p = \frac{1}{3}$ **(1) mark for correct *p* [AK]**

[6 marks]

Question 3. (continued)

(b) (ii) $Var(Y) = E(Y^2) - [E(Y)]^2$

$E(Y^2) = \sum y^2 P(Y = y) = 0 + \frac{1}{3} + \frac{1}{6} + \frac{9}{4} = \frac{33}{12}$

$Var(Y) = \frac{33}{12} - \left(\frac{5}{4}\right)^2 = \frac{57}{48}$

Standard Deviation (Y) = $\sqrt{\frac{57}{48}} = 1.09 \quad (3\ s.f.)$

Correct formula - 1 mark [CK]
Calculating $E(Y^2)$ – 1 mark [AK]
Correct substitution into variance formula – 1 mark [AK]
Square root of candidates' variance - 1 mark [CK]
Correct answer only 1 mark [AK]

[5 marks]

Total 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 4.

(a) (i)

- The test can have one of two possible results - either positive or negative.
- The trials are independent, as the results of one test does not affect the results of another patient.
- Each trial has the same probability.
- There is a finite number of trials.

1 mark for each point stated **[4 marks] [R]**

(ii) $X \sim Bin(5,0.3)$

1 mark each for stating both parameters **[2 marks][CK]**

(b) (i) $P(X = 1) = \binom{5}{1}(0.3)^1(0.7)^4 = 0.36015$

3 marks for all values correctly substituted in formula [AK]
1 for correct answer only [AK]
[4 marks]

(ii) $P(X < 2)$

$= P(X = 0) + P(X = 1)$

1 mark for *X* = 0 + X = 1

$=\binom{5}{0}(0.3)^0(0.7)^5 + 0.36015$

1 mark for correctly calculating P (*x* = 0) =

$0.16807 + 0.36015$ **1 mark for summation of "candidates" values (SOI)**

$= 0.52822$ **1 mark for "candidate's" answer**

1 use of summation [R]
1 mark for *x* = 0 [AK]
1 mark for *x* < 2 [AK]
1 for correct answer only [AK]
[4 marks]

Question 4. (continued)

(c) $E(X) = np = 5(0.3) = 1.5$

1 correct substitution into formulae
1 correct answer only

[2 marks] [AK]

(d) $Var(X) = npq = 5(0.3)(0.7) = 1.05$

1 correct substitution into formulae1
1 correct answer only

[2 marks] [AK]

(e) $X \sim Bin(100,0.3)$

The value of *n* (100) is large.

The value of *np* > 5 (*np*=30)

$$X \sim N(30,21)$$

$$P(X \leqslant 25) = P\left(Z \leqslant \frac{25.5 - 30}{\sqrt{21}}\right) = P(z \leqslant -0.982)$$

$1 - \phi(0.982)$

$1 - 0.8370 = 0.163$

2 marks 1 each for stating both parameters [CK]
1 mark for application of continuity correction [CK]
1 mark for correct standardization [AK]
1 mark for correct use of phi [AK]
1 mark for correct table values [CK]
1 mark for correct answer only [AK] **[7 marks]**

Total 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

SECTION C

MODULE 3: ANALYZING AND INTERPRETING DATA

Question 5.

(a) (i) $\bar{X} \sim N\left(62.2, \frac{12.96}{20}\right)$

12.96 or 3.6^2 **(1)**

N **(1)**

1 mark for the sigma squared/*n*
1 mark for it is N normal

[2 marks] [AK]

(ii) Less than 60 g

$P(\bar{X} < 60) = P\left(Z < \frac{60-62.2}{3.6/\sqrt{20}}\right)$ **Attempt to standardize (1) [CK]**

$= P(Z < -2.733)$ **Correct standardization (1) [AK]**

$= \phi(-2.733)$

= 1 - 0.9968 **use of ϕ for standardized value (1) [CK]**

= 0.0032 **Correct answer (1) [AK]**

[4 marks]

(iii) Between 63 g and 64 g

$P(63 < \bar{X} < 64) = P\left(\frac{63-62,2}{3.6/\sqrt{20}} < Z < \frac{64-62.2}{3.6/\sqrt{20}}\right)$

Stating that $\bar{X}$ is between 63 and 64 (1) [R]
Attempt to standardize (1) [CK]

$= P(0.994 < Z < 2.236)$ **correct standardization (1) [AK]**

$= \phi(2.236) - \phi(0.994)$ **use of ϕ for "his" standardized value (1) [CK]**

= 0.9873 - 0.8399

= 0.1474 **correct answer (1) [AK]**

[4 marks]

Question 5. (continued)

(a) (iv) For a 95% CI $-1.96 < \frac{\bar{X} - \mu}{3.6/\sqrt{n}} < 1.96$ **SOI (1) [R]**

$-1.96 < \frac{(\bar{X} - \mu)\sqrt{n}}{3.6} < 1.96$

$-1.96 \times 3.6 < \sqrt{n} < 1.96 \times 3.6$

using $\bar{X} - \mu = 1$ **1 mark [AK]**

simplification **1 mark [AK]**

$\sqrt{n} < 1.96 \times 3.6$ **1 mark for correct substitution [AK]**

n < 49.8

n = 49 **1 mark for correct answer [AK]**

[5 marks]

(v) 60 – 90% of 60 **SOI (1) [R]**

60 – 54

6 intervals do NOT contain μ **(1) [AK]**

[2 marks]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 5. (continued)

(b) (i) np = $\frac{10}{6} = 1.667$

1 mark for calculating the expected value (mean) [AK]

npq = $\frac{10}{6} \times \frac{5}{6}$ = 1.389 **1 mark for calculating the variance [AK]**

$\bar{X} \sim N\left(1.667, \frac{1.389}{50}\right)$ **1 mark for stating the distribution [R]**

$\bar{X} \sim N(1.667, 0.028)$ **[3 marks]**

(ii) = P($\bar{X}$ > 1.95) **1 mark for using continuity correction [R]**

= $P\left(Z > \frac{1.95-1.667}{\sqrt{0.028}}\right)$ **1 mark for attempt to standardize (CK)**

= P(Z >1.691) 1 mark for c**orrect standardization [AK]**

= $1 - \phi(1.691)$ **Use of ϕ (1) [CK]**

= 1 - 0.9546

= 0.0454 **1 mark for correct answer (1) [AK]**

[4 marks]

(iii) 0.0454 = 4.54 %

1 mark for changing values to percentage **[1 mark]**

TOTAL 25 marks

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 6.

(a) (i) H_0: There is no association between type of treatment and the survival of the trees. **(1)**

H_1: Type of treatment and the survival of trees are associated. **(1)**

[2 marks]; AK

(ii)

	Treatment		
Survival	**A**	**B**	**C**
< 3 months	**23.8**	**27.3**	**36.9**
≥ 3 months	**30.2**	**34.7**	**47.1**

1 mark for each correct value
[3 marks] [AK]

(iii) 2 degrees of freedom **correct answer (1) [AK]**

Critical value = 7.378 **correct reading from the tables (1) [CK]**

[2 marks]

Question 6. (continued)

(a) (iv)

Observed (O)	Expected (E)	$\chi^2 = (O - E)^2/E$
24	23.8	0.00168
30	30.2	**0.00132**
32	27.3	**0.80916**
30	34.7	0.63660
32	36.9	**0.65068**
52	47.1	0.50977
		2.609

$\chi^2 = \frac{(O-E)^2}{E}$ **SOI (1) [CK]**

1 mark each for correct calculations of χ^2 (3) [AK]

1 mark for test statistic (1) [AK] **[5 marks]**

(v) Since the test statistic is less than the critical value of χ^2 **(1)** accept H_0 and conclude there is no association between the type of treatment and the survival time of the fruit trees **(1)**.

[2 marks] [R]

(b) (i) Independent- v (the number of viewers) **[1 mark] [R]**

(ii) $b = \frac{S_{vc}}{S_{vv}}$ **1 mark SOI [CK]**

$b = \frac{594.05}{85.44} = 6.953$ **1 mark [AK]**

$a = \bar{c} - b\bar{v}$ **1 mark SOI [CK]**

$= 104.4 - 6.953 \times 4.92$

$= 70.192$ **1 mark for correct answer [AK]**

$a = 70.2 + 6.95v$ **1 mark for substituting values into c [AK]**

[5 marks]

Question 6. (continued)

(b) (iii) a is the number of sign-ups without advertising **(1)**

b is the of extra sign-ups per million viewers of advert **(1)**

[2 marks] [R]

NB: the gradient of slope is not accepted

(iv) $c = 70.192 + (6.953 \times 3.7)$ **1 mark for multiplying *b* by 3.7 [CK]**

$= 95.92$ **1 mark for adding c to *a* [CK]**

$= 96.0$ million 1 mark for **correct answer [AK]**

[3 marks]

TOTAL 25 marks

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

APPLIED MATHEMATICS

SPECIMEN

2022

TABLE OF SPECIFICATIONS

Paper 032

UNIT 1

Question	***Module***	***CK***	***AK***	***R***	***Total***
1	1-3	-	-	18	18
2	1-3	-	30	-	30
3	1-3	-	-	12	12
SUBTOTAL		**-**	**30**	**30**	**60**

TEST CODE **02105032-CASE**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 032

ALTERNATIVE TO SCHOOL BASED ASSESSMENT

CASE STUDY FOR THE ALTERNATIVE TO SCHOOOL-BASED ASSESSEMENT EXAMINATION

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. The Paper 032 paper will consist of THREE questions based on your analysis of the given case study.
2. Examine the case study carefully to prepare for your examination.

N.B. **Candidates are to receive this paper ONE week in advance of the date of the examination.**

CASE STUDY

A small production company employs 40 machine operators, working an eight-hour day, whose main function is to assemble components to be used in a further manufacturing process. Machines are placed into groups which are set to operate at different assembly speeds as shown in the following table.

Group	**A**	**B**	**C**	**D**	**E**
Assembly speed (components per minute)	6–8	9–11	12–14	15–17	18–20
Number of Components	15	15	40	20	10

Components from each group are checked at intervals and defective components are taken off the line. The owner of the company is of the opinion that the assembly speed, x components per minute, influences the number of defective components, y, found during inspection. He is also concerned about the uncertainty of the number of defective components that he should expect from each machine. As quality control manager, you are asked to investigate the uncertainty of the number of defective components produced and his assessment on whether the assembly speed of his machines influences the number of defective components produced.

TEST CODE **02105032**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 032

ALTERNATIVE TO SCHOOL-BASED ASSESSMENT

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of a case study and THREE questions. Answer ALL questions.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
6. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials

Mathematical formulae and tables **(Revised 2022)**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

“*”Barcode Area”*”
Sequential Bar Code

INSTRUCTION: Read the case and answer the questions that follow.

CASE STUDY

A small production company employs 40 machine operators, working an eight-hour day, whose main function is to assemble components to be used in a further manufacturing process. Machines are placed into groups which are set to operate at different assembly speeds as shown in the following table.

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1. (a) Explain why a chi-squared analysis is suitable for this investigation.

....

....

....

....

....

....

[3 marks]

(b) Suggest a suitable model for testing the probability of getting a given number of defective components from a sample of components produced.

....

....

....

....

....

....

[3 marks]

(c) State the independent variable and the dependent variable.

....

....

....

[2 marks]

(d) Explain why it is important to use a sample for your data collection.

....

....

....

....

....

....

[3 marks]

(e) Explain why a stratified random sample is BEST for the situation described.

....

....

....

....

....

....

[3 marks]

(f) Describe fully how stratified random sampling will be used to select the sample for this investigation.

....

....

....

....

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....

....

[4 marks]

Total 18 marks

GO ON TO THE NEXT PAGE

2. (a) Over a four-hour period, twenty machines sampled had the following number of defective components: 9, 9, 12, 13, 14, 13, 8, 17, 12, 18, 9, 13, 17, 15, 8, 12, 16, 12, 7, 19.

(i) Construct a frequency table using the data given on defective components. Classify the data into the groups in the case, A (6–8), B (9–11), C (12–14), D (15–17) and E (18–20).

[5 marks]

(ii) Construct a suitable chart or diagram to show this data.

[5 marks]

(b) For an assembly speed of 12 components per minute, it is believed that the probability of having a defective component is 0.2. Use an appropriate distribution to calculate the probability that in a sample of 15 components from this assembly speed, there will be at least 2 defective components.

[10 marks]

(c) The following data were collected after the inspection of the five groups over a four-hour period.

Number of Defective Components Found		2	3	4
Group	A	14	20	16
	B	12	15	23
	C	15	16	19
	D	27	12	11
	E	18	23	9

Use the chi squared test, at the 5% level, to analyze the data presented.

(i) State the null and alternative hypothesis

[2 marks]

(ii) Sketch a graph to display the rejection region

[3 marks]

GO ON TO THE NEXT PAGE

(iii) Calculate the chi-squared value.

[5 marks]

Total 30 marks

3. (a) State THREE key findings from the investigation. Support each finding with the results of your investigation.

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[3 marks]

(b) Outline ONE conclusion that can be made from the investigation.

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[2 marks]

(c) Outline ONE limitation of the investigation.

....

....

....

[2 marks]

(d) Outline ONE recommendation to improve the investigation.

....

....

....

[2 marks]

Total 12 marks

END OF TEST

02105020/KMS/CAPE/SPEC 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 1 – Paper 02

KEY AND MARK SCHEME

SPECIMEN 2022

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 1.

(a) Machines at this company operate at different assembly speeds and the machines also produce a number of defective components **(1).** The owner of this company is concerned that higher assembly speeds result in more defective components produced **(1).** The chi squared test will be used to determine whether there is a relationship or association between the variables **(1).** **[3 marks]**

(b) Since the owner is uncertain that the speed of the machine is responsible for the number of defective components produced **(1)** it will be necessity to conduct probability tests. The Binomial distribution will be used for this **(1)** since components are examined to see whether or not they are defective **(1).** **[3 marks]**

(c) **Independent variable** - the assembly speed **(1)**
Dependent variable - the number of defective components **(1)**
[2 marks]

(d) A sample will be necessary to reduce the time that the investigation will take **(1).** It will be a difficult task to check every machine **(1).** A sample will reduce the cost of the investigation **(1).** **[3 marks]**

(e) There are five groups in the population **(1).** A stratified random sample involves selecting your sample in proportion to strata sizes **(1).** This will ensure that samples are randomly selected from each group of components **(1).** **[3 marks]**

(f) A manufacturing day has 8 hours and there are 40 machines, operating at five different assembly speeds. Five groups of 8 machines will be created based on assembly speed.

Well defined strata - 1 mark
Sample Strata sizes - 1 mark
Sampling frame indicated – 1 mark
Selecting sample using a random method – 1 mark

[4 marks]

Total 18 marks [R]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 2.

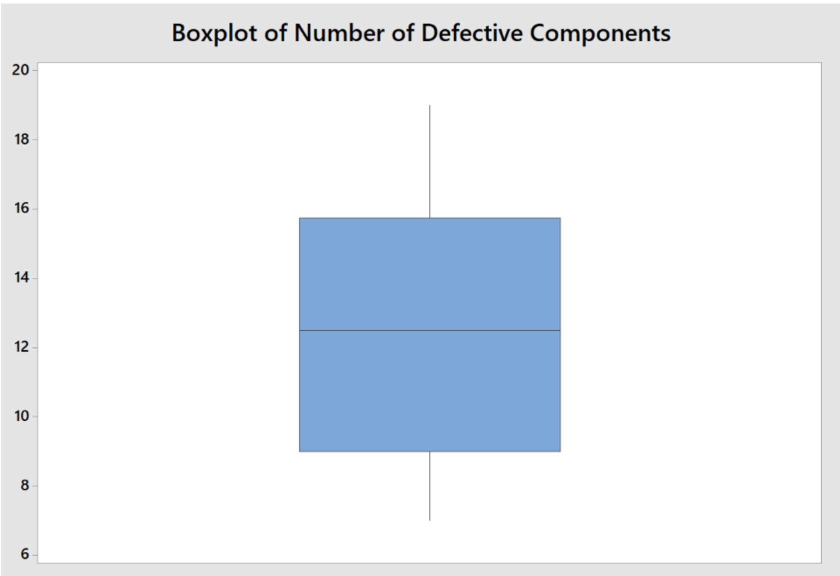
(a) (i)

Group	A (6-8)	B (9-11	C (12-14)	D (15-17)	E (18-20)
Frequency	3	3	8	4	2

Table is clearly written (unambiguous and systematic) (1 mark)
Appropriate headers (columns and rows) (1 mark)
Correct Frequencies (3 marks)
Award 3 marks for at least 3 correct
Award 2 marks for 2 correct
Award 1 marks for 1 correct

[5 marks]

(ii) **Award marks as follows**



Calculating $Q_1 = 9$ 1 mark
Calculating $Q_2 = 12.5$ 1mark
Calculating $Q_3 = 15.5$ 1 mark
Constructing the box and whisker plot 1 mark
Appropriate scale used 1 mark

[5 marks]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

(b) Use a Binomial distribution with n = 15 and p = 0.2 **(3 marks)**

Let X be the number of defective components **(1 mark)**

Then $P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)]$ = Correct $P(X \geq 2)$ **(2 marks)**

$1 - [{}^{15}C_0 \times 0.2^0 \times 0.8^{15} + {}^{15}C_1 \times 0.2 \times 0.8^{14}]$ (correct use of formula)

$= 1 - [1.801 \times 10^{-2} + 1.319 \times 10^{-1}]$ **(2 marks)**

$= 1 - 1.4995 \times 10^{-1}$ **(1 mark)**

= 0.85 (correct answer) **(1 mark)**

1 mark for using binomial
2 marks for parameters (1 mark each for the parameters (n and p))
1 mark for defining the random variable X
1 mark for correct formula (SOI)
1 mark for the $P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)]$
1 mark for calculating $P(X = 0)$
1 mark for calculating $P(X = 1)$
1 mark for subtracting the probabilities from 1
1 mark for the correct answer

[10 marks]

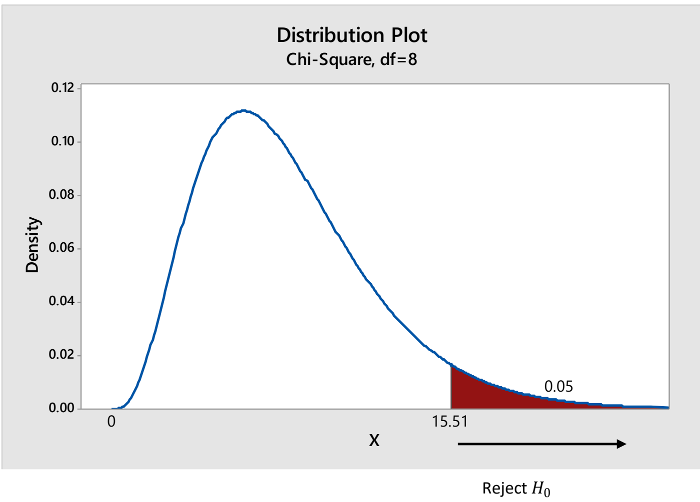
(a) (i)

H_0: There is no association between assembly speed and the number of defective components. **(1 mark)**

H_1: Assembly speed affects the number of defective components produced. **(1 mark)**

[2 marks]

(ii)



1 mark for the critical value 15.51
1 mark for the shape of the graph
1 mark for the region
[3 marks]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

(iii)

Number of defective components found		2 O	E	3 O	E	4 O	E	Total
Group	A	14	17.2	20	17.2	16	15.6	50
	B	12	17.2	15	17.2	23	15.6	50
	C	15	17.2	16	17.2	19	15.6	50
	D	27	17.2	12	17.2	11	15.6	50
	E	18	17.2	23	17.2	9	15.6	50
Total		86		86		78		250

Expected frequencies = $\frac{\textbf{row total} \times \textbf{column total}}{\textbf{grand total}}$ **(SOI) at least 3 cells correct**

1 mark

$\chi^2 = \sum \frac{(O-E)^2}{E}$ (SOI)

A = 0.595 + 0.456 + 0.010 = 1.061

B = 1.572 + 0.281 +3.510 = 5.363

C = 0.281 + 0.837 + 0.741 = 1.859

D = 5.584 + 1.572 + 1.356 = 8.512

E = 0.037 + 1.956 + 2.792 = 4.818

(1 mark each for at least 3 correct calculations)

χ^2 = 21.61 **(1 mark)**

[5 marks]

Total 30 marks [AK]

APPLIED MATHEMATICS

UNIT 1 - Paper 02

KEY AND MARK SCHEME

Question 3.

(a) The probability of at least two defective components is 0.85 **(1 mark)**
since $P(X \geq 2) = 1 - P(X < 2)$ **(1 mark)**
The chi-squared calculated value is 21.6 **(1 mark)**
Since $\chi^2_{calc} = \sum \frac{(\text{O}-\text{E})^2}{\text{E}}$ **(1 mark)**

The data is positively skewed **(1 mark)**
Since $Q_3 - Q_2 < Q_2 - Q_1$ **(1 mark)**

[6 marks]

(b) Since $\chi^2_{calc} = 21.6 > 15.51 = \chi^2_{0.05}(8)$, we reject H_0 and conclude that there is an association between the assembly speed and the number of defective components.

reject H_0 $\chi^2_{calc} = 21.6 > 15.51 = \chi^2_{0.05}(8)$ 1 mark
there is an association 1 mark
[2 marks]

(c) One limitation of the finding is that the data collected was restricted to a four – hour period which would provide a snapshot of what is happening and not necessarily an accurate picture.

Restricted four -hour period 1 mark
Less accurate 1 mark
[2 marks]

(d) One recommendation is conduct regular observations over a period of time to increase the accuracy of the data

Extend time 1 mark
Increased accuracy 1 mark
[2 marks]

Total 12 marks

TEST CODE **02205020**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

MATHEMATICAL APPLICATIONS

UNIT 2 − Paper 01

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have one hour and 30 minutes to answer them.
2. In addition to this test booklet, you should have an answer sheet.
3. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.
4. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
5. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   The mean of 5, 7, 9, 11 and 13 is

   (A) 5
   (B) 7
   (C) 8
   (D) 9

   Sample Answer

   Ⓐ Ⓑ Ⓒ ●

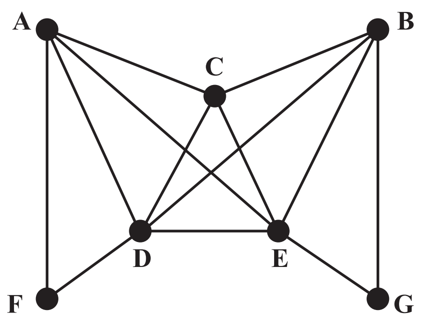
   The best answer to this item is “9 ”, so (D) has been shaded.
6. If you want to change your answer, erase it completely before you fill in your new choice.
7. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, go on to the next one. You may return to that item later.
8. You may do any rough work in this booklet.
9. The use of silent, non-programmable scientific calculators is allowed.

   Examination Materials:

   A list of mathematical formulae and tables. **(Revised 2019)**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

<u>Item **1**</u> refers to the following graph.



1. Which of the following is a path from vertex G to vertex D?

(A) GBECFAD
(B) GBECAFD
(C) GBCAEBD
(D) GEBCEAD

2. What is the contrapositive of the conditional statement "The home team misses whenever it is drizzling"?

(A) If it drizzling, then the home team misses.
(B) If the home team misses, then it is drizzling.
(C) If it is not drizzling, then the home team does not miss.
(D) If the home team wins, then it is not drizzling.

3. A proposition that is always false is a

(A) tautology
(B) contingency
(C) conjunction
(D) contradiction

4. Which of the following is De Morgan's law?

(A) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
(B) $\sim (P \wedge R) \equiv \sim P \vee \sim R$
(C) $P \vee \sim P \equiv$ True, $P \wedge \sim P \equiv$ False
(D) $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$

<u>Item **5**</u> refers to the following information.

Jim is drawing caricatures at a fair for 8 hours (480 minutes). He can complete a small drawing in 15 minutes and charges \$10 for that drawing. He can complete a larger drawing in 45 minutes and charges \$25 for that drawing. Jim hopes to make at least \$200 at the fair. Let x represent the number of small drawings and let y represent the number of large drawings.

5. Which of the following systems of inequalities BEST models the situation?

(A) $10x + 15y \leq 480$
$45x + 25y \geq 250$

(B) $10x + 25y \leq 200$
$15x + 45y \leq 480$

(C) $10x + 25y > 200$
$15x + 45y < 480$

(D) $10x + 25y \geq 200$
$15x + 45y \leq 480$

6. What is the name of the extra row or column that is added to balance an assignment problem?

(A) Regret
(B) Epsilon
(C) Dummy
(D) Extra

GO ON TO THE NEXT PAGE

7. Suppose you are using the Hungarian algorithm on the matrix shown below.

$$\begin{bmatrix} 4 & 7 & 5 \\ 10 & 18 & 14 \\ 12 & 8 & 19 \end{bmatrix}$$

Which of the following matrices would be the result after subtracting row minima AND subtracting column minima?

(A) $\begin{bmatrix} 0 & 3 & 1 \\ 0 & 8 & 4 \\ 4 & 0 & 11 \end{bmatrix}$

(B) $\begin{bmatrix} 4 & 7 & 5 \\ 10 & 18 & 14 \\ 12 & 8 & 19 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 3 & 0 \\ 0 & 8 & 3 \\ 4 & 0 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

8. The matrix below shows the time required for each combination of a worker and a job. The jobs are denoted by J1, J2, J3, J4 and J5: the workers by W1, W2, W3, W4 and W5.

	J1	**J2**	**J3**	**J4**	**J5**
W1	10	5	13	15	16
W2	3	9	18	13	6
W3	10	7	2	2	2
W4	7	11	9	7	12
W5	7	9	10	4	12

Each worker should perform exactly one job and the objective is to minimize the total time required to perform all jobs. What values would be in the 5th row of the matrix above after subtracting the row minimum?

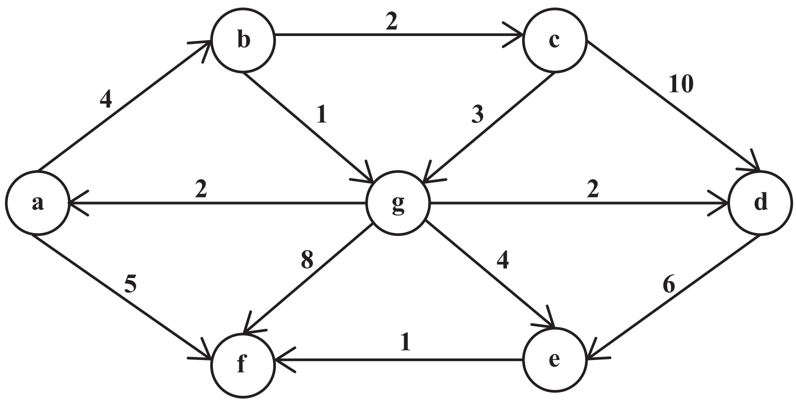
(A) 5 0 8 10 11
(B) 0 6 15 10 3
(C) 0 4 2 0 5
(D) 3 5 6 0 8

9. Activities R, S, and T are the immediate predecessors for activity W. If the earliest finishing times for the three activities are 10, 13 and 18 respectively, what will be the earliest starting time for activity W?

(A) 8
(B) 9
(C) 10
(D) 13

Item **10** refers to the following graph.

Consider the following graph.



10. If **b** is the source vertex, what is the minimum cost to reach vertex **f**?

(A) 8
(B) 9
(C) 4
(D) 6

Item **11** refers to the following information.

Let **p**, **q**, and **r** represent the following statements:

p: Sam had pizza last night.
q: Chris finished her homework.
r: Pat watched the news this morning.

11. What is the statement "Sam did not have pizza last night or Pat did not watch the news this morning." in symbolic form?

(A) $p \wedge \sim r$
(B) $\sim p \vee q$
(C) $\sim p \vee \sim r$
(D) $\sim p \wedge \sim q$

Items **12** and **13** refer to the following information.

A movie theatre has 300 seats and charges \$7.50 for adults and \$5.50 for children. The theatre expects to make at least \$2000 for each showing. Let x represent the number of adults and y represent the number of children.

12. The system of inequalities to model the situation is

(A) $x + y \leq 300$
$7.5x + 5.5y \geq 2000$

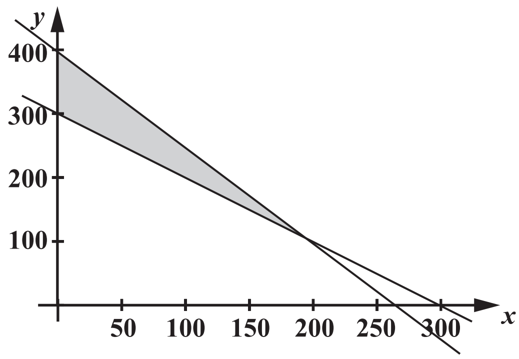
(B) $x + y < 300$
$7.5x + 5.5y \geq 2000$

(C) $x + y \leq 300$
$7.5x + 5.5y \leq 2000$

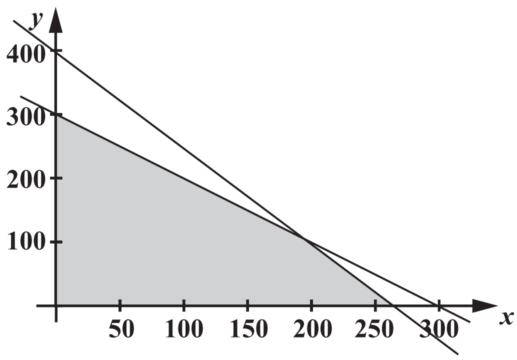
(D) $x + y > 2000$
$7.5x + 5.5y \leq 300$

13. In which of the following diagrams is the information given represented by the shaded region.

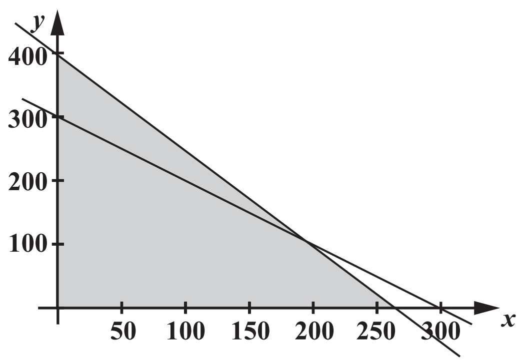
(A)



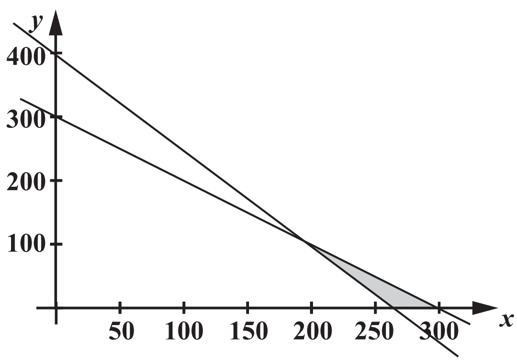
(C)



(B)



(D)



<u>Item **14**</u> refers to the following information.

Given the following original cost matrix

	J1	**J2**	**J3**
T1	482	437	437
T2	402	499	432
T3	502	518	502

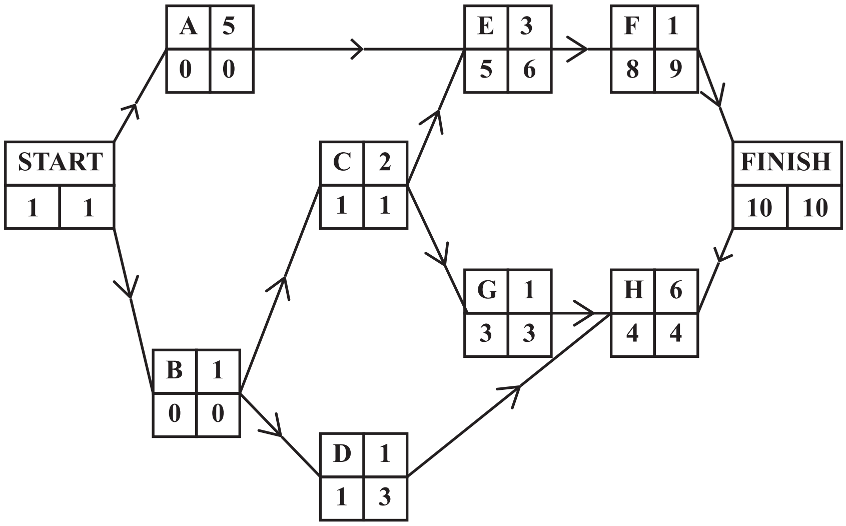
and the optimal assignment matrix

45	**0**	**0**
0	**97**	**30**
0	**16**	**0**

14. Which of the following would represent the optimal assignment?

(A) T1 → J1, T2 → J3, T3 → J2
(B) T1 → J2, T2 → J1, T3 → J3
(C) T1 → J3, T2 → J1, T3 → J2
(D) T1 → J3, T2 → J1, T3 → J1

Item **15** refers to the following network diagram.



15. Determine the critical path.

(A) A − E − F
(B) B − D − H
(C) B − C − G − H
(D) A − E − C − G − H

16. The events A and B are such that $P(A) = 0.44, P(B) = 0.48, P(A \cup B) = 0.71$. Events A and B are BEST described as

(A) Binomial
(B) Dependent
(C) Independent
(D) Mutually exclusive

17. The probability density function of the form $\sum_{i=1}^{N} P(X = x_i) = 1$ is applicable only to

(A) Binary variables
(B) Ordinal variables
(C) Discrete variables
(D) Continuous variables

18. If the probability density function of a continuous random variable is represented as $f(x)$, then $P(x \geq 2)$ is

(A) the height of the curve at $x = 2$
(B) the area under the curve at $x = 2$
(C) the area under the curve to the left of $x = 2$
(D) the area under the curve to the right of $x = 2$

19. In the steel industry, a manufacturer is interested in the number of flaws occurring in every 100 feet of steel sheet. The probability distribution that is most applicable to this situation is the

(A) normal distribution
(B) binomial distribution
(C) poisson distribution
(D) uniform distribution

20. How many ways can the letters of the word GERMAN be arranged so that the vowels only occupy even spaces?

(A) 36
(B) 48
(C) 96
(D) 144

21. S and T are two events such that $P(\overline{S}) = 0.4$ and $P(S \cap T) = 0.2$. Then $P(S \cap \overline{T})$ is equal to

(A) 0.2
(B) 0.4
(C) 0.6
(D) 0.8

22. Ramey is the leading goal scorer for the Pontalia football team. The probability that he will score 0, 1, 2, or 3 goals in any of his matches is 0.20, 0.35, 0.35, and 0.10. What is probability that Ramey will score less than 3 goals in an upcoming match?

(A) 0.20
(B) 0.55
(C) 0.90
(D) 1.00

23. The daily log of a supermarket shows that an average of 10 customers are served at the 'less than 10 items' checkout counter each hour. What is the probability that 15 customers arrive at this checkout counter in 1 hour?

(A) 0.03
(B) 0.05
(C) 0.11
(D) 0.91

24. The continuous random variable X has a probability density function, f, given by

$$f(x) = \begin{cases} Ax(1-x^2) & (0 \leq x \leq 1), \\ 0 & \textit{elsewhere}, \end{cases}$$

The values of A and μ are

(A) $A = \frac{1}{4}, \mu = \frac{8}{15}$

(B) $A = \frac{1}{4}, \mu = \frac{11}{225}$

(C) $A = 4, \mu = \frac{8}{15}$

(D) $A = 4, \mu = \frac{11}{225}$

Item **25** refers to the following information.

A firm wishes to determine for the population of employees if the days with the highest number of late entries occur with equal frequencies. A random sample of 60 supervisors revealed the days of a five-day work week with the highest number of employees arriving late. The results are shown in the table below.

Day of the week	**Mon**	**Tues**	**Wed**	**Thur**	**Fri**
No. of late employees	16	10	12	10	12

25. Using a 5% significance level, which of the following is **NOT** true?

(A) $\upsilon = 4$
(B) $E_i = 12$
(C) Critical Region: $\chi^2 < 9.488$
(D) H_0: number of late entries occur with equal frequency

26. X and Y are independent random variables with the distribution given by given by $X{\sim}N(2, 25)$ and Y is $N(3, 16)$. The variance of the distribution for $2X - 3Y$ is

(A) 48
(B) 96
(C) 148
(D) 244

27. A collection of large inflatable balloons has 5% defective balloons. Selecting a non-defective balloon on the 4th try will have a geometric distribution with

(A) $x = 3, p = 0.05$
(B) $x = 4, p = 0.05$
(C) $x = 3, p = 0.95$
(D) $x = 4, p = 0.95$

Items **28 and 29** refers to the following information.

Sunny Days Nuts claims that each bag of dried mixed fruits contains 100 pieces with equal amounts of pineapple, pecan, raisin, and banana pieces. Joe randomly purchased 10 bags of the mixed fruits to investigate this claim. A count of the fruit pieces in the 10 bags found 300 pineapple, 200 pecan, 275 raisin, and 225 banana pieces.

28. What is an appropriate null hypothesis for this investigation?

(A) The average of all fruit pieces is the same.
(B) The average of all fruit pieces is different.
(C) The proportion of all fruits pieces is different.
(D) The proportion of all fruit pieces is the same.

29. The chi-squared value with $\alpha = 0.05$ and 3 degrees of freedom is 7.81. What is the value of the χ^2 statistic and the correct conclusion to be made?

(A) $\chi^2 \approx 25$; not significant at the 0.05 level
(B) $\chi^2 \approx 25$; significant at the 0.05 level
(C) $\chi^2 \approx 81.25$; not significant at the 0.05 level
(D) χ^2 81.25; significant at the 0.05 level

GO ON TO THE NEXT PAGE

30. X is a continuous random variable that represents the score obtained on a test item. The scores are defined by the probability density function:

$$f(x) = \frac{2}{9}x(3-x), 0 \le x \le 3$$

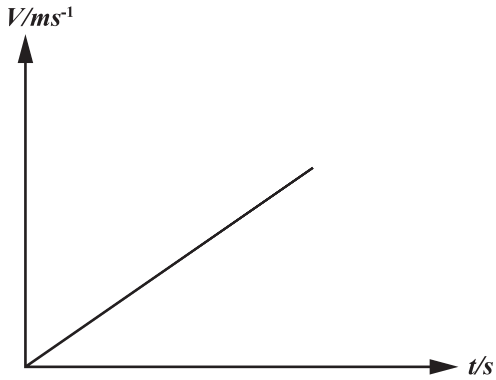
The probability that a student scores more than 1 on the test is

(A) 0.11
(B) 0.59
(C) 0.63
(D) 0.72

31. Two forces **F**1 and **F**2 have a resultant force **F**3. If **F**1 = 3**i** − 5**j** and **F**3 = −7**i** + 2**j**, then **F**2 is

(A) 4**i** − 3**j**
(B) −10**i** + 7**j**
(C) 10**i** − 7**j**
(D) −4**i** + 3**j**

<u>Item **32**</u> refers to the following velocity time graph which shows a particle moving in a straight line.



32. The gradient of the velocity time graph represents the

I. acceleration
II. rate of change of velocity
III. rate of change of displacement

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II and III

33. A particle is projected with a speed of $u\ ms^{-1}$ at an angle of θ^0 to the horizontal. What is the distance travelled horizontally?

(A) $\frac{2u \sin\theta}{g}$

(B) $\frac{u^2 \sin^2 \theta}{2g}$

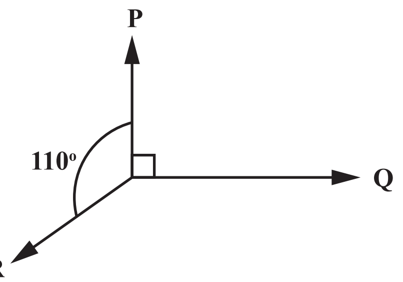
(C) $\frac{u^2 \sin 2\theta}{g}$

(D) $\frac{u^2 \sin\theta \cos\theta}{2g}$

34. A construction worker lifts a box of mass 15 kg to a height of 3 m. How much work is done by gravitational force?

(A) 450 J
(B) 45 J
(C) 30 J
(D) 150 J

35. A body is held in equilibrium by three forces, P, Q and R, as shown in the diagram. R has a magnitude of 45 N. What is the magnitude of the force P to one decimal place?



(A) 5.6 N
(B) 15.4 N
(C) 42.3 N
(D) 123.6 N

36. A constant force of 40 N pulls a small wooden block along a rough horizontal floor. The force acts a angle of θ^0 to the horizontal and the block moves at a constant speed of 30 ms$^{-1.}$ Given that the work done by this force is 900 J in 10 s, what is the value of θ?

(A) 4.3^0
(B) 41.4^0
(C) 48.6^0
(D) 85.7^0

37. An object of mass 4 kg lies on a rough horizontal surface where μ is the coefficient of friction between the block and surface. A force of magnitude of 20 N acts downwards on the block where $\sin a = \frac{3}{5}$. If the block remains at rest, then

(A) $\mu \le \frac{4}{13}$

(B) $\mu \ge \frac{4}{13}$

(C) $\mu \le \frac{3}{14}$

(D) $\mu \ge \frac{3}{14}$

38. Particles A, with mass 0.5 kg, and B, with mass, 0.4 kg are attached to the ends of a light inextensible string. The string which is taut passes over a smooth pulley. The system is released from rest and the particles move vertically. What is the magnitude of the resultant force exerted on the pulley?

(A) $0N$

(B) $\frac{20}{9}N$

(C) $\frac{40}{9}N$

(D) $\frac{80}{9}N$

GO ON TO THE NEXT PAGE

39. A golfer hits a ball with an initial speed of 30 ms^{-1} an angle of 40^0 above the hortizontal. What is the vertical component of initial velcoity of the ball, in ms^{-1}?

(A) 19.3
(B) 23.0
(C) 25.2
(D) 30.0

40. A golfer hits 2 similar balls with the same force. One ball was hit at an angle of 30^0 and the 2nd ball at an angle of 45^0. Compared to the ball hit at 30^0, the ball fired at 45^0 has

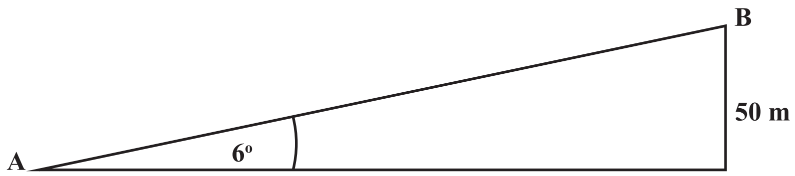
(A) shorter range, more time of flight
(B) shorter range, less time of flight
(C) longer range, less time of flight
(D) longer range, more time of flight

---

41. A car travel at a constant speed of 54 kmh^{-1}. The car's engine produce a driving force of 6000 N in order to keep the speed constant. What is the power developed by the engine?

(A) 30000 W
(B) 90000 W
(C) 324000 W
(D) 1166400 W

Item **42** refers to the following diagram.



42. A car of mass 1500 kg proceeds up a hill inclined at 6^0 to the horizontal with *B* being 50 m higher than A. If the speed of the car is constant, and the work done against resistance is 240 KJ,what is the work done by the car's engine?

(A) 71000 J
(B) 718000 J
(C) 750000 J
(D) 990000 J

43. A particle moves such that its velocity, v, is given by, $v = 2t^2 - 11t + 15$. If a and x represents the particle's acceleration and displacement respectively after t seconds which of the following are true?

I. $x = \int (2t^2 - 11t + 15)\, dt$

II. $\alpha = 4t - 11$

III. at t = 2.5 s the particle will be instantaneously at rest

(A) I and II only
(B) II and III only
(C) I and III only
(D) I, II and III

44. Two bowling balls, each with a mass of 9 kg are travelling toward each other. The bowling ball moving to the right has a speed of 4 ms^{-1} and the bowling ball moving to the left has a speed of 3 ms^{-1}. What is the total momentum before the collision?

(A) −27 Ns
(B) −9 Ns
(C) 9 Ns
(D) 36 Ns

45. Which of the following objects has the greatest momentum?

(A) A tractor trailer at rest
(B) A 60 kg man walking
(C) A fast bowler bowling a ball
(D) A sports car exceeding the speed limit on the highway

END OF TEST

Item	Specific Objective	Key		Item	Specific Objective	Key
1	2.1.3.1	B		26	2.2.2.2	A
2	2.1.4.4	D		27	2.2.2.5	D
3	2.1.4.3	D		28	2.2.3.3	A
4	2.1.4.4	B		29	2.2.4.5	B
5	2.1.1.1	D		30	2.2.3.1	B
6	2.1.2.1	C		31	2.3.1.2	B
7	2.1.2.4	A		32	2.3.2.1	A
8	2.1.2.4	D		33	2.3.2.3	C
9	2.1.3.4	C		34	2.3.4.3	A
10	2.1.3.3	D		35	2.3.1.6	B
11	2.1.4.1	B		36	2.3.4.1	D
12	2.1.1.3	C		37	2.3.1.5	A
13	2.1.1.2	D		38	2.3.1.5	D
14	2.1.2.1	D		39	2.3.3.3	A
15	2.1.4.7	D		40	2.3.3.3	A
16	2.2.1.4	B		41	2.3.4.4	B
17	2.2.2.1	C		42	2.3.4.3	D
18	2.2.3.1	D		43	2.3.2.5	D
19	2.2.4.1	C		44	2.3.2.7	C
20	2.2.1.2	D		45	2.3.2.7	C
21	2.2.1.2	B				
22	2.2.2.1	C				
23	2.2.2.4	A				
24	2.2.3.4	A				
25	2.2.4.4	C				

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

APPLIED MATHEMATICS

SPECIMEN

2022

TABLE OF SPECIFICATIONS

Paper 02

UNIT 2

Question	***Module***	***CK***	***AK***	***R***	***Total***
1	1	6	14	5	25
2	1	6	14	5	25
3	2	6	14	5	25
4	2	6	14	5	25
5	3	6	14	5	25
6	3	6	14	5	25
SUBTOTAL		**36**	**84**	**30**	**150**

SPECIMEN 2022

TEST CODE **02205020**

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

MATHEMATICAL APPLICATIONS

UNIT 2 – Paper 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections. Each section consists of TWO questions.
2. Answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials:

Mathematical formulae and tables **(Revised 2022)**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

"*"Barcode Area"*"
Sequential Bar Code

SECTION A

MODULE 1: DISCRETE MATHEMATICS

Answer BOTH questions.

1. (a) (i) Construct the truth table for the proposition $\mathbf{x} \Rightarrow \mathbf{y}$.

[2 marks]

(i) Hence determine the truth value of the statement, if $3 \times 5 = 15$, then $6 + 2 = 7$

[2 marks]

(b) Using the laws of Boolean algebra, show that $\mathbf{p \vee (p \wedge q) = p}$

[4 marks]

(c) Represent the Boolean expression

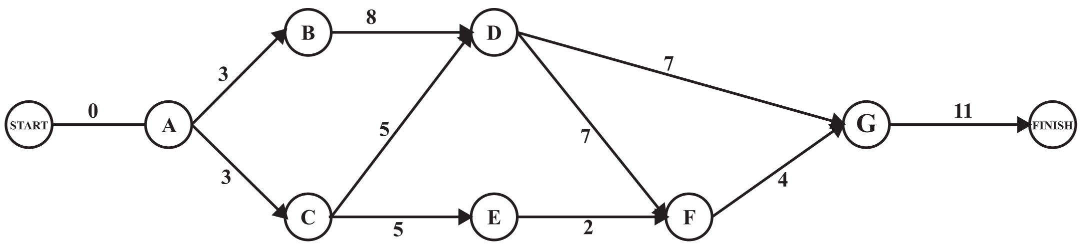
(i) $\mathbf{r \wedge (s \vee \sim t)}$ as a logic circuit, using only AND, OR and NOT gates

[4 marks]

(ii) $(w \wedge x) \vee y \vee z$ as a switching circuit.

[3 marks]

(d) The following diagram is an activity network relating to the assembly of an item. The number on each arc is the time taken, in minutes, to complete the activity.



(i) Construct a precedence table to represent the information in the activity network above.

[3 marks]

GO ON TO THE NEXT PAGE

(ii) Complete the following table to show the earliest start time and the latest start time of EACH activity.

Activity	**Earliest Start Time**	**Latest Start Time**
A		
B		
C		
D		
E		
F		
G		

[5 marks]

(iii) Determine the critical activities and the duration of the critical path.

....

....

[2 marks]

Total 25 marks

NOTHING HAS BEEN OMITTED.

2. (a) Paint-It-Right factory produces two types of paint, gloss and emulsion, in batches of 50-gallon containers. The factory takes four hours to produce a 50-gallon container of the gloss paint while it takes two hours to produce a 50-gallon container of the emulsion paint.

It takes five labour-hours to label and check a batch of gloss paint for consistency while it takes four labour-hours to check a batch of emulsion paint for consistency.

The factory has 35 hours of machine time and 55 labour-hours available each day.

Both types of paints must be produced each day.

The profit on each batch of gloss paint is \$150 and on each batch of emulsion paint is \$125.

(i) Clearly define the variables to be used in setting up this problem as a linear programing model.

[2 marks]

(ii) Define the TWO major constraints to be used in setting up this problem.

[2 marks]

(iii) Write the FOUR inequalities of the constraints that must be used in setting a linear programing model of the information given.

[3 marks]

(iv) Write the objective function from this information that will maximize the profit of this paint producing process.

....

....

[2 marks]

(v) On the graph sheet provided **on page 9**, draw a clearly labelled graph representing the four inequalities. **[5 marks]**

GO ON TO THE NEXT PAGE

(vi) On the graph plotted **on page 9**, clearly identify the feasible region to satisfy the inequalities. **[1 mark]**

Using the graph plotted **on page 9**, determine

(vii) the number of batches of each type of paint that will maximize the profit function

[2 marks]

(viii) the maximum profit that can be made on the production of each batch of these two types of paints.

....

....

[1 mark]

(b) In order to finish an obstacle course competition, teams of three persons are asked to complete three different tasks. They must work as a team; however, each task can only be completed by one team member. Each task is scored out of 30 and the team with the highest score wins.

Three friends form a team so they can enter the competition. The team of three is selected so as to maximize the combined scores of the members while assigning each member to a task.

They each perform the tasks of the course and their scores are shown in the table below.

	Task X	***Task Y***	***Task Z***
Adrian	25	28	29
Brian	28	22	27
Corey	23	24	26

Use the Hungarian algorithm to determine the task to which each is assigned. Your response **must** include the maximum combined score.

[7 marks]

Total 25 marks

SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Answer BOTH questions.

3. (a) (i) Determine the number of different ways of arranging the letters of the word SATISFY if there are no restrictions.

[2 marks]

(ii) Calculate the probability that the two S's remain together in a word.

[3 marks]

(b) Two independent random variables, X and Y, are such that E[X] = 6, E[Y] = 8, Var[X] = 2 and Var[Y] = 4.

Determine

(i) E[X +Y]

[2 marks]

(ii) $E[3X - 2Y]$

[3 marks]

GO ON TO THE NEXT PAGE

(iii) $\text{Var}[3X - 2Y]$

[3 marks]

(c) A continuous random variable, X, has the probability density function, *f*, given by

$$f(x) = \begin{cases} k\,(4 - x) & 0 \le x \le 3 \\ 0 & \textit{otherwise}\,. \end{cases}$$

(i) Determine the value of *k*.

[3 marks]

(ii) Calculate $P\,(X > 1)$.

[3 marks]

(iii) Construct the distribution function $F(x)$.

[3 marks]

(iv) Hence determine the median value of X.

[3 marks]

Total 25 marks

4. (a) A cloth manufacturer estimates that faults occur randomly in the production process at a rate of 2 faults every 10 metres of cloth.

(i) Calculate the probability that there are exactly 2 faults in a 10-metre length of cloth.

[3 marks]

(ii) Calculate the probability of at least 3 faults in a 30-metre length of cloth.

[3 marks]

(iii) A person bought 5 metres of the cloth. Determine the probability of getting 2 faults in that piece of cloth.

[3 marks]

(b) A Trivia quiz is made up of 15 multiple-choice questions, each with four options. It is assumed that each option has the same probability of being correct, and the questions are independent of each other. Let the random variable X represent the number of correct questions that a student taking the quiz is likely to obtain.

(i) State the distribution that may be modelled by this situation, giving its parameter(s).

....

....

[2 marks]

(ii) Determine the number of correct questions that a student is expected to obtain.

[2 marks]

(iii) Calculate the probability of getting exactly four questions correct.

[2 marks]

(c) A cubical die is to be tested for bias by analysing the results of 120 throws of the die. The number of times that each score was obtained is shown in the following table.

score	1	2	3	4	5	6
frequency	26	17	20	13	21	23

(i) Complete the following table to the expected frequency for each score, assuming that each score is equally likely to occur.

Score	1	2	3	4	5	6
Expected frequency						

[2 marks]

GO ON TO THE NEXT PAGE

“*”Barcode Area”*”
Sequential Bar Code

(ii) State the null and alternate hypotheses.

....

....

....

....

[2 marks]

(iii) Carry out a goodness of fit analysis to test the hypotheses stated in (c) (ii), using a 10% level of significance.

[5 marks]

(iv) State your conclusion clearly.

....

....

[1 mark]

Total 25 marks

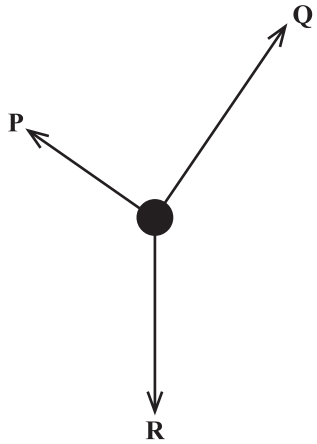
SECTION C

MODULE 3: PARTICLE MECHANICS

Answer BOTH questions.

(Where necessary, take g = 10 ms^{-2}.)

5. (a) The following diagram shows three forces *P*, *Q* and *R* acting a point. *P* is inclined at 30º to the horizontal and *Q* is perpendicular to the to *P*.



Given that *R* = 13 N and the system of forces is in equilibrium,

(i) show the angles between the forces on the diagram above

[3 marks]

(ii) calculate the forces P and Q using Lami's theorem.

[5 marks]

GO ON TO THE NEXT PAGE

(b) A block of mass 30 kg rests on a rough plane which is inclined at an angle θ to the horizontal, as shown in the diagram. A force, T, acts on the block along the line of greatest slope. The coefficient of friction between the plane and the block is 0.35 and $\theta = 50°$.

(i) Draw a carefully labelled diagram showing all the forces acting on the block.

[3 marks]

(ii) Determine the value of T when the block is just about to slip up the plane, the system is in equilibrium and the force of friction would be acting down the plane.

[5 marks]

(iii) Determine the value of T when the block is just about to slip down the plane, the system is in equilibrium and the force of friction would be acting up the plane.

[3 marks]

GO ON TO THE NEXT PAGE

(c) A truck moves in a straight line with speed u ms^{-1}. The truck retards uniformly for 15 seconds, then maintains a constant speed for 15 seconds. It comes to rest after a uniform retardation in 10 seconds. If both retardations are equal and the total distance travelled during the 40 seconds is 250 metres, calculate the value of u.

[6 marks]

Total 25 marks

6. (a) Two cars, A and B, are moving in the same direction on a smooth horizontal road. Car A has a mass of 2000 kg and the mass of car B is 1200 kg. Initially A, moving with speed 25 ms^{-1}, "is catching up with" B, whose speed is u ms^{-1}. Immediately after the cars collide, B has a speed of 20 ms^{-1}. Given that the impulse acting on B has magnitude 6000 Ns,

(i) determine the value of u

[3 marks]

(ii) calculate the loss in kinetic energy after the collision.

[5 marks]

Car B subsequently collides with a stationary car, C, of mass 1500 kg. B comes to a complete stop after the collision while C moves with a speed of x ms^{-1}.

(iii) Determine the value of x.

[3 marks]

GO ON TO THE NEXT PAGE

(b) A golfer hits a golf ball from a point, W, to a point, Z. Z is on the same horizontal level as W. The ball is projected from W at a speed of 95 ms^{-1}, and at an angle of $\alpha°$ above the horizontal.

(i) Given that the ball hits the ground at Z which is 750 m from W, calculate the two possible values of α.

[4 marks]

(ii) Given that $\alpha = 30°$, calculate the times when the ball is 100 m above the ground.

[3 marks]

(iii) Given that $\alpha = 45$, determine the magnitude and direction of the velocity of the ball at the instant when it is vertically above Z.

[7 marks]

Total 25 marks

END OF TEST

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 2 – Paper 02

KEY AND MARK SCHEME

SPECIMEN 2022

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

SECTION A

MODULE 1:

Question 1.

(a) (i) The proposition $\mathbf{x} \Rightarrow \mathbf{y}$

x	**y**	**x ⇒ y**
T	T	T
T	F	F
F	T	T
F	F	T

OR

x	**y**	**x ⇒ y**
0	0	1
0	1	1
1	0	0
1	1	1

1 mark each for a pair of correct row entries

[2 marks] [CK]

(ii) If $3 \times 5 = 15$, then $6 + 2 = 7$

Let ***x*** *represent* $3 \times 5 = 15$ so this is true
Let ***y*** *represent* $6 + 2 = 7$, and this is false

1 mark for correct statements [CK]

Therefore, $x \Rightarrow y$ must be false.

1 mark for conclusion (R)
[2 marks]

Question 1. (continued)

(b) Using the LHS

$\mathbf{p} \vee (\mathbf{p} \wedge \mathbf{q})$

$= (p \wedge 1) \vee (p \wedge q)$ identity law

$= p \wedge (1 \vee q)$ distributive law

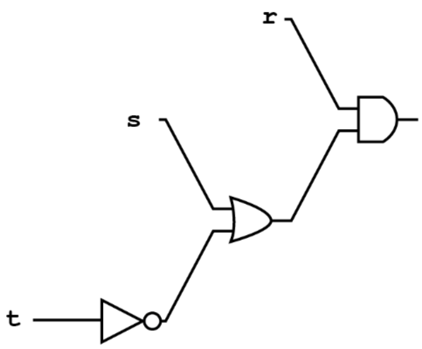
$= p \wedge 1$ identity law

$= p$ identity

1 mark for each step

[4 marks, AK]

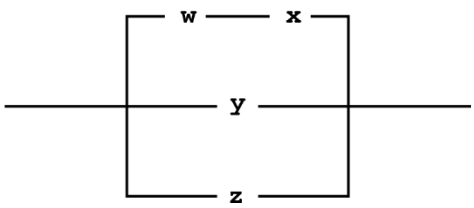
(c) (i) **r** ∧ (**s** ∨ ~ **t**) as a logic circuit, using only AND, OR and NOT gates.



1 mark for each correct gate [AK]
1 mark for correct sequencing [R]

[4 marks]

(ii)



1 mark for w and x in series [AK]
1 mark for 3 switches in parallel [AK]
1 mark for correct diagram [R]

[3 marks]

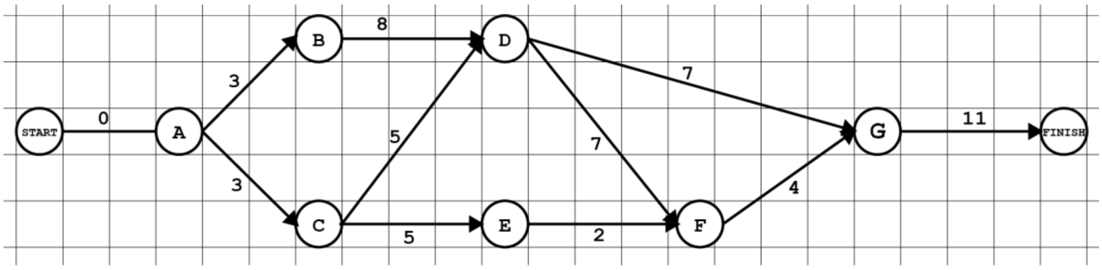
APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 1. (continued)

(d) The following diagram is an activity network relating to the assembly of an item. The number on each arc is the time taken, in minutes, to complete the activity.



(i) Construct a precedence table to represent the information in the activity network above.

A	-
B	A
C	A
D	B, C
E	C
F	D, E
G	D, F

1 mark for correct entry for activity A [CK]
1 mark for every 3 other correct preceding events [R]
[3 marks]

(ii) Complete the following table to show the earliest start time and the latest start time of EACH activity.

Activity	Earliest Start Time	Latest Start Time
A	0	0
B	3	3
C	3	6
D	11	11
E	8	16
F	18	18
G	22	22

1 mark for correct entries for activity A [CK]
1 mark each for every 3 correct entries in earliest and late start time columns [AK]
[5 marks]

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 1. (continued)

(iii) Determine the critical activities and the duration of the critical path.

Critical path:	Start-A-B-D-F-G-FINISH	**(1 MARK)**	**[CK]**
Duration	= 33 MINUTES	**(1 MARK)**	**[AK]**

[2 marks]

Total 25 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 2.

(a) (i) Let x represent the amount of gloss paint. **1 mark**
Let y represent the amount of emulsion paint. **1 mark**

[2 marks] [CK]

(ii) Labour hours
Machine time

[2 marks] [CK]

(iii)

$4x + 2y \le 35$	machine time	**1 mark correct coefficients AK**
$5x + 4y \le 55$	labour hours	**1 mark correct inequalities R**
$x \ge 1,\ y \ge 1$	minimum requirements	**1 mark correct constraints R**

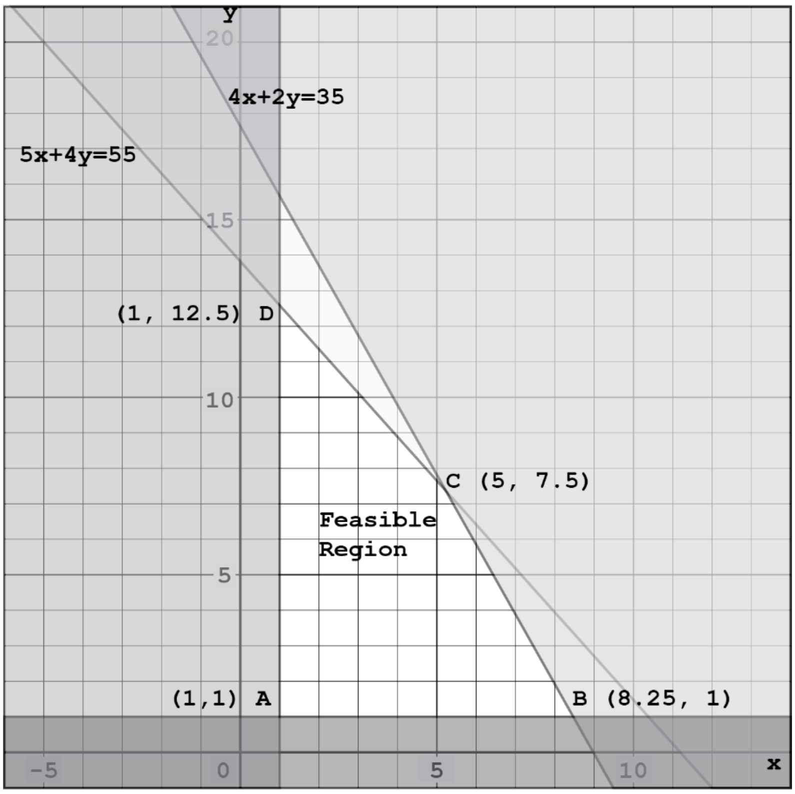
[3 marks]

(iv) Objective function $z = 150x + 125y$

correct x coefficient **1 mark** **R**
correct y coefficient **1 mark** **R**

[3 marks] [R]

(v)



1 mark for every correctly drawn and labelled inequality
1 mark for correctly identifying the feasible region

[5 marks] [AK]

Question 2. (continued)

(vi)

A (1,1)	150(1) + 125(1) = 275
B (8.25, 1)	150(8.25) + 125(1) = 1362.5
C (5, 7.5)	150(5) + 125(7.25) = 1687.5
D (1, 12.5)	150(1) + 125(12.5) = 1712.5

1 mark each for a pair of correct calculations
1 mark for identifying correct maximization values

[2 marks] [AK]

(vi) The maximum profit generated is $1712.50
1 mark for identifying maximum profit [CK]

(b)

	x	y	z	min
A	-25	-28	-29	-29
B	-28	-22	-27	-28
C	-23	-24	-26	-26

1 mark for negating all values [AK]
1 mark for correctly choosing the minimum values [AK]

	x	y	z
A	4	1	0
B	0	6	1
C	2	2	0
min	0	1	0

1 mark for correct subtraction of values [AK]

	x	y	z
A	4	0	0
B	0	5	1
c	2	1	0

1 mark for correct subtraction of values [AK]
1 mark for shading with minimum shading [R]

Allocation: Adrian – Task Y
Brian – Task X
Corey – Task Z **1 mark for the correct allocations [CK]**

Maximum score = 28+28+26 = 82 **1 mark for correct answer [AK]**

[7 marks]

Total 25 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Question 3.

(a) (i) $\frac{7!}{2!} = 2520$

1 mark for division by 2! [CK]

1 mark for correct answer [AK]
[2 marks]

(ii) $\frac{6!}{2520}$ **1 mark for correct numerator [R]**

$= \frac{720}{2520}$ **1 mark for division [AK]**

$= \frac{2}{7}$ **1 mark for correct answer [AK]**

[3 marks]

(b) (i) E (X + Y)

E(X) + E(Y) = 6 + 8 **1 mark for addition of expected values [AK]**

= 14 **1 mark for correct answer [AK]**

[2 marks]

(ii) E [3X – 2Y]

= 3E[X] – 2E[Y] **1 mark for distributing E [CK]**

= 3 x 6 - 2 x 8 **1 mark for multiplication [AK]**

= 18 – 16 **1 mark for subtracting [AK]**

= 2

[3 marks]

(iii) Var [3X – 2Y]

= 9Var[X] + 4Var[Y] **1 mark for a^2Var(X) [CK]**

= 9 x 2 + 4 x 4

= 18 +16 **1 mark for addition of values [AK]**

= 34 **1 mark for correct answer [AK]**

[3 marks]

Question 3. (continued)

(c) (i) $\int_0^3 k(4-x)dx = 1$ **1 mark for integral = 1 [CK]**

$k\left(4x - \frac{x^2}{2}\right)\Big|_0^3 = 1$ **1 mark for integrating [AK]**

$k\ (12 - \frac{9}{2}) = 1$

$k = \frac{2}{15}$ **1 mark for correct answer [AK]**

[3 marks]

(ii) $= \frac{2}{15}\int_1^3 (4-x)dx$ **1 mark for substituting for k [CK]**

$= \frac{2}{15}\left(4x - \frac{x^2}{2}\right)\Big|_1^3$ **1 mark for integrating between 1 and 3 [R]**

$= \frac{2}{15}\left(12 - \frac{9}{2} - 4 + \frac{1}{2}\right)$ **1 mark for correct integration1 mark [AK]**

$= \frac{8}{15}$

[3 marks]

(iii) $P\ (X < t) = \frac{2}{15}\int_0^t (4-x)\,dx$ **1 mark [CK]**

$\frac{2}{15}\left(4x - \frac{x^2}{2}\right)\Big|_0^t$

$\frac{2}{15}\left(4t - \frac{t^2}{2}\right)$ **1 mark [AK]**

$F(x) = \begin{cases} 0 & x < 0 \\ \frac{2}{15}\left(4x - \frac{x^2}{2}\right) & 0 \le x \le 3 \\ 1 & x > 3 \end{cases}$ **1 mark [R]**

[3 marks]

(iv) $P(X < t) = 0.5$ **1 mark [R]**

$\frac{2}{15}\left(4x - \frac{x^2}{2}\right) = 0.5$

$8x - x^2 = \frac{15}{2}$

$x = \frac{8 \pm \sqrt{64-30}}{2}$ **1 mark [AK]**

$x = \frac{8 \pm 5.83}{2}$

$x = 13.83/2$ **1 mark for inadmissible [R]**

or $x = 1.085$

median $= 1.085$

[3 marks]

Total 25 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 4.

(a) (i) X is the number of faults in the 10-metre length of cloth.

$X \sim Po(2)$ **1 mark (SOI) [CK]**

$P(X = 2) = \frac{e^{-2}2^2}{2!}$ **1 mark for correct use of formula [AK]**

$= \frac{4e^{-2}}{2}$

$= 2e^{-2}$

$= 0.2706 \sim 0.271$ **1 mark for correct answer [AK]**

[3 marks]

(ii) In 30 metres of cloth, $X \sim Po(6)$

change of λ1 mark [R]

$P(X \text{ at least } 3) = 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$ **1 mark [AK[**

$= 1 - [e^{-6} + 6e^{-6} + 18e^{-6}]$

$= 1 - 25e^{-6}$ **1 mark [AK]**

$= 0.938$

[3 marks]

(iii) In 5 metres of cloth, $X \sim Po(1)$ **1 mark for change of λ [R]**

$P(X = 2) = \frac{e^{-1}1^2}{2!}$

1 mark for correct use of formula [AK]

$= \frac{e^{-1}}{2}$

$= 0.1839$ **1 mark for correct answer [AK]**

[3 marks]

(b) (i) $X \sim Bin(15, 0.25)$

1 mark for binomial
1 mark for both parameters
[2 marks] [CK]

(ii) $E[X] = np = 15 \times 0.25$ **1 mark for multiplying parameters [CK]**

$= 3.75$ **1 mark for correct answer [AK]**

[2 marks]

(iii) P (at exactly 4 questions correct)

$= {}^{15}C_4 (0.25)^4(0.75)^{11}$ **1 mark correct application of formula [AK]**

$= 0.225$ **1 mark for correct answer [AK]**

[2 marks]

Question 4. (continued)

(c) (i)

score	1	2	3	4	5	6
Expected frequency	20	20	20	20	20	20

2 marks for All values correct [AK]
1 mark for at least 4 values correct [AK]

[2 marks]

(i) H_0: the results follow a uniform distribution
H_1: the results do not follow a uniform distribution.
1 mark for correct null and alternate hypothesis [AK]

Using a Chi square distribution **1 mark [R]**

At the 10% level of significance the critical region is

$\chi^2 \geq 9.236$ **5 degrees of freedom SOI** **1 mark [AK]**

correct table value **1 mark [CK]**

$\chi^2_{\text{calculated}} = \sum \frac{(O-E)^2}{E} = \frac{36}{20} + \frac{9}{20} + 0 + \frac{49}{20} + \frac{1}{20} + \frac{9}{20}$

$= \frac{104}{20}$ 1 mark for **correct formula [CK]**

$= 5.2$ 1 mark for **correct answer** [**AK**]

Since 5.2 < 9.236, the calculated value of χ^2 falls in the accepted region **(1 mark)**. Therefore, accept the hypothesis that each score is likely to occur an equal number of times **(1 mark)**.

2 marks [R]

[8 marks]

Total 25 marks

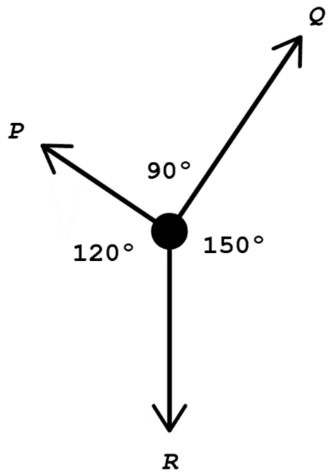
APPLIED MATHEMATICS

UNIT 2 - Paper 02

KEY AND MARK SCHEME

Question 5.

(a) (i)



1 mark for each angle [AK]
[3 marks]

(ii) $Lami's\ theorem: \frac{P}{\sin 150^0} = \frac{Q}{\sin 120^0} = \frac{R}{\sin 90^0}$ **(SOI)**

$\frac{P}{\sin 150^0} = \frac{13}{\sin 90^0}$

$P \sin 90^0 = 13 \sin 150^0$

$P = \frac{13\ sin 150^0}{\sin 90^0} = 6.5\ N$

$\frac{Q}{\sin 120^0} = \frac{13}{\sin 90^0}$

$Q \sin 90^0 = 13 \sin 120^0$

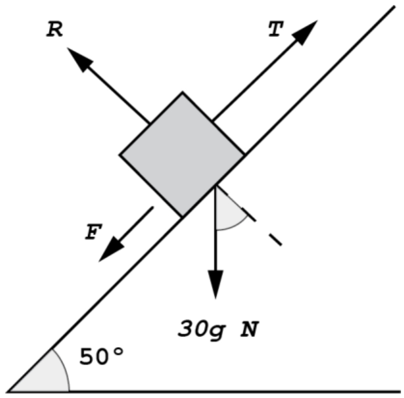
$Q = \frac{13\ sin 120^0}{\sin 90^0} = 11.3\ N$

1 mark for stating Lami's (CK)
1 mark for substituting into Lami's Theorem (AK)
1 mark for calculating P (AK)
2 marks for calculating Q (2 AK)

[5 marks]

Question 5. (continued)

(b) (i)



[3 marks] [CK]

(ii) when the block is just about to slip up the plane the system is in equilibrium and the force of friction would be acting down the plane.

Resolving parallel to the plane:
$T = F + 30g\sin 50^0$
Resolving perpendicular to the plane:
$R = 30g\cos 50^0 = 192.8\ N$
The force of friction, $F = \mu R = 0.35 \times 192.8 = 67.5\ N$
So: $T = 67.5 + 229.8 = 297.3\ N$

1 mark for correctly resolving parallel to the plane (R)
1 mark for equation in terms of F and T (CK)
1 mark for correctly resolving perpendicular to the plane (R)
1 mark for correct F (AK)
1 mark for T (using his F) (AK)

(iii) when the block is just about to slip down the plane the system is in equilibrium and the force of friction would be acting up the plane. (SOI)

Resolving parallel to the plane:
$T = 30g\sin 50^0 - his\ F$

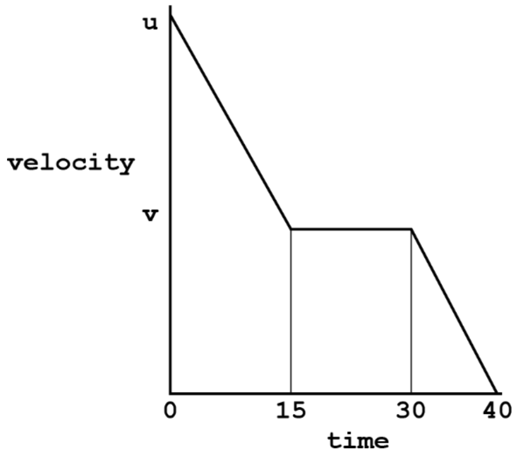
So: $T = 229.8 - 67.5 = 162.3\ N$

1 mark resolving parallel to the plane (R)
1 mark for substitution AK
1 mark for calculating T (AK)

[3 marks]

Question 5. (continued)

(c)



Both retardations are equal:

$$\frac{u-v}{15}=\frac{v-0}{10}$$

$$10u-10v=15v-0$$
$$10u=25v$$
$$v=0.4u \quad \dots\dots(1)$$

1 mark for equating retardations (R)
1 mark for calculating *v* in terms of *u* (CK)

Total distance travelled is 250 m:

$$\frac{1}{2}(u+v)(15)+(v)(15)+\frac{1}{2}(10)(v)=250$$

$$15u+15v+30v+10v=500$$
$$15u+55v=500 \dots\dots\dots(2)$$

Substitute (1) into (2):

$$15u+55(0.4u)=500$$

$$37u=500 \Rightarrow u=\frac{500}{37}=13.5\,ms^{-1}$$

1 mark for calculating the total area (R)
1 mark for simplifying the equation (AK)
1 mark for substitution (AK)
1 mark for calculating the value of *u* (AK)

[6 marks]
TOTAL 25 marks

Question 6.

(a) (i) Impulse acting on B = change in momentum of B (SOI)

$$6000 = 1200(20) - 1200(u)$$

$$\Rightarrow u = \frac{1200(20) - 6000}{1200} = 15\ ms^{-1}$$

1 mark for formula for impulse (CK)
1 mark for substituting into formula for impulse (AK)
1 mark for calculating *u* (AK)

[3 marks]

(ii) Let the velocity of A after collision $= v\ ms^{-1}$

$impulse\ of\ A = -impulse\ on\ B$

$-6000 = 2000(v) - 2000(25)$

$$\Rightarrow v = \frac{-6000 + 2000(25)}{2000} = 22\ ms^{-1}$$

1 mark for relationship between impulses (R)
1 mark for substituting into formula for impulse (AK)
1 mark for calculating *v* (AK)

$loss\ in\ kinetic\ energy = initial\ K.E. - final\ K.E.$ (SOI)

$$= \frac{1}{2}(2000)(25^2) + \frac{1}{2}(1200)(15^2) - \frac{1}{2}(2000)(22^2) - \frac{1}{2}(1200)(20^2)$$

$$= 625000 + 135000 - 483000 - 240000 = 37000\ J$$

1 mark for formula for loss in K.E. (CK)
1 mark for calculating the loss in K.E. (AK)

[5 marks]

(iii) Momentum before collision = momentum after collision (SOI)

$1200(20) + 1500(0) = 1200(0) + 1500(x)$

$x = 16\ ms^{-1}$

1 mark for principle of conservation of linear momentum [CK]
1 mark for substituting [AK]
1 mark for correct answer [AK]
[3 marks]

Question 6. (continued)

(b) (i) Horizontally: we use $s = ut + \frac{1}{2}at^2$

$750 = 95\cos\alpha\, t \;\ldots\ldots(1)$

Vertically: $v = u + at$

$-95\sin\alpha = 95\sin\alpha - 10t$

$t = 19\sin\alpha \ldots\ldots(2)$

1 mark for stating equation (1) (AK)
1 mark for stating equation (2) (AK)

Substitute (2) into (1):

$750 = 95\cos\alpha\,(19\sin\alpha)$

$$\sin 2\alpha = \frac{1500}{1805} = 0.831$$

$2\alpha = \sin^{-1}(0.831) = 56.2^0, 123.8^0$

$\alpha = 28.1^0, 61.9^0$

1 mark for correct $\sin 2\alpha$ (CK)
1 mark for his values of α (R)

[4 marks]

(ii) Given that $\alpha = 30^0$,

calculate the times when the ball is 100 m above the ground.

Vertically: we use $s = ut + \frac{1}{2}at^2$

$$100 = 95\sin 30^0\, t - \frac{1}{2}(10)t^2$$

$$5t^2 - 47.5t + 100 = 0$$

$$t = \frac{47.5 \pm \sqrt{47.5^2 - 4\times 5\times 100}}{2\times 5}$$

$$= \frac{47.5 \pm \sqrt{2256.25 - 2000}}{10}$$

$$t = \frac{47.5 \pm \sqrt{256.25}}{10} = 6.35\,sec, 3.15\,sec$$

1 mark for using formula (CK)
1 mark for the correct quadratic equation in terms of t. (AK)
1 mark for his values of t (AK)

[3 marks]

Question 6. (continued)

(iii) Horizontally: we use $v = u + at$ and $s = ut + \frac{1}{2}at^2$

$v_1 = 95\cos 45^0 = \frac{95\sqrt{2}}{2} = 67.18\ ms^{-1}$

$750 = \frac{95\sqrt{2}}{2}t \Rightarrow t = \frac{150\sqrt{2}}{19}$

Vertically: $v = u + at$

$-v_2 = 95\sin 45^0 - 10t$

Substitute the value of t into above:

$-v_2 = 95\sin 45^0 - 10\left(\frac{150\sqrt{2}}{19}\right)$

$v_2 = 10\left(\frac{150\sqrt{2}}{19}\right) - \frac{95\sqrt{2}}{2} = 44.47\ ms^{-1}$

$Hence: v = \sqrt{\left(\frac{95\sqrt{2}}{2}\right)^2 + (44.47)^2} = \sqrt{6490.07} = 80.6\ ms^{-1}$

$direction\ of\ velocity = \tan^{-1}\left(\frac{44.47}{67.18}\right) = 33.5^0$

1 mark for calculating v_1 (R)
1 mark for calculating t (R)
1 mark for writing formula with negative v_2 (R)
1 mark for calculating v_2 (AK)
1 mark for formula for v (CK)
1 mark for calculating v (AK)

[7 marks]

TOTAL 25 marks

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CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

APPLIED MATHEMATICS

SPECIMEN

2022

TABLE OF SPECIFICATIONS

Paper 032

UNIT 2

Question	***Module***	***CK***	***AK***	***R***	***Total***
1	1	-	10	10	20
2	2	-	10	10	20
3	3	-	10	10	20
SUBTOTAL		**-**	**30**	**30**	**60**

TEST CODE **02205032-CASE**

SPECIMEN 2022

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 2 – Paper 032

ALTERNATIVE TO SCHOOL BASED ASSESSMENT

CASE STUDY FOR THE ALTERNATIVE TO SCHOOOL-BASED ASSESSEMENT EXAMINATION

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. The Paper 032 paper will consist of THREE questions based on your analysis of the THREE given case studies as follows:

   Question 1: DISCRETE MATHEMATICS

   Question 2: PROBABILITY AND DISTRIBUTIONS

   Question 3: PARTICLE MECHANICS

2. Examine the cases carefully to prepare for your examination.

N.B. Candidates are to receive this paper ONE week in advance of the date of the examination.

Module 1 – Discrete Mathematics

CASE STUDY

A store sells two types of laptops – A and B. The store owner pays \$500 for each unit of laptop A and \$800 for each unit of laptop B. The store owner has found over time, twice as many of laptop A as laptop B are sold. From past sales it is estimated that between 5 and 15 of laptop A and between 2 and 5 of laptop B may be sold each month. The plan is not to invest more than \$10,000 in inventory of these laptops.

The store owner estimates that a profit of \$200 can be made on the sale of one unit of laptop A while a unit of laptop B will yield a profit of \$300. The store owner needs to be advised on the number of each type of laptop that should be sold to maximize the profit next month.

Module 2 – Probability and Distributions

CASE STUDY

The Ministry of Works wishes to investigate the number of car accidents in a particular stretch of highway. Police reports on the number of car accidents a day was collected for 100 weeks and tabularised below. A chi squared test was carried out at a 5 % level of significance.

Number of car accidents	0	1	2	3	4	5
Number of weeks	30	34	23	10	3	0

Module 3 – Particle Mechanics

CASE STUDY

A Form 4 physical education class is learning about golf and is interested in determining the optimum angle that a golfer needs to hit the golf ball in order to achieve the maximum possible horizontal displacement.

The experiment is carried out on a perfect day in which there are no clouds, and the wind speed is minimal. The mass of the ball is negligible and the speed at which the ball is hit is kept constant.

TEST CODE **02205032**

SPECIMEN 2022

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

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APPLIED MATHEMATICS

STATISTICAL ANALYSIS

UNIT 2 – Paper 032

ALTERNATIVE TO SCHOOL-BASED ASSESSMENT

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of THREE case studies and THREE questions as follows:

   Question 1: DISCRETE MATHEMATICS
   Question 2: PROBABILITY AND DISTRIBUTIONS
   Question 3: PARTICLE MECHANICS

2. ANSWER ALL questions.

3. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.

4. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.

5. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

6. Do NOT write in the margins.

Examination Materials

Mathematical formulae and tables **(Revised 2022)**
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

"*"Barcode Area"*"
Sequential Bar Code

NOTHING HAS BEEN OMITTED.

MODULE 1: DISCRETE MATHEMATICS

INSTRUCTION: Read the case and answer the questions that follow.

CASE STUDY

A store sells two types of laptops — A and B. The store owner pays \$500 for each unit of laptop A and \$800 for each unit of laptop B. The store owner has found over time, twice as many of laptop A as laptop B are sold. From past sales it is estimated that between 8 and 15 of laptop A and between 2 and 5 of laptop B may be sold each month. The plan is not to invest more than \$10,000 in inventory of these laptops.

The store owner estimates that a profit of \$200 can be made on the sale of one unit of laptop A while a unit of laptop B will yield a profit of \$300. The store owner needs to be advised on the number of each type of laptop that should be sold to maximize the profit next month.

1. (a) State ALL variables and constraints to be used in setting up the problem.

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[7 marks]

(b) (i) On the grid provided **on page 5**, draw all the lines defined by the constraints and shade the feasible region which satisfies ALL constraints using a scale of 1 cm to represent 5 units on both axes. **[4 marks]**

(ii) State the objective function.

[1 mark]

(iii) State the optimal vertices of the graph.

[2 marks]

(iv) Calculate the solutions for each vertex.

[2 marks]

(v) Determine the optimal solution to the objective function.

[1 mark]

(c) (i) State ONE conclusion which can be made from the results of your analysis.

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[1 mark]

(ii) State ONE recommendation to improve the investigation.

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[2 marks]

Total 20 marks

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NOTHING HAS BEEN OMITTED.

MODULE 2: PROBABILITY AND DISTRIBUTIONS

INSTRUCTION: Read the case and answer the questions that follow.

CASE STUDY

The Ministry of Works wishes to investigate the number of car accidents in a particular stretch of highway. Police reports on the number of car accidents a day was collected for 100 weeks and tabularised below. A chi squared test was carried out at a 5% level of significance.

Number of car accidents	0	1	2	3	4	5
Number of weeks	30	34	23	10	3	0

2. (a) Using examples from the case, outline how the given data meets THREE conditions of the Poisson distribution.

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[6 marks]

(b) (i) On the grid provided **on page 9**, construct a bar graph to present the data given.

[4 marks]

(ii) Calculate the expected value of the number of accidents correct to 1 decimal place.

[1 mark]

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(iv) Calculate the test statistic.

[4 marks]

(c) (i) State ONE valid conclusion based on the results of your calculation. Give ONE reason to support your conclusion.

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[2 marks]

(ii) State ONE limitation of the investigation. Give ONE reason to support your conclusion.

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[2 marks]

Total 20 marks

MODULE 3: PARTICLE MECHANICS

INSTRUCTION: Read the case and answer the questions that follow.

CASE STUDY

A Form 4 physical education class is learning about golf and is interested in determining the optimum angle that a golfer needs to hit the golf ball in order to achieve the maximum possible horizontal displacement.

The experiment is carried out on a perfect day in which there are no clouds, and the wind speed is minimal. The mass of the ball is negligible and the speed at which the ball is hit is kept constant.

3. (a) (i) Using examples from the case, outline how the information presented meets the TWO assumptions of projectile motion.

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[4 marks]

(ii) Identify ONE independent variable and ONE dependent variable.

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[2 marks]

(b) (i) Construct a suitable diagram to illustrate the situation described.

[3 marks]

(ii) Write an expression, in terms of t seconds, for the horizontal distance travelled by the ball.

[2 marks]

(iii) Write an equation of the vertical distance travelled.

[1 mark]

(iv) Determine the maximum horizontal range.

[4 marks]

(c) (i) State ONE conclusion which can be made from the results of your analysis. Give ONE reason to support your conclusion.

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[2 marks]

(ii) State TWO recommendations for improving the investigation.

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[2 marks]

Total 20 marks

END OF TEST

APPLIED MATHEMATICS

UNIT 2 - Paper 032

KEY AND MARK SCHEME

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

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STATISTICAL ANALYSIS

UNIT 2 – Paper 032

KEY AND MARK SCHEME

SPECIMEN 2022

APPLIED MATHEMATICS

UNIT 2 - Paper 032

KEY AND MARK SCHEME

Question 1

(a) **Variables and Constraints**

x = Number of laptop A and y = Number of laptop B **(1 mark)**

$500x + 800y \leq 10000$ money to be invested (**1 mark)**

$x \geq 2y$ ratio of stock for each type of laptop **(1 mark)**

$8 \leq x \leq 15$ number of laptops **(1 mark)**

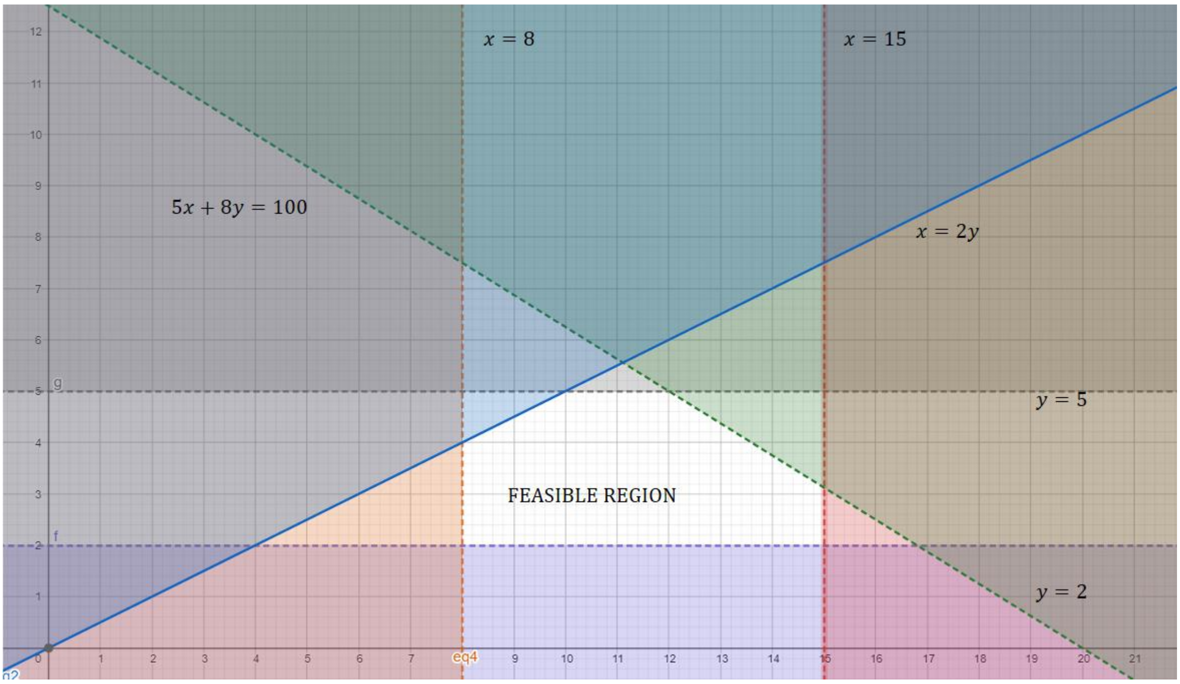
$2 \leq y \leq 5$ number of laptops B **(1 mark)**

$x, y \geq 0$ **(1 mark)**

x, y are integers **(1 mark)**

[7 marks; R]

(b) (i) **Graph**



Sketching $5x + 8y = 100$ **(1 mark)**

Sketching $x = 2y$ **(1 mark)**

Sketching the lines $x = 8$ and $x = 15$

Sketching the lines $y = 2$ and $y = 5$

(1 mark for sketching all four lines)

(1 mark for shading the correct feasible region)

[4 marks; AK]

Question 1 continued

(ii) **Objective Function**

Maximise: $P = 200x + 300y$

[1 mark; AK]

(iii) **Optimal Vertices**

(8, 2), (8, 4) (10, 5), (12, 5), (15, 2), (15, 3)

1 mark for every 3 correct values

[2 marks; AK]

(iv) **Solutions for each vertex**

200(8) + 300(2) = 2200

200(8) + 300(3) = 2500

200(10) + 300(5) = 3500

200(12) + 300(5) = 3000

200(15) + 300(2) = 3600

200(15) + 300(3) = 3900

1 mark for every 3 correct values

[2 marks; AK]

(V) **Optimal Solution**

Maximum value X = 15, Y = 3

[1 mark; AK]

(c) (i) **Conclusions**

The maximum profit is $ 3900

[1 mark; R]

(ii) **Recommendations**

A more in-depth study of the financials to determine other factors that contribute to the profit obtained can provide a more accurate estimate of the profit.

[2 marks; R]

Total 20 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 032

KEY AND MARK SCHEME

Question 2

(a) **Conditions of Poisson**

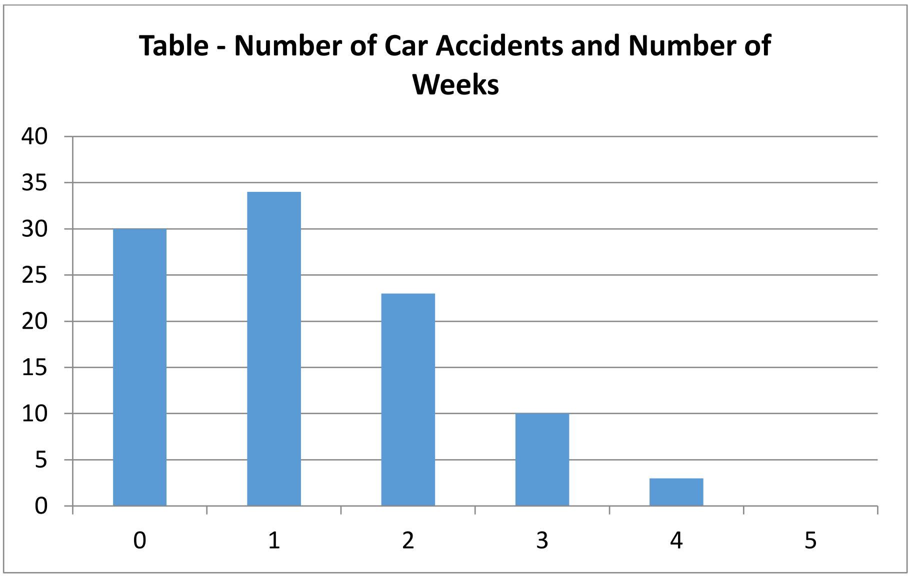
Car accidents are randomly occurring **[2 mark]**

Car accidents are independent events. **[2 mark]**

The data collected can be modelled as the average number of occurrences in a specified time. **[2 mark]**

[6 marks; R]

(b) (i) **Graph**



[4 marks; AK]

(ii) **Critical Value**

$E(X) = \frac{\Sigma fx}{\Sigma f}$ **[1 mark]**

$= \frac{122}{100} = 1.2$ **[1 mark]**

[2 marks; AK]

Question 2 continued

(iii)

Number of car accidents	0	1	2	3	4	5
Observed Frequencies	30	34	23	10	3	0
Expected Frequencies	30.1	36.1	21.7	8.7	2.6	0.8

1 mark for at least 3 correct expected frequencies

Combining expected frequencies since $E_i < 5$ **[1 mark]**

O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
30	30.1	0.0003
34	36.1	0.122
23	21.7	0.0779
10	8.7	0.194
3	3.4	0.047

[1 mark] for all values correct

$$\chi^2_{calc} = \sum \frac{(O_i - E_i)^2}{E_i}$$

$= 0.4412$ **[1 mark]**

[4 marks; AK]

(c) (i) **Conclusions**

do not reject H_0 **[1 mark]**

Since $\chi^2_{calc} = 1.11 < 7.815 = \chi^2_{0.05}(3)$, **[1 mark]** conclude that the data does not follow a Poisson Distribution.

[2 marks; R]

(ii) **Limitations**

One limitation is that the data collected may not have been accurate **[1 mark]** as not all accidents that may have occurred would

[2 marks; R]

Total 20 marks

APPLIED MATHEMATICS

UNIT 2 - Paper 032

KEY AND MARK SCHEME

Question 3

(a) (i) **Assumptions of Projectile Motion**

The mass of the basketball in this study is **negligible**, hence the basketball can be modelled as a **particle. 2 marks**

The experiment is conducted on a cloudless day and wind speed is at a minimum thus reducing **the effects of air resistance. 2 marks**

[4 marks; R]

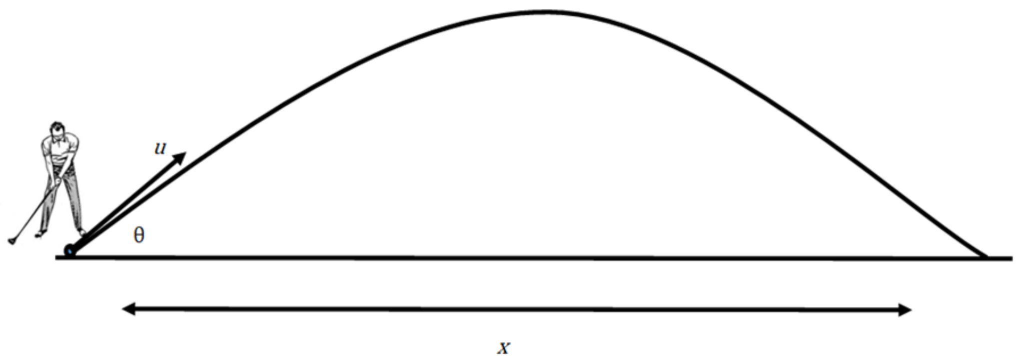
(ii) **Variables.**

Independent – angle of trajectory

Dependent – horizontal range of the ball

[2 marks; R]

(b) (i) **Diagram.**



1 mark for indicating the horizontal distance.

1 mark for indicating u and θ.

1 mark for trajectory of the golf ball.

[3 marks; AK]

(ii) **Expression for Horizontal distance travelled**

$x = (u\cos\theta)t$ **1 mark for** $(u\cos\theta)$ and **1 mark for multiplying by t**

[2 marks; AK]

(iii) **Vertical Distance**

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

[1 mark; AK]

Question 3 continued

(iv) **Maximum horizontal range.**

When $y = 0$

$0 = (u\sin\theta)t - \frac{1}{2}gt^2$

$0 = t\left(usin\,\theta - \frac{1}{2}gt\right)$

$t = 0$ or $t = \frac{2u\sin\theta}{g}$

1 mark for obtaining the values of t

Substituting $t = \frac{2u\sin\theta}{g}$ into (i)

1 mark for substituting $t = \frac{2u\sin\theta}{g}$ into (i)

$x = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$

$= \frac{u^2\sin 2\theta}{g}$

1 mark for obtaining $\frac{u^2\sin 2\theta}{g}$

Maximum horizontal range $= \frac{u^2}{g}$

1 mark for $R = \frac{u^2}{g}$

[4 marks; AK]

(c) (i) **Conclusions**

The optimum angle to achieve the maximum horizontal displacement (range) is 45^0. **[1 mark]** For the range to be a maximum, $\sin 2\theta$ has a to be a maximum. The maximum value of $\sin 2\theta = 1$ and this occurs when $\theta = 45^0$. **[1 mark]**

[2 marks; R]

(ii) Recommendations

- A speed gun should be used ensure that every time the ball is hit, the speed remains the same. **[1 mark]**
- An indoor golf simulator can be used to get a more accurate determination of the angle of projection and horizontal displacement. Also, this will reduce the cost and time to conduct the experiment. **[1 mark]**

[2 marks; R]

Total 20 marks

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

APPLIED MATHEMATICS
(Including Statistical Analysis)

INTEGRATED MATHEMATICS

Statistical Tables

and

List of Formulae

Revised April 2022

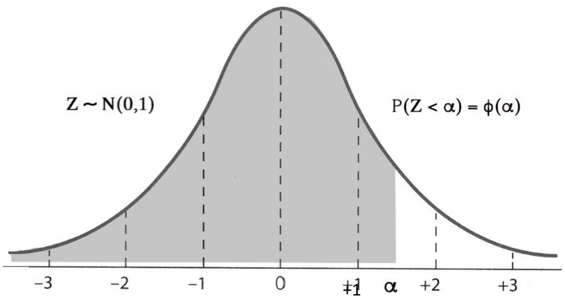
DO NOT REMOVE FROM THE EXAMINATION ROOM

Table 1: The Normal Distribution Function

If Z is a random variable, normally distributed with zero mean and unit variance, then ϕ (z) is the probability that Z $\leq z$. That is, ϕ(z) = P(Z $\leq$ z).

The function tabulated below is ϕ(z), and is shown diagrammatically as

Standard Normal Distribution (area to the left of α)

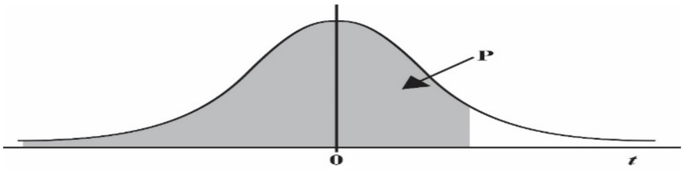


The Distribution Function, ϕ(z)

Z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5 ADD	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6590	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0 7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9046	0.9148	0.9429	0.9441	1	2	4	1	2	4	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	1	2	3	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	1	2	3	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	1	1	2	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9760	0.9767	1	1	2	1	1	2	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	0	1	1	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	0	1	1	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	0	1	1	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	0	1	1	2	2	2
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	0	0	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Table 2: *t*-Distribution

If *T* has a *t*-distribution with *v* degrees of freedom then, for each pair of values of *p* and *v*, the table gives the value of *t* such that P(T ≤ *t*) = P

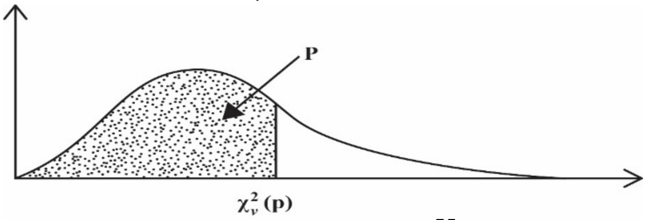


Critical Values for the *t*-distribution

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
V = 1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	4.317	5.208	5.959
12	0.695	1.356	1.782	2.179	2.681	3.055	4.029	4.785	5.408
13	0.694	1.350	1.771	2.160	2.650	3.012	3.833	4.501	5.041
14	0.692	1.345	1.761	2.145	2.624	2.977	3.690	4.297	4.781
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Table 3: Percentage Points of the x^2 Distribution

If X is a random variable, distributed as X^2 with v degrees of freedom then p is the probability that $X \leq \chi_v^2$ (p), where the values of the percentage points χ_v^2 (p), are tabulated in the table below. p is shown diagrammatically (when $v \geq 3$) as



Critical Values for the X^2- distribution

P	.01	.025	.050	.900	.950	.975	. 990	.995	.999
v = 1	0.0001571	0.0009821	0.003932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7 779	9.488	11.14	13.28	14 86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.52
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.646	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.72	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28 41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	104.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.76	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

Table 4: Random Sampling Numbers

18	11	36	26	88	81	11	33	64	08	23	32	00	73	04
57	33	88	37	26	10	79	91	36	03	07	52	55	84	61
72	02	11	44	25	45	92	12	82	94	35	35	91	65	78
89	83	98	71	74	22	05	29	17	37	45	65	35	54	44
44	88	03	81	30	61	00	63	42	46	22	89	41	54	47
68	60	92	99	60	97	53	55	34	01	43	40	77	90	19
87	63	49	22	47	21	76	13	39	25	89	91	38	25	19
44	33	11	36	72	21	40	90	76	95	10	14	86	03	17
60	30	10	46	44	34	19	56	00	83	20	53	53	65	29
03	47	55	23	26	90	02	12	02	62	51	52	70	68	13
09	24	34	42	00	68	72	10	71	37	30	72	97	57	56
09	29	82	76	50	97	95	53	50	18	40	89	40	83	29
52	23	08	25	21	22	53	26	15	87	93	73	25	95	70
43	78	19	88	85	56	67	56	67	16	68	26	95	99	64
45 0	69	72	62	11	12	18	25	00	92	26	82	64	3	
21	72	97	04	52	62	09	54	35	17	22	73	35	72	53
65	95	48	55	12	46	89	95	61	31	77	14	24	14	41
51	69	76	00	20	92	58	21	24	33	74	08	66	90	61
89	56	83	39	58	22	09	01	14	04	14	97	56	92	97
72	63	40	03	07	02	62	20	11	50	11	98	23	80	99

FORMULAE

PURE MATHEMATICS

For the quadratic equation: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n - 1)d, \qquad S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

For a geometric series:

$$u_n = a r^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1},\ r > 1, \qquad S_n = \frac{a(1 - r^n)}{1 - r},\ r < 1, \qquad S_\infty = \frac{a}{1 - r},\ |r| < 1$$

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 +,\ \ldots\ + b^n,$$ where n is a positive integer.

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(r-1)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots r}x^r + \ldots \text{ where } n \text{ is a real number and } |x| <|$$

Summations:

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1). \qquad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1). \qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Complex numbers:

$z^n = (\cos x + i \sin x)^n = \cos nx + i \sin nx$, where n is an integer and x is real

$e^{ix} = \cos x + i \sin x$ where x is real

$[r(\cos x + i \sin x)]^n = r^n(\cos nx + i \sin nx)$

Maclaurin's series:

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all real } x$$

$$\text{In}\,(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r!} + . \quad (-1 < x \le 1)$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots - \frac{x^r}{r!} - \ldots \qquad (-1 \le x < 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all real } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + . \quad \text{for all real } x$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^r(0) + \ldots$$

Taylor's series:

$$f(x) = f(a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)^2}{2!} + f'''(a)\frac{(x-a)^3}{3!} + + f^r(a)\frac{(x-a)^r}{r!} + \ldots$$

The trapezium rule $\int_a^b y\,\mathrm{d}x = \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + .. + y_n - 1)\}$,

$h = \frac{b-a}{n}$, where n is the number of intervals (strips)

The Newton-Raphson iteration $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

TRIGONOMETRY

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Arc length of a circle: $s = r\theta$, (θ measured in radians)

Area of a sector of a circle: $\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$

$$\text{If } \tan \frac{a}{2} = t \text{, then } \sin \alpha = \frac{2t}{1 + t^2} \text{ and } \cos \alpha = \frac{1 - t^2}{1 + t^2}$$

Trigonometric Identities:

$$cos^2 \alpha + sin^2 \alpha \equiv 1, \; 1 + tan^2 a = sec^2 a, 1 + cot^2 a = cosec^2 a$$

$$\sin(\alpha \pm \beta) \equiv \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) \equiv \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) \equiv \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \qquad \alpha \pm \beta \neq (k + \frac{1}{2})\pi$$

$$cos\, 2\alpha = cos^2 \alpha - sin^2 \alpha = 2\, cos^2 \alpha - 1 = 1 - 2\, sin^2 \alpha$$

$$sin\, 2\alpha = 2\, sin\, \alpha\, cos\, \alpha$$

$$\sin \alpha + \sin \beta \equiv 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta \equiv 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta \equiv 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta \equiv 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2} \quad \text{or} \quad -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

STATISTICS

Frequency distributions

$$Mean\ \overline{x} = \frac{\sum fx}{\sum f}$$

$$Standard\ Deviation\ \sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{\sum f(x - \overline{x})^2}{\sum f}} = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n}$$

Median $Q_2 = \left(\frac{n+1}{2}\right)^{th}$ *value*

Grouped data

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$ *where* x *=midpoint of each class, f is the frequency of each class.*

Median $= l + \left(\frac{\frac{N}{2} - f_0}{f_1}\right) w$ *where,*

l= lower limit of the median class

N = total frequency

f_0 = frequency of class preceding the median class

f_1 = frequency of median class

w = width of median class

Mode = $l + \left(\frac{f_1 - f_2}{2f_1 - f_0 - f_2}\right)w$

Where ,

l = lower limit of the modal class

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

w = width of the modal class

Measures of spread or dispersion

Standard deviation for the population

s.d = $\sqrt{\frac{\sum fx^2 - \frac{(\sum fx)^2}{n}}{n}}$

where x = midpoint of each class

f = the frequency of each class

n = population size.

unbiased estimator of the variance of X is $\hat{\sigma}^2 = \frac{n}{n-1}\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

Product Moment Correlation Coefficient, *r*

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left[n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

Covariance Formula $= \frac{S_{x_i y_i}}{S_{x_i} S_{y_i}}$ where $S_{x_i y_i}$ is the co-variance of x and y,

$S_{x_i} S_{y_i}$ is the product of the standard deviation of x and y respectively

Regression line *y* on *x*

$y = a + bx$ passing through $(\overline{x}, \overline{y})$ where

$$\overline{x} = \frac{\sum x}{n} \text{ and } \overline{y} = \frac{\sum y}{n}$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$b = \frac{S_{xy}}{S_{xx}}$$, where S_{xx} is the variance of x.

$$a = \overline{y} - b\overline{x}$$

MECHANICS

Uniformly accelerated motion

$$v = u + at, \quad s = \frac{1}{2}(u + v)t, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

Motion of a projectile

Equation of trajectory is:

$$y = x \tan\theta - \frac{gx^2}{2V^2 \cos^2\theta}$$

$$= x \tan\theta - \frac{gx^2}{2V^2}(1 + \tan^2\theta)$$

Time of flight = $\frac{2V \sin\theta}{g}$

Greatest height = $\dfrac{V^2 \sin^2 \theta}{2g}$

Horizontal range = $\dfrac{V^2 \sin 2\theta}{g}$, maximum range = $\dfrac{V^2}{g}$ for $\theta = \dfrac{\pi}{4}$

Lami's Theorem

$\dfrac{F_1}{\sin \alpha} = \dfrac{F_2}{\sin \beta} = \dfrac{F_3}{\sin \gamma}$, where F_1, F_2, F_3 are forces acting on a particle

and α, β, γ are the angles vertically opposite F_1, F_2, F_3, respectively

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2006

APPLIED MATHEMATICS

APPLIED MATHEMATICS

CARIBBEAN ADVANCED PROFICIENCY EXAMINATIONS
MAY/JUNE 2006

GENERAL COMMENTS

INTRODUCTION

The revised Applied Mathematics syllabus was followed this year for the second time. Of the one hundred and forty one candidates registered for the examination, one hundred and thirty three wrote all the required papers of Option C, while one candidate wrote only Paper 02 of Option C. One candidate wrote all the required papers of Option B and five wrote all the required papers of Option A. One candidate wrote the Alternative to the SBA Paper in Option C.

This is a one-Unit course comprising three papers and three options. However a candidate is required to take only ONE Option. Papers 01 and 02 were examined externally, while Paper 03 was examined internally by the teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to the Unit were 40 per cent, 40 per cent and 20 per cent respectively.

The three options are Option A, Option B and Option C.

Option A consists of Discrete Mathematics, Probability and Distributions and Statistical Inference. Option B consists of Discrete Mathematics, Particle Mechanics and Rigid Bodies, Elasticity, Circular and Harmonic Motion. Option C consists of Discrete Mathematics, Probability and Distributions and Particle Mechanics.

GENERAL COMMENTS

Approximately 91 per cent of the candidates obtained Grades I – V while 7 per cent obtained grade VI and 2 per cent obtained grade VII. The standard of work seen from most of the candidates in this examination was above average. Again this year, candidates appeared to be well prepared in Discrete Mathematics and generally answered the questions well. All of the candidates answered all of the questions in this section. There were a number of candidates who appeared to be well prepared in all three of their modules. However, there were a few candidates who seemed not well prepared in modules 2 and 3 of their respective option. In general, there were a large number of areas of strength displayed by many candidates, nevertheless, candidates need to pay more attention to their algebraic manipulation.

Areas of strength displayed by most candidates on this paper were as follows:

- conversion from logic symbols to words
- use of the distributive law
- construction of truth tables
- calculation of the earliest start time
- identification of the number of degrees from a vertex
- definition of a trail and a path
- construction of an activity network
- identification of the given lines and feasible region in a linear programming problem
- drawing the line $x + y = 5$
- formulation of the null and alternative hypotheses in words and symbols
- calculation of test statistics
- naming and justifying the use of given distributions
- calculating probabilities of events combined by unions and intersections using appropriate formulae

- calculation of P(A / B)
- calculating the expected value and variance of a linear combination of two independent random variables
- applying the formula for the binominal distribution
- using the Poisson distribution as an approximation to the binomial distribution.

Area weakness/errors exhibited by the candidates were as follows:-

- ability to convert from worked to symbols
- ability to simplify $c \wedge \sim c$
- omission of the non-negativity constraints in the linear programming problem
- calculation of the latest start time and hence the critical path
- ability to distinguish between the inverse, converse and contrapositive
- ability to calculate the mean for a given distribution.
- ability to calculate the number of degrees of freedom for a goodness-of-fit using a Poisson distribution of unknown mean
- justifying the position distribution as an approximation to the binominal distribution
- interpretation of the notation $E(Y^2)$ as $[E(Y)]^2$

Internal Assessment

Generally the topics chosen were suitable for candidates at this level. Again this year, most of the projects submitted by candidates were from the Discrete Mathematics Section. These projects were appropriate and were given adequate treatment by candidates. There was an understanding of what was required in the assessment but a number of candidates who tried to incorporate Mathematics from two or more modules including Module 1 experienced difficulty in doing so. It appeared that the hands-on approach in which candidates were afforded the opportunity to apply their mathematics in real life situations served them in good stead. Candidates appeared to have developed an overall appreciation of the mathematical concepts used in their projects and many used them with confidence. A few candidates were even able to go above and beyond what was expected of them.

Teachers' marking of the projects showed that they understood the criteria used and are well aware of the type of assessment used. There was generally close agreement between the marks awarded by the teachers and those by the CXC moderator.

Paper 01 Option C
SECTION A (Module 1: Discrete Mathematics)
Same for Options A and B

Question 1

This question tested the candidates' ability to

(a) formulate

(i) simple propositions;
(ii) the negative of simple propositions;
(iii) compound propositions in symbols or in words;
(iv) compound propositions that involve conjunctions, disjunctions and negatives;
(v) conditional and bi-conditional propositions

(b) Use the laws of Boolean algebra (distributive, commutative, associative and the de Morgan's) to simplify Boolean expressions.

This question was fairly well done. Candidates were able to convert from logic symbols to words, use the distributive law and generally the order of propositions was followed. The difficulties manifested were in converting from words to symbols and in simplifying c ^ ~c as well as 0 V(c ^ d).

In Part (ii) some candidates omitted the use of brackets, while others placed them in the incorrect position.

Answers

1.

(a) (i) a) If there are clouds in the sky, then it is raining OR There are clouds in the sky, so it is raining. (ii) $q \Rightarrow (r \vee \sim p)$

(b) $c \wedge (\sim c \vee d)$

$= (c \wedge \sim c) \vee (c \wedge d)$

$= o \vee (c \wedge d)$

$= c \wedge d$

Question 2

This question tested the candidates' ability to

(i) identify linear programming problems;
(ii) identify the objective function of a linear programming problem;
(iii) use the concept of slack variables and of basic and non-basic variables.

This question was reasonably well done. In many cases, the objective, in this case maximise was omitted. A few candidates did not completely write the objective function, (C =3x + 2y), while some others used the letter P instead of C. There were some cases where it was evident that the candidates were unclear as to what is an objective function and what is the constraint. Most candidates were able to interpret the table and convert it to a linear programming problem. The non-negativity constraints were generally omitted when setting up the constraints.

Answers

2.

Maximise $C = 3x + 2y$

Subject to
$x + y \leq 6$
$2x + 3y \leq 14$
$x \geq 0,\ y \geq 0$

Question 3

This question tested the candidates' ability to establish the truth value of:

(i) compound propositions that involve conjunctions, disjunctions and negations;
(ii) represent a Boolean expression by a switching of logic circuit;

This question was generally well done. Candidates were able to correctly construct truth tables for a ∧ b and a v b. Most candidates were able to identify the correct proposition for the burglar alarm system and the computer system as well as to give the valid explanation.

Answers

3.

(a)	a	b	a v b	a ^ b	OR	a	b	a v b	a ^ b
	T	T	T	T		1	1	1	1
	T	F	T	F		1	0	1	0
	F	T	T	F		0	1	1	0
	F	F	F	F		0	0	0	0

(b) (i) a v b, since the alarm would sound if at least one of the switches closed.

(ii) a ∧ b, since access could only be obtained if both switches are closed.

Question 4

This question tested the candidate's ability to

(i) calculate the earliest and latest start times;
(ii) identify the critical path in a simple activity network.

This question was satisfactorily done. Candidates were able to calculate the earliest start time (EST), however the latest start time (LST) seemed to have posed problems for a large number of them. Consequently, they had difficulty obtaining the critical path. Even in those instances where EST and LST were correctly calculated, it was unclear whether the candidates knew how to obtain a critical path as they were unable to identify the correct critical path.

Answers

4.

(a)	Activity	Earliest Start Time (No. of Days)	Latest Start Time (No. of Days)
	A	0	0
	B	8	8
	C	8	10
	D	19	19
	E	14	14
	F	20	20

(b) Critical path: ABEDF

Question 5

This question tested the candidate's ability to explain and use the terms, trails and paths.

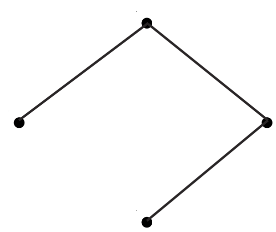
This question was fairly well done. In Part (a) (i) many candidates were able to identify that there was a repeat of an edge and a vertex in the case of the trail and the path respectively, but they did not identify the edge or the vertex that was repeated.

In Part (a) (ii) all candidates were able to state the degree of the vertices from the given diagram. In Part (b) many candidates were unable to draw a graph with the required number of degrees. Some candidates sketched two different graphs to display the information.

Answers

5.

(a) (i) a)	An edge is contained more than once in the sequence: BE						
b)	A vertex is included more than once in the sequence: E						
(ii)		Vertex	A	B	C	D	E
		Degree	2	3	3	2	4
(b)	Two examples of possible graphs:						



Paper 01 Option C
SECTION B (Module 2: Probability and Distributions)
Same for Option A

Question 6

This question tested the candidates' ability to model practical situations in which the binomial geometric and Poisson distribution are suitable.

This question was generally well done. The majority of candidates were familiar with the names of the distributions, and with the exception of the Poisson distribution, were able to state the parameters correctly.

Answers

(a) A is a binomial distribution n = 20 and p = 0.73
(b) B is a geometric distribution with p = 0.005
(c) C is a Poisson distribution with $\lambda = 2$

Question 7

This question tested the candidates' ability to:

(i) calculate probabilities of events combined by unions and intersections using appropriate formulae;
(ii) calculate $P(A|B)$
(iii) use the formula for Var(X) where X follows a geometric distribution.

In Part (a), candidates were able to use a correct formula to obtain $P(A \cap B)$ and then use this result to obtain the required value for $P(A \cap B)$.
In Part (b), candidates were generally able to state $q/p^{2-} = 5/16$ but surprisingly, many then incorrectly wrote q = 5 and p^2=16. The invalidityof p = - 4, was stated by the majority of them.

Answers
(a) 2/3
(b) 4/5

Question 8

This question tested the candidates' ability to calculate

(i) $E(Y^2)$ given E(Y) and Var (Y);
(ii) the expected value and variance of a linear combination of two independent random variables;

Most candidates were able to find

E(X) given $E(X^2)$ and Var (X) as well

and E(5X – 3Y) given E(X) and E(Y).

However the calculation of Var (2X – Y) posed greater problems to candidates who incorrectly wrote

$Var(2X - Y) = 4Var(X) - Var(Y)$ or $2 Var(X) - Var(Y)$ or $E(2X^2 - Y^2) - E(2X - Y)^2$

Answers

(a) 1.84
(b) -2.2
(c) 10
(d) 1.2

Question 9

This question tested the candidates' ability to

(i) model the practical situation by the binomial distribution;
(ii) apply the formula for the binomial distribution;
(iii) justify and use the Poisson distribution as an approximation to the binomial distribution.

This question was generally well done by the candidates. A few candidates neglected to put a negative sign in the probability distribution table (i.e. -2) or they found the variance instead of the standard deviation.

Answers

(a) 0.273
(b) 0.268
(c) 0.751

Question 10

This question tested the candidates' ability to identify and use the geometric distribution.

This question was not well done. One candidate did not respond. Candidates incorrectly identified the distribution as a binomial or Poisson distribution. Of the 2 candidates who correctly identifying the distribution as geometric, one incorrectly stated the formula for $P(X=2)$ as q^2p.

Answer

62.88

Paper 01 Option C
SECTION C (Module 3: Particle Mechanics)
Same for Module 2 Option B

Question 11

This question tested the candidates' ability to apply the principle of linear momentum to two particles moving in a straight line.

This was a popular question attempted by a majority of candidates. They were able to use the concept of conversation of momentum, but in many cases experienced difficulty taking the direction of motion of body into account.
A common error was – Total momentum = (4 * 9) + (5*5),
instead of (4*9) – (5*5)

Stating the direction of the combined mass after impact was often neglected.

Answers

11/9 the bodies move in the direction of the 4 kg mass after collision.

Question 12

This question tested the candidates' ability to apply Newton's Laws of motion to a particle moving on an inclined plane with constant acceleration.

There appeared to be a great reliance on memory, rather than an understanding of resolving a force in a particular direction. For example, the presence of the force P was often neglected, and the normal section R, equated to 1200g Cos 30^{o}. Only the high scoring candidates were able to solve the problem.

Answers

7820 N

Question 13

This question tested the candidates' ability to find

(i) use a velocity-time graph to calculate displacements;
(ii) apply the appropriate equation of motion for constant acceleration in a straight line to calculate the time taken.

Few candidates used the diagram to solve the problem, but the majority proceeded to use the equations of motion for a body moving with constant acceleration. This approach led to failure in Part (c), when they assumed that the acceleration was constant for the first 16 m.

Answers

$10\ \frac{2}{3}$ s

Question 14

This question tested the candidates' ability to find

(i) the constant acceleration
(ii) the tension in the wire, for a body of given mass, connected by a weightless wire and moving vertically upwards from rest.

Candidates were able to use correctly the equation of motion to determine the acceleration. Using this value for acceleration, they then applied Newton's Law to find the tension in the wire, during acceleration and during the deceleration periods.

Answers

(a) (i) 2.4 ms^{-2} (ii) 61000 N (b) 37000 N

Question 15

This question tested the candidates' ability to use the equations of motion of a projectile.

Given the initial velocity u, and angle of projection Ü candidates were required to

(a) show that the horizontal range is $\frac{u^2 \sin 2d}{g}$

(b) calculate the angle of projection for which the range is a maximum
(c) determine the time of flight.

This question proved to be difficult, as candidates seemed to know the results but were unable to prove them or apply them to solve the problem.

Answers

(b) 45^0
A. 57.7 s

Paper 02 Option C
SECTION A (Module 1: Discrete Mathematics)
Same for Options A and B

Question 1

This question tested the candidates' ability to

(a) formulate

(i) the negative of simple propositions;
(ii) compound propositions in symbols or in words;
(iii) compound propositions that involve conjunctions, disjunctions and negatives;

(b) establish the truth value of a compound proposition that involves conjunctions, disjunctions and negatives.

(c) state the converse, contrapositive and inverse of implications of propositions.

(d) use truth tables to

(i) determine whether a proposition is a tautology or a contradiction;
(ii) establish the truth value of the converse, contrapositive and inverse of implications of propositions;
(iii) determine equivalent propositions.

(e) use the activity network algorithm in drawing a network diagram to model a real world problem.

Generally this question was well done. In Part (a) many candidates were unable to correctly identify the propositions using the given terms inverse, converse and contrapositive.

In Part (b) most candidates were able to construct correctly the truth tables for $x \Rightarrow y$ and $\sim x \Rightarrow \sim y$. Candidates were able to identify the statements $x \Rightarrow y$ and $\sim x \Rightarrow \sim y$ as logically equivalent, but a few of them had problems clearly communicating the answer.

In Part (c), some candidates introduced unnecessary additional symbols to express the given proposition. Having constructed the truth table for this proposition it became evident that some candidates did not understand the significance of getting all zeros in the answer.

In Part (d), no candidate displayed the activity network algorithm to construct the activity Network. Nevertheless many candidates were still able to correctly construct the activity network, while a few candidates incorrectly placed the duration on the network. There were those who did not include the direction by using arrows and did not give the correct precedents to D.

Answers

1.

(a) (i) Inverse

(ii) Converse

(iii) Contrapositive

(b) (i)

X	Y	~X	~Y	X ⇒ Y	~Y ⇒ ~X
T	T	F	F	T	T
T	F	F	T	F	F
F	F	T	F	T	T
F	F	T	T	T	T

OR

X	Y	~X	~Y	X ⇒ Y	~Y ⇒ ~X
1	1	0	0	1	1
1	0	0	1	0	0
0	1	1	0	1	1
0	0	1	1	1	1

(iii) x ⇒ y and ~y ⇒ ~x are logically equivalent, since the corresponding terms in their truth tables are the same.

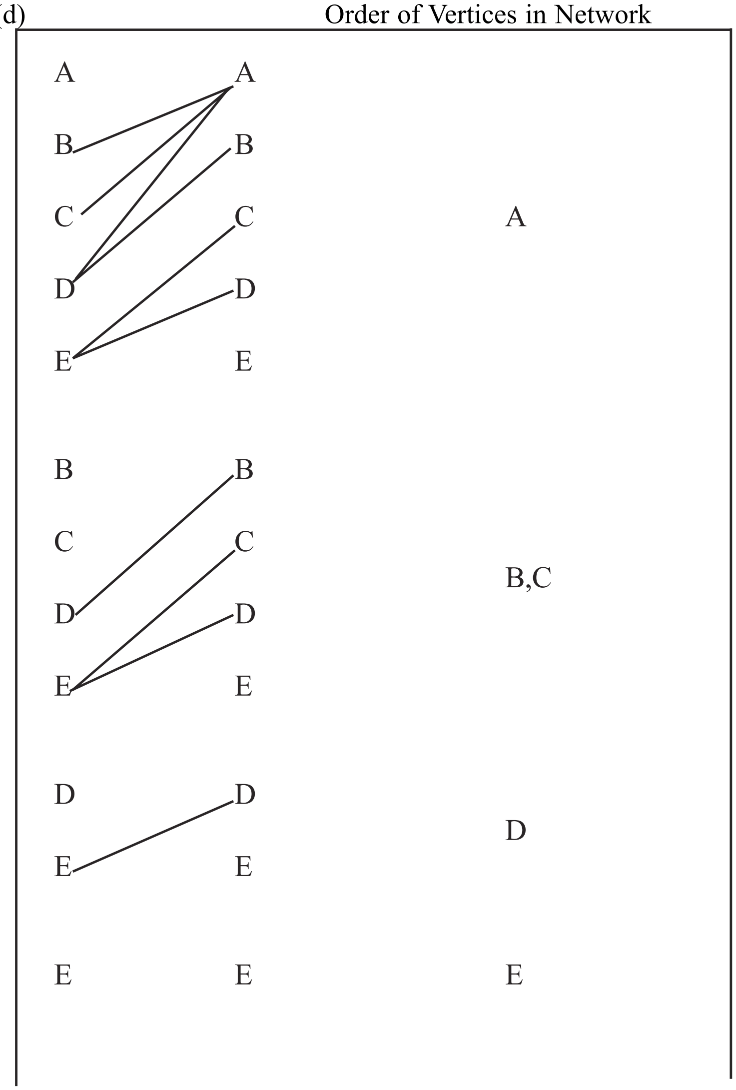
(c) (ii)

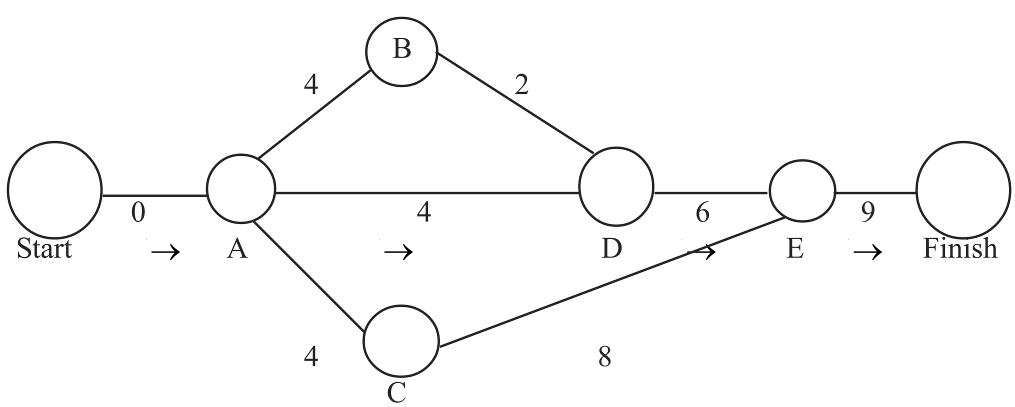
P	~P	P^ ~P	OR	P	~P	P^ ~p
T	F	F		1	0	0

(i) P∧ ~P

Since all the terms in its truth table are F10, ‘To have your cake and eat it’, that is, p∧ ~p is a contradiction.

(d) Order of Vertices in Network





Question 2

This question listed the candidates' ability to

(i) identify and graph linear inequalities in two variables;
(ii) determine the solutions set that satisfies a set of linear inequalities in two variables;
(iii) determine the feasible region of a linear programming problem;
(iv) formulate linear programming model in two variables from real world data.

Generally this question was fairly well done. In Part (a), the answers given were not in a structured format, that is

Minimize … write the objective function here
Subject to … give the constraints here

The objective function and constraints were mixed up, hence variables were not clearly and completely defined. Some constraints were not represented as an inequality but rather as an equality. Non-negativity constraints were generally omitted. Candidates need to exercise caution when simplifying inequalities. Some candidates did not seem to understand the concepts of "at least" and "at most".

Part (b) was generally well done. Lines were labeled accurately, but some candidates were unable to determine the feasible region correctly. Those who obtained the incorrect feasible region used $x + 2y \leq 7$ instead of $x + 2y \geq 7$. All candidates were able to correctly draw the line $x + y = 5$, however few candidates were able to correctly obtain A (the maximum) and B (the minimum) using the line $x + y = 5$. In several instances, the A and B identified were not in the feasible region. Both methods of determining $60x + 24y \geq 1200$ optimal points need to be taught, that is by considering intersection of parallel lines with the vertices as well as considering the value of the objective function at the vertices of the feasible region.

Answer

2.

(a)

x: no. of large trucks used
y: no. of small trucks used

Minimize $240x + 100y$

Subject to

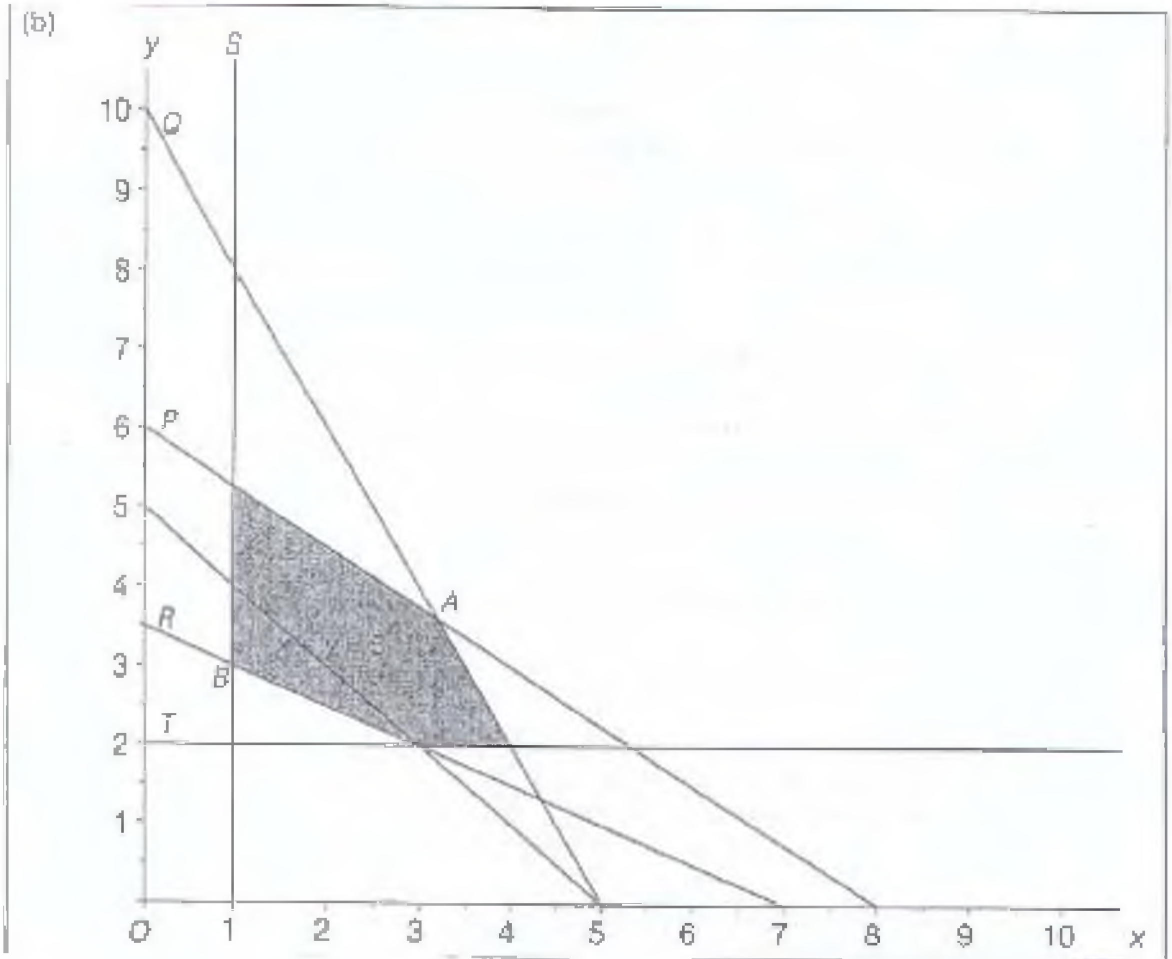
$x + y \leq 50$

$x \leq 25$

$y \geq 15$

$x \geq 0, y \geq 0$

x, y integers


(b)
y
S
Q
P
R
A
B
T
10
9
8
7
6
5
4
3
2
1
O
1
2
3
4
5
6
7
8
9
10
x

Paper 02 Option C
SECTION B (Module 2: Probability and Distributions)
Same for Option A

Question 3

This question tested the candidates' ability to;

(i) use a cumulative distribution function to solve problems involving probabilities;
(ii) calculate probability, expected value, and probability density function for a cumulative distribution function;
(iii) apply the properties of probability density function to determine the value of the constant;
(iv) calculate probability, for a probability distribution function.

This question was not as well done as expected. In part (a) (i) only two students were able to state the value of P(X = ½) correctly. Instead candidates integrated between the limits ½ and 1. For Part (ii), candidates seemed unfamiliar with F(x) and so replaced it with f(x) and then proceeded to integrate between limits ½ and ¾.

In Part (iii), many candidates correctly differentiated F(x) to get f(x), but were unable to obtain a full score because they did not define the probability density function over the entire range, that is for $x < 0$ and $x \geq 0$.

$\int_1^{y_n}$

Candidates generally performed better in Part (b) than Part (a). They were able to show that $k = \frac{4}{15}$, and many of them were able to find the expected value of Y, however some of those who were unable to find E(Y) defined it as $\sum y$ f(y) and did not complete the solution, while a few recovered.

In Part (c) the concept of $P(Y \leq y_n)$ as $f(y)dy = \frac{n}{100}$ was not well demonstrated. Many candidates did not include the lower limit, but these correctly wrote the value of the probability as 0.3.

Answers

(a) (i) 0 (ii) 0.28125 (iii) $x + \frac{1}{2}$ $\quad 0 \leq x \leq 1$
0 otherwise

(b) (ii) 1.65 (iii) 1.53

Question 4

This question tested the candidates' ability to:

(i) calculate the number of ordered arrangements of n objects taken r at a time without restrictions;
(ii) calculate the number of selections of n objects taken r at a time with restrictions;
(iii) calculate the probability of an event A in a possibility space S using independent events and the formula $P(A) = \frac{n(A)}{n(S)}$ where n(A) and n(S) are obtained from appropriate counting techniques.

Some good responses were obtained in this question. Some candidates gave a number for the answer with no intermediate steps and in cases where these values were incorrect, the candidate ended up losing all of the marks. Candidates need to be encouraged to display all steps in their working to gain full marks.

In Part (a) (i), some candidates interpreted the 12 students like 12 objects of which 7 were of one type and 5 of another type and so gave their answer as
12!/ (7!5!) instead of 12!

Many candidates were unable to deal with the various restrictions

In Part (b), there were a few candidates who calculated the probability using 'with replacement' rather than 'without replacement'.

Answers

(a)
(i) 479001600
(ii) 39916800
(iii) 3628800
(iv) 7257600
(v) 224985600

(b) 35/66

Paper 02 Option C
SECTION C (Module 3: Particle Mechanics)
same for Module 2 Option B

Question 5

This question tested the candidates' ability to

(i) apply Newton's Laws of motion to a system of two particles, connected by a light inextensible string passing over a smooth fixed light pulley moving on an inclined plane with constant acceleration;
(ii) use the work-energy theorem to solve problems.

The question was attempted by a majority of candidates all of whom attempted to apply Newton's Law. They however failed to take into account the component of the weight along the plane, and too often the equation T = 12a was seen, instead of T - 12 g sin α = 12a.

(b) Given the mass of a vehicle, and its distance traveled in accelerating from a given speed to a greater speed, candidates were required to determine the average tractive force of the vehicle.

Few candidates applied the principle of 'work done = change in Kinetic Energy".
The majority used the equation of motion $v^2 = u^2 + 2as$ to calculate the acceleration, and then applied: Force = mass x acceleration.

Answers (a) (i) 2.94 ms^{-2} (ii) 54.9 N (iii) 116 N

(b) 805 N

Question 6

The question tested the candidates' ability to:

(i) apply $a = v\dfrac{dv}{dx}$
(ii) using Newton's Laws of motion to a constant mass moving in a straight line
(iii) formulate a first order differential equation to model the linear motion of the mass when the applied force is proportioned to its velocity.

The candidates were given the mass of a body which was moving forward with velocity v ms^{-1} against a variable force of resistance proportional to v^2, producing a forward constant thrust of given magnitude. It was required to (a) Sketch a diagram to show the forces acting on the system and the direction of the acceleration.

(b) Show that the motion of the body may be modeled by the differential equation $ds/dv = \dfrac{14v}{36 - v^2}$ given that the body starts from rest and that $a = 0$ ms^{-2} when $v = 6ms^{-1}$.

Part (a) was well done by candidates, who drew correct diagrams showing clearly the forces and direction of motion.

(b) Few candidates were able to solve this problem, as it appeared that they were not familiar with expressing acceleration in the form $v\,dv/ds$.

Paper 01 Option B
SECTION C (Module 3: Rigid Bodies)

Question 11

This question required candidates to calculate the elastic potential energy in a spring, given its natural length, stretched length, and modulus of elasticity. This question was well done, and did not present any difficulty as candidates were familiar with, and able to apply the necessary formula to obtain the result.

In the second part of the question, it was required to find the velocity of a particle which was attached to the spring, and released from rest to pass through a given point.

The principle of conservation of energy was applied, and the velocity calculated.

Answer

(a) 2.43 J (b) 1.27 ms^{-1}.

Question 12

Candidates were required to consider the motion of a particle attached to one end of a light inelastic string, and moving in a horizontal circle, while a second particle was hanging freely from the other end of the string which passed through a fixed ring

Difficulty was experienced by candidates in considering the circular motion of the particle. They appeared to be unfamiliar with the direction and magnitude of the acceleration for motion in a circle. This resulted in a poor performance on this question.

Answer

3.96 ms^{-1}

Question 13

Candidates were required to determine the ratio of the radius to the height, of a regular cylinder, attached to a hemisphere of equal radius, so that the centre of mass of the solid formed is on the plane of the intersection of two figures.

This was not a popular question. Candidates did not attempt to take moments at any stage. If the centre of mass of the solid is at G, then equating the moments about G to 0, provides the equation from which is determined that the ratio is $1 : \sqrt{3}$

Question 14

This question described a uniform ladder with one end resting on smooth horizontal ground, and the other resting against a rough vertical wall. The ladder is kept in equilibrium by a force P applied to the end of the ladder on the ground. Candidates were required to (a) draw a diagram showing the forces acting on the ladder (b) show that the least magnitude of P required to prevent the ladder from slipping down the wall is equal to a given expression.

Part (a) was well done, showing forces clearly. In Part (b) majority of candidates were able to resolve in horizontal and vertical directions correctly. However, they experienced difficulty in attempting to take moments.

Question 15

In this question, it was stated that the depth of water in a harbour may be modelled by simple harmonic motion with equation $d^2x/dt^2 = -\omega^2 x$ where x, is the depth of water at time t.

Given the depth of water at low tide and at high tide, and the times of occurrence of these on a particular day, candidates were asked to -

a. state the value of the amplitude a.
b. state the value of the period, and hence determine the value of ω
c. write an equation expressing x in terms of t, if $x = a$, when $t = 0$

This was not a popular choice. Candidates clearly associated SHM with vibrating springs or swinging pendulums, and not with tides.

Answers

(a) 3 (b) Period = 12.5 hrs, $\omega = 4\pi/25$ (c) $x=3 \cos 4\pi/25\, t$

Paper 02 Option B
SECTION C (Module 3: Rigid Bodies)

Question 5

Given a particle suspended from a fixed point by light elastic string of given modules of elasticity, candidates were required to investigate the motion, showing that while the string is taut the particle will describe SHM.

This was not a popular question. Candidates experienced difficulty from the outset, as they were unable to determine a starting point from which to work towards a solution.
For example: Let the particle be a distance x from O at time t. This would then allow them to apply Hooke's Law to obtain an expression for T, in terms of x and a.

At this point Newton's Law will be applied, and the required differential equation for SHM seen.

Answers

(b) $\sqrt{(2ga)}$ (e) $2\pi\sqrt{(a/4g)}$

Question 6

This question considered the motion of a body P from rest at the top of O of a smooth sphere, centre C. Candidates were required to use the energy equation to find an expression for the normal force acting on the body, and the value of the angle, OCP, at which the particle leaves the surface of the sphere.

Candidates were unable to obtain an expression $r\omega$, for the velocity of the particle moving with circular motion. They also experienced difficulty with the acceleration $r\omega^2$, towards the centre of the sphere. Resulting from this was their inability to obtain the energy equation and the normal equation of motion.

Answers

(c) 48.2^0

Paper 02 Option A
SECTION B (Module 3: Statistical Inference)

Question 5

This question tested the candidates' ability to:

(i) formulate null and alternative hypotheses;
(ii) carry out χ^2 goodnes-of-fit test for a Poisson distribution of unknown mean

This question was quite well done. Most candidates had difficulty calculating the mean of the data given. Problems were also experienced calculating the number of degrees of freedom for a goodness-of-fit using the Poisson distribution when the mean was unknown. These candidates used $n-1$ rather than $n-2$. Most of the candidates were able to calculate the expected frequencies, but care must be taken to ensure that the total of these equal to the total of the observed frequencies. Generally candidates were able to state the null and alternative hypotheses and calculate the test statistic.
Candidates correctly combined classes in the case where the expected frequency was less than 5.

Answers

(a) 1.7 (b) H_0:X~P_o(1.7) (c) 18.268,31.056,26.398,14.959,6.3575,2.9615

(d) (i) 3, (ii) 7.815 (e) 4.121 (f) accept H_0 and conclude that $X \sim P_o(1.7)$

Question 6

This question tested the candidates' ability to formulate hypotheses and test the null hypothesis for the population mean using a $p-$value approach when a sample is drawn from a normal distribution using a z test.

Almost all candidates were able to formulate null and alternative hypotheses in words and symbols and name the distribution and justify its use. Some candidates experienced difficulty in obtaining the correct p-value because of inaccurate calculation.

Answers

(a) H_0: $\mu \geq 5$ (b) since the sample size $n = 40 > 30$ is large by central limit theorem a normal distribution is used in the test and in calculation the p-value. (c) 0.0174 (d) reject H_0 if $p<0.05$
(b) reject H_0 and calculate that the mean weight loss is less than 5kg.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2007

APPLIED MATHEMATICS

APPLIED MATHEMATICS

MAY/JUNE 2007

INTRODUCTION

The revised Applied Mathematics syllabus was followed this year for the third time. Of the one hundred and fifty-nine candidates registered for the examination, one hundred and twenty-one wrote Option C, twenty–seven wrote Option B and eleven wrote Option A. One candidate wrote the Alternative to the SBA Paper in Option C.

This is a one-Unit course comprising three papers and three Options. However, a candidate is required to take only ONE Option. Papers 01 and 02 were examined externally, while Paper 03 was examined internally by the teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to the Unit were 40 per cent, 40 per cent and 20 per cent respectively.

The three options are Option A, Option B and Option C.

Option A consists of Discrete Mathematics, Probability and Distributions and Statistical Inference. Option B consists of Discrete Mathematics, Particle Mechanics and Rigid Bodies, Elasticity, Circular and Harmonic Motion. Option C consists of Discrete Mathematics, Probability and Distributions and Particle Mechanics.

GENERAL COMMENTS

Acceptable grades, Grades I to V, were obtained by 79 per cent of the candidates writing Option C, 33 per cent of the candidates writing Option B and 64 per cent of the candidates writing Option A. The standard of work seen from many candidates in this examination was above average. Again this year, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions and generally answered the questions well. Many candidates did not do well in the Mechanics module. Despite this fact, there were a number of candidates who appeared to be well prepared in all three of their modules. In general, there were a large number of areas of strength displayed by many candidates. Nevertheless, many candidates still need to pay more attention to their algebraic manipulation.

Areas of strength displayed by most candidates on this paper were as follows:

- conversion from words to logic symbols
- construction of truth tables
- calculation of the earliest start time
- identification of the number of degrees for a vertex
- formulation of the null and alternative hypotheses in symbols
- calculation of test statistics
- naming and justifying the use of given distributions
- calculating probabilities of events combined by unions and intersections using appropriate formulae
- calculating the expected value and variance of a linear combination of two independent random variables
- applying the formula for the binomial distribution
- use of the Poisson, geometric and normal distributions to solve problems.

The areas of the course which the candidates found difficult were as follows:

- inability to convert from words to symbols
- manipulation of the rows in the simplex method
- omission of the non-negativity constraints in the linear programming problem
- calculation of the latest start time and hence the critical path
- inability to construct an activity network algorithm
- inability to calculate the number of degrees of freedom for a goodness-of-fit using a Poisson of unknown mean
- inability to resolve forces correctly

Internal Assessment

Generally, the size of the samples submitted were adequate and marks were entered correctly onto the AMAT1-3 forms.

This year, the overall presentation and quality of samples submitted were satisfactory. Candidates chose topics which were relevant to the objectives of the syllabus and, in most cases, were suitable to their level. In fact, a few candidates excelled in their assignment, employing techniques beyond the level expected.

Projects equally incorporated Discrete Mathematics with Particle Mechanics and Discrete Mathematics with Statistics and Probability, while a small number of candidates did their project on only one of the 3 modules.

Samples were generally well done, with 71 per cent being adequately handled, 24 per cent were successfully accomplished, while the remaining 5 per cent were vague.

Not all candidates gave full descriptions of their plan for carrying out their tasks. 81per cent gave articulate descriptions of their investigation. The majority showed clear evidence of doing purposeful mathematics in relation to collecting of data. However, the integration and use of diagrams and illustrations could have been more effectively utilized.

The majority of candidates appeared to have a better understanding of the syllabus; they were successful at applying their mathematical knowledge appropriately and within context.

Few candidates included in their evaluation the insights into problems encountered in their investigation or what was done to resolve them. The majority of candidates also disregarded or perhaps did not understand what was meant by “suggest (2 or more) ideas for future study of the assignment’s topic and suggestions for improvement.”

Most candidates had little or no difficulty with grammar or structuring of their statements. However, some candidates lost marks for lack of recording their actions at each stage of the investigation, as well as during calculations.

Teachers’ assessments of assignments were generally appropriate since the majority of the marks were in agreement with that of the moderator. Overall, both teachers and candidates displayed a satisfactory understanding of the assessment criteria and the ranking of the candidates was reliable in most of the cases.

Paper 01 Option C
SECTION A (Module 1: Discrete Mathematics)
same for Options A and B

Question 1

This question tested the candidates' ability to:

(a) construct truth tables for given compound propositions that involve conjunctions, disjunctions and negatives

(b) use the result obtained from the truth table, in Part a, to simplify the given proposition.

This question was well done by most candidates. In Part (a) candidates were able to correctly construct the truth tables for $p \Rightarrow q$ and $\sim p \vee q$ and most of these candidates were able to correctly explain why these two propositions are equivalent.

In Part (b) candidates were able to use the truth table to simplify the given proposition. Some candidates used the result obtained in Part (a) along with the distributive law and the complement law to simplify the expression.

Answers

1 (a)

p	q	~p	$p \Rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

OR

p	q	~p	$p \Rightarrow q$	$\sim p \vee q$
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$p \Rightarrow q$ and $\sim p \vee q$ are logically equivalent since the corresponding terms in their truth tables are the same.

(b) $(p \vee q) \wedge (p \Rightarrow q) = (p \vee q) \wedge (\sim p \vee q)$
$= (p \wedge \sim p) \vee q$
$= 0 \vee q$
$= q$

OR

p	q	$p \vee q$	$p \Rightarrow q$	$(p \vee q) \wedge (p \Rightarrow q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

OR

p	q	$p \vee q$	$p \Rightarrow q$	$(p \vee q) \wedge (p \Rightarrow q)$
1	1	1	1	1
1	0	1	0	0
0	1	1	1	1
0	0	0	1	0

$(p \vee q) \wedge (p \Rightarrow q) = q$ since the corresponding terms in their truth tables are the same

Question 2

This question tested the candidates' ability to formulate compound propositions in symbols in terms of the given propositions **w, x, y** and **z** and the logical connectives $\wedge$, $\vee$ and ~.

This question was reasonably well done. Candidates were able to answers Parts (a) and (c) correctly. In Part (b) some candidates, incorrectly expressed the proposition as $\sim y \wedge w$. In a few instances, candidates used the incorrect symbol for the proposition.

Answers

(a) $z \wedge x$
(b) $\sim(y \wedge w)$
(c) $x \vee y$

Question 3

This question tested the candidates' ability to

(a) construct an activity network showing the cost associated with allocating three individuals *A, B* and *C* to three tasks *P, Q* and *R*
(b) identify which one of the vertices *P, Q* and *R* has the highest degree.

This question was well done, with candidates being able to represent individuals and tasks as vertices and costs as edges. It should be noted that very few candidates displayed the correct directions on the edges.

Part (b) was well done by all candidates who were able to identify the vertex with the highest degree.

Question 4

This question tested the candidates' ability to formulate a linear programming problem in two variables from real-world data.

The performance on this question was disappointing. Most candidates were unable to correctly obtain the objective and objective expression as well as the correct constraints. Many candidates gave the constraints as $\begin{array}{l} x+7y \leq 110 \\ 5x+y \leq 80 \end{array}$ instead of $\begin{array}{l} x+7y \geq 110 \\ 5x+y \geq 80 \end{array}$

Some candidates also used the equality symbol instead of the inequality symbol. Candidates correctly indicated that *x* and *y* are non-negative constraints, but did not indicate that they must be integers.

Answers

Minimize $x + y$
Subject to $x + 7y \geq 110$
$5x + y \geq 80$

$x, y, \geq 0$
$x, y,$ integers

Question 5

This question tested the candidates' ability to:

(a) to determine a Boolean expression for a given logic circuit
(b) identify which one of the given circuits is equivalent to an OR and a NOT gate.

This question was fairly well done. In Part (a) many candidates were able to identify the correct expression for circuits *A*, *B*, *C* and *D*. However, circuit *E* proved challenging to a number of candidates. Only a few of those who were able to write down the expression, simplified it correctly.
.
In Part (b) most candidates were able to identify the NOT gate, but few were able to identify the OR gate.

Answers

(a) A: $\sim r$
B: $\sim (r \vee s)$ OR $\sim r \wedge \sim s$
C: $r \vee s$
D: $\sim (\sim r \vee \sim s)$
$= r \wedge s$

E: $\sim (\sim r \vee \sim (r \vee s))$
$= r \wedge (r \vee s)$
$= r$

(b) (i) Circuit 2 functions as an OR gate
(ii) Circuit 1 functions as a NOT gate

Paper 01 Option C
SECTION B (Module 2: Probability and Distributions)
same for Option A

Question 6

This question tested the candidates' ability to calculate the number of ordered arrangements of n objects taken r at a time, with and without restrictions.

This question was generally fairly well done. In Part (a), many candidates gave the answer as 7! instead of $\frac{7!}{2!}$, while in Part (b), most candidates were able to fix the first and last positions, but encountered difficulty arranging the remaining letters. Errors seen were $3x\frac{4!}{2!}x2$ and $3x5!x2$

Answers

(a) 2520

(b) 360.

Question 7

This question tested the candidates' ability to:

(a) state TWO properties that must be satisfied by a discrete probability distribution,
(b) use the given probability distribution table to obtain the value of the constant *a*, as well as to determine probability.

In Part (a), a few candidates gave the properties for a binomial distribution, but most candidates indicated that the sum of the probability is one.
In Part (b) (i), all candidates were able to find the value of *a* correctly. The error made in

Part (b) (ii) was $P(2.2 \le X \le 4) = P(2 \le X \le 4)$

Answers

(a) $0 \le P(X = x) \le 1$ and $\sum P(X = x) = 1$

(b) (i) $a = 0.35$ (ii) 0.4

Question 8

This question tested the candidates' ability to apply the Poisson formula in solving problems.

This question was quite well done. Part (a) was well done by all candidates. Part (b) posed problems for a number of candidates who used $\lambda = 2$ instead of $\lambda = 4$.
Another error made in this question by candidates was to find $P(Y \ge 3) = 1 - P(X \le 3)$ using $\lambda = 2$

Answers

(a) 0.271
(b) 0.762

Question 9

This question tested the candidates' ability to:

(a) model a practical situation by the geometric distribution
(b) calculate the expected value and variance of the geometric distribution
(c) calculate probability using the geometric distribution.

This question was not done as well as expected. Part (a) was well done by a large number of the candidates, however there were a few candidates who identified the distribution as binomial, while one candidate identified it as a Poisson distribution.

In Part (b) almost all candidates were able to calculate E(*X*) and Var(*X*) correctly.

Part (c) posed difficult to a number of candidates who wrote P(*X* < 26) = 1- P(X > 26) rather than

P(*X* < 26) = 1- P(X > 25) = $1 - q^{25}$ where $q = \dfrac{29}{30}$

Answers

(a) Geometric distribution with $p = \frac{1}{30}$

(b) (i) E(X) = 30 and Var(X) = 870.

(ii) 0.572

Question 10

This question tested the candidates' ability to use the cumulative distribution function to

(a) find the value of a constant
(b) calculate probability
(c) determine the probability density function, f(x)
(d) calculate the expected value for a linear combination of variable X.

This question was well done by a number of candidates. However, the common error seen in Part (a) was the use of integration to find the constant a. For some candidates this error continued in Part (b).

In Part (c), all candidates recognised that they needed to differentiate in order to find the probability function f(x), however it was noted that some of these candidates did not indicate the value of f(x) when x< 1 and when x > 2.

Part (d) was well done by almost all candidates.

Answers

(a) $f(x) = \frac{3}{8}x^2 \quad 1 < x \leq 2$

$0 \quad$ otherwise

(b) 0.19225

Paper 01 Option C
SECTION C (Module 3: Particle Mechanics)
same for Module 2 Option B

Question 11

This question tested the candidates' ability to find the magnitude and direction of a third force Q, if R is the resultant of P and Q, given the magnitude and direction of two forces P and R.

This question was attempted by the large majority of candidates but was not well answered. They however interpreted the question as "a body in equilibrium under the action of forces P, Q, and R." Few candidates produced a triangle of forces showing R as the resultant of P and Q.

Answer

Q = 373 N; Direction - An angle of 19.57^{o} to the vertical.

Question 12

This question tested the candidates' ability to construct a velocity-time graph from data, and use it to determine the distance travelled, and the acceleration of a body.

Few candidates used the graph to find the time, given the acceleration. They did not relate the gradient to the acceleration. They however were more familiar with associating the area under the graph with distance traveled. The question was generally well answered.

Answer

Total time = $3\frac{1}{3}$ seconds.

Total distance = $6\frac{2}{3}$ metres.

Question 13

This question tested candidates' ability to:

(a) apply the principle of conservation of momentum to the direct impact of two inelastic bodies moving in a straight line.
(b) calculate the kinetic energy of a body.

This was a popular and well answered question. Candidates demonstrated their knowledge of the principle and it was evident that they had adequate practice in solving this type of problem.

Answer

(i) velocity before impact = 337.4 ms^{-1}
(ii) 11.8 J

Question 14

This question tested the candidates' ability to:

(a) find the tension and acceleration, given two particles connected by a light inextensible string passing over a smooth, weightless pulley
(b) find the distance travelled by one of the particles before coming to rest.

In attempting to solve this problem, many candidates failed to indicate the forces acting on the particles. This led, in many cases, to errors when they attempted to apply Newton's equation, Force = mass × acceleration. The net force was too often incorrect.

In part (b), candidates experienced difficulty as they tried to apply the principle of conservation of energy to determine the required height reached by the 8 kg mass.

Answer

(a) Acceleration = 1.96 ms^{-2}.
Tension = 94.08 N

(b) 0.24 m

Question 15

This question tested the candidates' ability to:

(a) express resistance in terms of speed,
(b) calculate the maximum speed, given the power developed by an engine, its speed and resistance.

This question presented little difficulty to candidates. They knew that at the maximum speed, the pull of the engine was equal to the resistance, and were familiar with the definition of power. They were able to derive the correct equations and solve the problem.

Answer :

(a) R = 30 v
(b) Maximum speed = 51.9ms^{-1} .

Paper 02 Option C
SECTION A (Module 1: Discrete Mathematics)
same for Options A and B

Question 1

This question tested the candidates' ability to use the Simplex method to solve a linear programming model in two variables.

This question was well done by some of the candidates, while the responses obtained from the remainder of the candidates were disappointing. Some of the latter candidates were only able to complete one tableau correctly, while others completed only two of them with some of the rows incorrectly calculated. In setting up the first tableau some candidates omitted to rewrite the objective function as $P - 5x - 2y = 0$. Those candidates entered 1 5 2... in the tableau rather than 1 -5 -2...

All candidates put in the two slack variables. Many arithmetic errors were seen in this question.

Answer

Operations

x	y	s_1	s_2	RHS
-5	-2	0	0	0
4	-1	1	0	4
4	3	0	1	36

Operations: $\frac{1}{4}R_2$

x	y	s_1	s_2	RHS	Operations
-5	-2	0	0	0	
1	$-\frac{1}{4}$	$\frac{1}{4}$	0	1	$R_1 + 5R_2$
4	3	0	1	36	$R_3 - 4R_2$

Operations

P	x	y	s_1	s_2	RHS
1	0	$-\frac{13}{4}$	$\frac{5}{4}$	0	5
0	1	$-\frac{1}{4}$	$\frac{1}{4}$	0	1
0	0	4	-1	1	32

$\frac{1}{4}R_3$

P	x	y	s_1	s_2	RHS
1	0	$-\frac{13}{4}$	$\frac{5}{4}$	0	5
0	1	$-\frac{1}{4}$	$\frac{1}{4}$	0	1
0	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	8

Operations

$R_1 + \frac{13}{4}R_3$

$R_1 + \frac{13}{4}R_3$

P	x	y	s_1	s_2	RHS
1	0	0	$\frac{7}{16}$	$\frac{13}{16}$	31
0	1	0	$\frac{13}{16}$	$\frac{1}{16}$	3
0	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	8

Operations

Valid termination of Simplex Method P= 31

Question 2

This question tested candidates' ability to:

(a) use the activity network algorithm to construct an activity network diagram in a real-world situation

(b) from a given activity network diagram

(i) calculate the earliest start time, latest start time and float time

(ii) identify the critical path

(iii) use the critical path in decision making

In Part (a), although the question clearly indicated that the network algorithm should be used, very few candidates did so. It was found that less than 3 per cent of the candidates used this method to answer the question. The result was that the activity network was poorly constructed by the majority of candidates. The major errors seen were

1. Activity C was positioned after activities F and G
2. Activities F and G were not obliquely opposite each other. The same occurred for activities B and E.
3. Activity D, though clearly the last activity, was found within the activity diagram.
4. The times between activities were incorrectly recorded.

In Part (b), the responses to this part of the question showed that candidates were more comfortable interpreting an activity network than constructing it.

Candidates were able to determine the earliest start time for the activities. A few had some difficulty in correctly stating the latest start time.

Candidates used a variety of diagrams to display the earliest, latest and float times, but the tabular form was the most popular. Though the float time in quite a few instances was incorrect, most knew that the difference of the two times was required.

Over 80 per cent of the candidates were able to quote the critical path correctly. Furthermore, over 60 per cent were able to determine the minimum completion time of the project. Very few candidates related it to the time associated with critical path and added one month to the time associated with the critical path.

Paper 02 Option C
SECTION B (Module 2: Probability and Distributions)
same for Option A

Question 3

This question tested candidates' ability to:

(a) (i) use the Normal Distribution X ~ N (μ, σ^2)

(ii) use $z = \dfrac{x-\mu}{\sigma}$

(iii) use the Probability Distribution tables

(b) (i) use the Binomial Distribution Y ~ B (n , p)

(ii) state the values of the parameters n and p

(c) (i) use the inverse of the formula $z = \dfrac{x-\mu}{\sigma}$

(ii) use algebra to simplify.

In Part (a) the majority of candidates were able to identify the distribution as Normal and correctly stated its parameters. These candidates were able to standardize correctly, but a few of them used the continuity correction factor in error.

In Part (b) the majority of the candidates were able to recall and use the binomial distribution formula. However, problems were encountered by the candidates in the substitution process. The minority did not identify the correct values for the parameters *n* and *p*. Overall the question was well done and the majority of the candidates obtained full marks.

In Part (c) some candidates were able to interpret the question correctly. The most common error was in the interpretation of $P\left(Z > \dfrac{167-a}{5}\right) = 0.825$. These candidates expressed the previous equation as follows:

$$1 - P\left(Z < \frac{167-a}{5}\right) = 0.825 \quad \Rightarrow \quad P\left(Z < \frac{167-a}{5}\right) = 1 - 0.825 = 0.175$$

They then encountered the problem of finding the z-value for 0.1750 in their standard normal tables. Candidates who expressed it this way failed to recognize that $P\left(Z > \dfrac{167-a}{5}\right) = 0.825$ means that the z-value for $\dfrac{167-a}{5}$ must be negative (because 0.825 is greater than 0.5) . Those who expressed it as $P\left(Z < \dfrac{a-167}{5}\right) = 0.825$ were able to obtain the correct answer. The candidates who avoided the error stated above obtained full marks.

Answers

(a) 0.497 (b) 0.167

(c) 172 cm to 3 sig. fig

Question 4

This question tested candidates ability to:

(a) use the probability function f(x) =P(X=x) where f is a simple polynomial or rational function
(b) calculate the expected values E(X)
(c) calculate the variances Var (X)
(d) calculate E (aX+bY), where X and Y are independent random variables.
(e) calculate Var(aX+bY) where X and Y are independent random variables.
(f) apply the properties f(x) ≥0 and $\int_{-\infty}^{+\infty} f(x)dx = 1$, where f is a probability density function (f will be restricted to simple polynomials)

In Part (a) the majority of candidates were able to calculate the exact value of a. They found the solution by integrating and had no difficulties in obtaining a final solution.

In Part (b) candidates did not recall the formula correctly and used other methods to arrive at a solution. These methods were not accurate and it was reflected in the overall performance. Approximately 60 per cent of the candidates found E (X) correctly, but a number of these same candidates experienced difficulty calculating Var (X).

In Part (c) most candidates correctly calculated E(Y) and Var(Y) . However, a few candidates incorrectly used the formula when calculating Var(Y) . These candidates subtracted E(Y) instead of E $(Y)^2$

In Part (d) some candidates failed to recall E(5) =5. These candidates gave E(5)=0 and so were not able to arrive at the correct answer.

In part (e) the formula was incorrectly used as candidates omitted to square the coefficients of the variables when calculating the variance.

Answer

(a) 1 (b) E(X)=3/4, Var(X)=3/80 (c) E(Y)=2, Var(X)=½ (d) 193/20 (e) 93/20

Paper 02 Option C
SECTION C (Module 3: Particle Mechanics)
same for Module 2 Option B

Question 5

This question tested candidates' ability to:

(a) model the motion of a projectile as a particle moving under constant gravitational force neglecting air resistance

(b) use the equations of motion of a projectile to determine the time of flight, the horizontal range, the greatest height reached and the Cartesian equation of the trajectory of a projectile.

Part (a) was attempted by 90per cent of the candidates. Most of them got the correct solution, but a few students used a variety of methods to find the greatest height rather than doing a simple subtraction (10 m – 2 m = 8 m).

Part (b) was poorly done by most of the same candidates. Some candidates who misinterpreted Part (a) and used 45^0 as the angle to attain maximum height, went on and get the same 45^0 for Part (b).

In Part (c), candidates were required to use the following equations: $x = ut\cos\theta$ and $x = ut + \frac{1}{2}at^2$ to find the height above the floor. The performance on this part of the question was poor. Common errors were: using the formula; $\frac{u\sin 2\theta}{g}$ instead of $\frac{u^2\sin^2\theta}{g}$ for greatest height.

Answers

(a) 8 m

(b) 44.1^0

(c) 6.76 m

Question 6

This question tested candidates' ability to:

(a) resolve forces

(b) use the principle that when a particle is in equilibrium, the vector sum of its forces is zero or the sum of the components in any direction is zero

(c) solve problems involving concurrent forces in equilibrium.

In part (a), most of the candidates produced diagrams, but some of them left out important details such as the directions of the tensions in the strings and the angles. A few candidates used Lami's theorem correctly at point Q. Most candidates used resolution of forces.

Part (b) (i) was well done by most of the candidates.

Part (c) was well done. M could be found by resolving at R or using applying Lami's theorem at point R

Answer

(b) (ii) 12.1 kg

Paper 02 Option A
SECTION B (Module 3: Statistical Inference)

Question 5

This question tested the candidates' ability to:

(a) justify the use of the z- test.
(b) formulate null and alternative hypotheses
(c) (i) define a Type I error
(ii) state the value of P(Type I error)
(d) state the decision rule
(e) Calculate the test statistic.

This question was well done by most candidates, who were able to justify the use of the z- test, formulate null and alternative hypotheses, define a Type I error, state the value of P(Type I error), state the decision rule but a few of the difficulty calculating the test statistic.

Answers

(a) In this test of the difference between two populations means, data are derived from random, independent samples taken from two normal distributions of known/equal variance.

(b) X_1 = scores of females candidates
X_2 = scores of male candidates
$H_0 = \mu_1 = \mu_2$
$H_1 = \mu_1 > \mu_{21}$

(c) (i) A Type I error is made when the null hypothesis is rejected when it is actually true.
(ii) P(Type I error) = 0.05 OR 5%

(d) Reject H_0 if z test > 1.645

(e)
$$z\text{ test } = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{190.3 - 185.2 - 0}{12.4\sqrt{\frac{1}{60} + \frac{1}{55}}}$$

$$\approx 2.203 \text{ to 3dp}$$

(f) Reject H_0 since 2.203 >1.645 and conclude that there is sufficient evidence at the 5 % level of significance that the score of females candidates are greater that the scores of male candidates in the CAPE exam.

Question 6

This question tested the candidates' ability to test the difference between two population means taken from normal distributions of known variances using a t- test.

This question was well done by almost all candidates, who were able to carry out the test correctly and obtain full marks. A few candidates gave the incorrect t value.

Answers

X: masses of packets of margarine (in grams)
H_0: $\mu = 250$
H_1: $\mu \neq 250$

$\Sigma x = 2004.4$
$\Sigma x^2 = 502\ 245.72$

$v = 8 - 1 = 7$
Reject H_0 if $|t_{test}| > 3.499$

$$\overline{x} = \frac{\Sigma x}{n}$$
$$= \frac{2004.4}{8}$$
$$= 250.55$$

$$= \frac{1}{n-1}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right]$$

$$= \frac{1}{7}\left[502245.72 - \frac{(2004.4)^2}{8}\right]$$

$$= \frac{43.3}{7} \approx 6.185714286$$

$$t_{test} = \frac{\bar{x} - \mu}{\frac{\hat{\sigma}}{\sqrt{n}}}$$

$$= \frac{250.55 - 250}{\sqrt{\frac{6.185714286}{8}}}$$

$$\approx 0.625 \text{ to 3sf}$$

since $|t_{test}| \not> 3.499$, accept H_0 and conclude that there is sufficient evidence at the 1 % level of significance that the machine produces 250g packets of margarine.

Paper 01 Option B
SECTION C (Module 3: Rigid Bodies)

Question 11

This question tested the candidates' ability to:

(a) draw a diagram showing clearly the forces acting on the rod
(b) find the inclination of the rod to the horizontal
(c) calculate the force exerted by the peg on the rod
(d) calculate the vertical component of the force exerted by the plane on the rod at its point of contact with the ground.

Given a uniform rod resting in equilibrium, with a point of the rod on a smooth peg, and one of its ends on a rough horizontal plane.

The candidates experienced difficulty inserting the forces correctly. For example, the direction of the force exerted on the rod by the smooth peg was drawn in the vertical direction, instead of perpendicular to the rod, by the majority of candidates.

Another weak area was that of "taking the moment of a force about a point". They were not taking the product of the force and the **perpendicular distance** of the force from the point.
This question was not well answered.

Answer

(a) (ii) 30 degrees.
(b) (i) 75.1 N (ii) 65 N

Question 12

This question tested the candidates' ability to calculate the period of oscillation of a particle moving with SHM , given its velocity and distance from the centre of oscillation in two specific cases.
This did not turn out to be a popular question. Those who attempted it seemed to be unfamiliar with the equation $v^2 = \omega^2 (a^2 - x^2)$, and the period of oscillation in terms of ω , i.e.
$T = \frac{2\pi}{\omega}$. This resulted in low scores for the candidates.

Answer T = 1.6 sec.

Question 13

This question tested the candidates' ability to calculate the distance of the centre of mass of the composite body from the base given a solid hemisphere surmounted by a solid cone of equal radius..

Candidates did not display any knowledge of a satisfactory method of solving this problem. The problem was easily solved by using "moment of composite body = sum of moments of hemi sphere and cone". Instead of this approach, candidates calculated the mean of the distances of C.G. of hemisphere and cone from the base. It was not a popular question.

Answer: 13 cm

Question 14

This question tested candidates' ability to use Hooke's law and apply Newton's law.

This proved to be an easy question for candidates, as they experienced little difficulty quoting Hooke's law and finding the correct resultant force to be used in applying Newton's law of motion. .

Answer : Acceleration $= 7.13 ms^{-2}$.

Question 15

This question tested candidates' ability to solve a problem in which a body was performing circular motion on a rough horizontal surface.

This proved to be one of the easier questions for the candidates. They showed an appreciation of the acceleration $r\omega^2$, towards the centre of the circle, and correctly stated that on the point of slipping, $F = \mu R$.

Answer :

(a) 0.9 8 (b) 1 N (c) 1.02

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2008

APPLIED MATHEMATICS
(TRINIDAD AND TOBAGO)

APPLIED MATHEMATICS

MAY/JUNE 2008

INTRODUCTION

The revised Applied Mathematics syllabus was followed this year for the first time. The revised Applied Mathematics syllabus is a two-Unit course comprising three papers. Papers 01, multiple choice items, and Paper 02, essay questions, were examined externally; while Paper 03 was examined internally by the teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each Unit were 30%, 50% and 20% respectively.

Unit 1: Statistical Analysis consists of Collecting and Describing Data, Managing Uncertainty and Analysing and Interpreting Data.

Unit 2: Mathematical Application consists of Discrete Mathematics, Probability and Distributions and Particle Mechanics.

GENERAL COMMENTS

In Unit 1 93 per cent of the candidates obtained Grade I-V. In Unit 2 approximately 95 per cent of the candidates obtained acceptable grades, Grades I – V, while nine per cent obtained Grade VI. The standard of work from most of the candidates in this examination was above average. Again this year, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions and generally answered the questions well.

However, the questions on particle Mechanics were not well answered. As in previous years, candidates need to pay more attention to their algebraic manipulation.

APPLIED MATHEMATICS

MAY/JUNE 2008

PAPER 02 - UNIT 1

Question 1

This question was attempted by the majority of the candidates. For the most part high scores were allocated to candidates.

Part (a)(i) of the question was well attempted with few errors. Most candidates were awarded three or two marks.

Part (a)(ii) of the question, stimulated candidates thoughts on the differences between cluster sampling and stratified random sampling. Most candidates had a fairly good understanding of stratified random sampling but far too many were not adequately familiar with cluster sampling.

Part (a)(iii) was well done.

In part (b), most candidates scored full marks in compositing the angles for the pie chart and were able to successfully construct the pie charts. Many, however, did not follow the instructions to use a radius of 4 cm.

The majority of the candidates scored high marks in part (c)(ii), where they were required to compute the standard deviation, however, a minority of them made mistakes in the arithmetic. Teachers are encouraged to give the students more practice in this area.

Question 2

This question was generally well done.

In part (a), many candidates left out the first row with the "0" cumulative frequency. A few candidates did not attempt do the cumulative frequency table.

In part (b), quite a few candidates plotted mid points rather than the upper class boundaries, so their graphs were translated to the left, hence they got incorrect values from the graph in (c). Also, many candidates did not close their cumulative frequency curves by plotting the "0" cumulative frequency. A few candidates used rulers to draw a polygon instead of a smooth curve.

In part (c)(i), since "35" was in between the small blocks, few candidates had problems reading off the values – they instead read the nearest integer values.

Part (c)(ii) was well done by most candidates. A few used "120", the upper class limit of the last call interval instead of using the sum of the frequencies to find the position of the quantities.

In part (c)(iii), some candidates incorrectly read the values from the graph and did not compute the position of the upper and lower quartiles correctly.

Part (d), most candidates' box and whisker plots had the correct shape. A few of them did not show the whiskers. Some started and ended their whiskers at the lower and upper quartiles respectively.

Part (e) was well done by most of the candidates. Some common mistakes were:

- Candidates divided by "5" instead of "Σf"
- Candidates used mid points instead of boundaries.

In part (f), the majority of candidates had some idea of what the answer should be. However, a few confused positively and negatively skewed.

<u>Question 3</u>

This question tested candidates' knowledge and application in the following topics:

- probability
- mutually exclusive events
- independent events
- mean and standard deviation.

This question was done well by the majority of the candidates.

All candidates who attempted question 3, did Part(a)(i) well.

In part (ii), most candidates scored at least 1 mark. Many candidates only stated the possible combinations of colours rather than all the possible arrangements.

In part (b), most candidates understood when two events are mutually exclusive and independent and had little problems with this section.

In part (c), the majority of candidates had a generally good understanding of the probability concepts and often times were able to use a venn diagram to help solve the problems on this part of the question.

In (d)(ii), most candidates have a basic understanding of expected value, but too many of them fell down in the determination of Var (x) and the standard deviation.

<u>Question 4</u>

In part (a), most candidates were able to state the three conditions that described a binomial distribution.

In part (b), most candidates were able to state which experiment may be modeled by a binomial distribution, with some not clearly stating a valid reason.

Parts (c)(i) and (ii) were done exceptionally well.

In part (d), most candidates answered this section well, with few candidates not being able to show the appropriate region. E.g. $P(-0.75 < Z < 1) = \Phi(0.75) - \Phi(1)$.

Question 5

Question (a)(i)a. was answered correctly by most candidates.

Part (a)(i)b. was generally not well answered due to use of incomplete formula – missing $\frac{n}{n-1}$ or $\frac{1}{n-1}$ and at times no $\sqrt{\ }$.

In part (a)(ii)a., most candidates used x^2 value rather than t-test. Most could identify the appropriate tail but a few forgot to apply the negative sign although they used the less than (<)sign.

Part (a)(ii)b. was done fairly well with the major error being the numerator, candidates used (45 – 43) instead of (43 – 45).

Part (a)(ii)c. was well done based on their critical region and computed value.

Part (b)(i) was fairly well done with the main error being 12/180 instead of 1/9.

In part (b)(ii), there was weakness in determining the correct parameter. Some wrote an English statement rather than using symbols as required by the question.

Part (b)(iii) was well done by most candidates.

In part (b)(iv), most candidates did not apply the continuity correction factor and use of incomplete formula.

In part (b)(v), the conclusion was well stated in most cases based on their critical region and computed value.

Question 6

Part (a) was well done by the majority of candidates.

In part (b), calculating the product moment correlation coefficient was well done. The interpretation of the correlation coefficients was not clearly stated. A possible interpretation is "There is a very strong degree of positive (linear) correlation between x only.

In part (c)(i), finding the regression line of y on x in the form $y = a + bx$, was well done, with the exception of a few rounding off errors.

In part (c)(ii), the drawing of the line was correctly drawn by many, but few did a "line of best fit".

In part (d)(i) and (ii) candidates did not interpret the regression coefficient, b, and the constant a, in respect to the context of the question.

Part (e)(i) was well done by the majority of candidates.

In part (e)(ii), the reliability of the value obtained was not clearly stated, since some candidates could not conclude that the value fell outside of the range.

UNIT 2

PAPER 02

General Comments

Question 1

This question was not well done by most candidates.

In part (a), many candidates did not use the term "maximize". Some left out the non-negativity constraints and did not state that "x and y are integers".

Part (b) was well done. Most of the candidates were able to identify the feasible region. A few interchanged the axes.

In part (c), many candidates did not write down the coordinates of the vertex and did not test all the vertices. Some candidates only tested the point they thought would yield the maximum value. Some candidates used the simplex method instead of using the graph. Quite a few candidates did not treat (0, 0) as a valid vertex in the feasible region.

Question 2

This question was reasonably well done.

Most candidates got part (a) correct. However, a few incorrectly stated "he not is tall and happy". The solution is "He is not tall and he is not happy" or "He is neither tall nor happy".

Part (b) was not well done. Quite a few candidates used truth tables, but did not identify the truth values of the statements.

The truth tables in part (c) were well done by most candidates. However, a few did not have a correct concept of a contradiction – they showed that they believed that if it is not a contradiction, then it must be a tautology.

In part (d)(i), some candidates confused switching and logic circuits. Some tried to use truth tables instead of drawing switching circuits. A few candidates confused parallel and series circuits.

In part (d)(ii), a few candidates used the "NOT' gate without the circle ("o"). A few candidates confused the "AND" and the "OR" gates.
Part (e) was very poorly done. A large number of candidates tried to use truth tables rather than use the laws of Boolean algebra as instructed. A few candidates confused complement and idempotent laws. Some did not use the distributive and deMorgan's laws appropriately. A possible solution is

$(A\wedge B) \vee (A\wedge \sim B) \vee (\sim A\wedge \sim B)$
$= [A\wedge (B\vee \sim B)] \vee (\sim A\wedge \sim B)$

$= (A\wedge 1) \vee (\sim A\wedge \sim B)$
$= (A\vee (\sim A\wedge \sim B)$
$= (A\vee \sim A)\wedge (A\vee \sim B)$
$= 1\wedge (A\vee \sim B)$
$= A\vee \sim B$

Question 3

In part (a), generally candidates were able to identify odd numbers and numbers greater than 500 000. Most of the errors came in determining repetition for the other 4 digits.

In part (b), most candidates correctly recognized this as a combination.

In part (b)(i), most candidates were able to write $^{13}C_4$.

Part (b)(ii) was well done by most candidates.

In part (b)(iii), although many candidates were able to identify 3 girls and 1 boy or 4 girls and 1 boy, many did not state both groups or they added the combinations rather than multiply.

For example the 3 girls and 1 boy was calculated as $6C_3 + 7C_1$ instead of $6C_3 \times 7C_1$. The correct answer was $6C_3 \times 7C_1 + 6C_4 \times 7C_0 = 155$.

In part (c), many candidates did not recognized that if A and B are independent

$P(A/B) = P(A)$

These candidates calculated P(B) and then used $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

Candidates recognized that since A and B are independent then
$P(A' \cap B') = P(A') - P(B')$

Few candidates used $P(A' \cap B') = 1 - P(A \cup B)$

In part (d), most candidates were able to identify that the correct solution and then gave $P(A \cap B) \neq 0$.

Some candidates however, confused the condition for independence with the condition for mutually exclusive.

Question 4

In part (a)(i), there were some candidates who attempted to integrate they were able to write

$\int_1^4 k\, x\, dx = 1$

however, none of these candidates integrated correctly.

All of the other candidates obtained the correct answer.

In part (a)(ii), again those candidates who attempted to integrate reached as far as stating

$\int_1^4 x\, f(x) = E(x)$.

Most other candidates correctly calculated $E(x) = \Sigma x\, P(X = x) = 3$.

Some candidates missed the final mark by incorrectly calculating $\frac{30}{10} = 3.5$ or 3.6.

In part (a)(iii), many candidates gave just 1 or 2 combinations. Those who got the 3 combinations correct proceeded to correctly finish the equation.

In part (a)(iv), few candidates used the table as a calculation for the values that Y can take.

Few candidates obtained all the values of Y giving values of 3, 4, 5, 6, 7 only. Several candidates appeared not to understand the concept of adding two independent variables. Calculating the probabilities for the values of Y is therefore a problem.

The solution is shown in the table below:

Y	2	3	4	5	6	7	8
P(Y=y)	1/100	1/25	1/10	1/5	1/4	6/25	4/25

In part (v), most candidates were able to write

$E(Y) = E(X_1) = E(X_2)$

but still had problems substituting values for $E(X_1)$ and $E(X_2)$.

Candidates did not understand what is meant by X_1 and X_2. In calculating Var (Y) some candidates calculated the $E(Y^2)$ from the table.

Few candidates used the property Var (Y) = 2 Var (X).

Part (a)(vi) E(3X + 2Y)

Though candidates were able to say E(3X + 2Y)
= 3 E(x) + 2 E(Y)

many did not complete the questions because they could not make the substitution for E(Y).

One candidate recognized

$E(Y) = 2\ E(X)$
$\therefore\ 3E(X) + 2\ E(Y)$
$= 3\ E(X) + 4\ E(X)$
$= 7\ E(X)$.

For Part (b) $P(X \geq 2)$, most candidates interpreted this correctly as $1 - P(X < 2)$
Few candidates wrote $P(X < 2) = P(X = 0) + P(X = 1) + P(X = 2)$

Most candidates were able to correctly apply the Poisson formula.

Question 5

The question tested candidates' ability to apply the principle of conservation of

(i) energy and
(ii) linear momentum

involving the collision of two bodies moving in a straight line.

The candidates correctly equated the momentum before and after impact, to obtain the velocity of the combined masses after the collision.

However, some incorrectly equated the kinetic energies before and after impact, and therefore obtained incorrect values for the velocity.

Part (b)(ii) presented difficulty to many candidates, who appeared not to know where to start. The required distance could have been obtained by equating the work done and the loss of kinetic energy.

The time required to bring the particle to rest may then be obtained by

(i) Applying the formula $F = ma$ to obtain acceleration a and
(ii) using $v = u - at$, when $v = 0$ to calculate t.

Question 6

This question tested knowledge of the motion of a projectile.

Candidates showed that they were familiar with this type of question and the performance was good.

In part (a), candidates quoted the expression for the greatest height and used it correctly. A small number of the candidates attempted to use the equation of the projectile, and then substitute $y = 0$.

This clearly showed a lack of understanding of "greatest height".

In part (b), expressions for the horizontal and vertical distances were correctly stated and the correct values obtained.

In part (b)(iii), there was a misprint of u cos t α for u cosπ5 α.

The majority of the candidates did not notice the difference and proceeded to find u cos α. Full marks were however awarded to all candidates attempting this part of the question.

Parts (iv) – (vi) were generally well done.

In part (v), candidates recognized that

$$u^2 = u^2 \sin^2\alpha + u^2 \cos^2\alpha, \text{ and}$$

used this to calculate u.

However, incorrect values of u sin α and u cos α, resulted in an incorrect value for u in a few cases.

In part (vi), the method, $\tan \alpha = \dfrac{u \sin \alpha}{u \cos \alpha}$, was recognized, and all candidates applied this to the selection.

Approximately 24 per cent of the candidates were awarded a score of less than 10 out of 25.

INTERNAL ASSESSMENT

UNIT 1

Again this year, the overall presentation and quality of the samples submitted were satisfactory. Generally, candidates chose topics that were suitable to their level, and were relevant to the objectives of the syllabus. There were a few candidates who employed techniques that went beyond the level expected.

Project Title:

Some project titles did not clearly indicate what the project was about. Candidates had problems with making the title relate to the project, for example, "internal versus external exams". There is need to have school teachers clarify this a bit more with candidates!

Purpose of Project:

Variables were not clearly defined. In some cases no variables were stated.

In cases where there were investigations to be done, the purpose of the project was not stated. There was a clear lack of responses to guide the project to a suitable conclusion.

Method of data Collection:

In many cases in a lot of cases, there was no sampling. Candidates used a fair amount of secondary data. Candidates must state how they arrived at the data in addition to stating the data.

Presentation of Data:

In general the presentation of data was well done. Use of more sophisticated A' level standard methods, like box and whisker plots, stem and leaf, plots is required.

Statistical Knowledge/Analysis of Data:

There was inappropriate use of concepts, for example, linear regression was attempted without calculation of the final equation of the line and the correlation coefficient was used with simplistic methods. In some cases calculations were done that were unrelated to the statement of task. For example, correlation coefficients were calculated without any justification.

Discussion of Finding/Conclusion:

This section was often ignored or candidates were not prepared enough to relate the results to the purpose of the project.

Communication of Information:

In only a minority of cases did candidates lose marks here. However, we would recommend that candidates familiarize themselves with the technical terms of statistics to produce a higher level of analysis and to state the conclusions more efficiently.

List of References:

In some cases candidates failed to state title, author, reference numbers or dates of publication.

UNIT 2

PAPER 02

MATHEMATICAL APPLICATIONS

INTERNAL ASSESSMENT

Generally the topics chosen were suitable for candidates at this level. Again this year, most of the projects submitted by candidates were from the Discrete Mathematics and Probability and Distribution sections. These projects were appropriate and were given adequate treatment by candidates. There was an understanding of what was required in the assessment but a number of candidates who tried to incorporate Mathematics from two or more modules including Module 1 experienced difficulty in doing so. It was noted that the majority of the topics chosen were based on the distribution of a product within small communities and "get rich quick" schemes.

It appeared that the hands-on approach in which candidates were afforded the opportunity to apply their Mathematics in real life situations served them in good stead. Candidates appeared to have developed an overall appreciation of the mathematical concepts used in their projects and many used them with confidence. A few candidates were even able to do more than what was expected of them.

Teachers' marking of the projects showed that they understood the criteria used and are well aware of the type of assessment used. There was generally close agreement between the marks awarded by the teachers and those by the CXC Moderator.

RECOMMENDATIONS

It is recommended that candidates practice more problems involving the use of the normal distribution and questions from the Mechanics module. Exercises involving the use of algebraic manipulation are strongly recommended. It is necessary for teachers to complete the entire syllabus so that candidates can answer all of the questions adequately.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2008

APPLIED MATHEMATICS (REGION EXCLUDING TRINIDAD AND TOBAGO)

APPLIED MATHEMATICS

MAY/JUNE 2008

INTRODUCTION

The revised Applied Mathematics syllabus was followed this year for the first time. The revised Applied Mathematics syllabus is a two-Unit course comprising three papers. Papers 01, multiple choice items, and Paper 02, essay questions, were examined externally; while Paper 03 was examined internally by the teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each Unit were 30%, 50% and 20% respectively.

Unit 1: Statistical Analysis consists of Collecting and Describing Data, Managing Uncertainty and Analysing and Interpreting Data.

Unit 2: Mathematical Application consists of Discrete Mathematics, Probability and Distributions and Particle Mechanics.

GENERAL COMMENTS

Approximately 91 per cent of the candidates registered for the Unit 2 Mathematical Applications obtained acceptable grades, Grades I – V, while nine per cent obtained Grade VI. The standard of work from most of the candidates in this examination was above average. Again this year, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions and generally answered the questions well. Very few candidates answered all of the questions in this section. There were a number of candidates who appeared to be well prepared in all three of their modules.

Particle Mechanics was not well answered. In general, there were a large number of areas of strength displayed by many candidates. As in previous years, candidates need to pay more attention to their algebraic manipulation.

UNIT 1

PAPER 02

STATISTICAL ANALYSIS

GENERAL COMMENTS

MODULE 1

Question 1

This question tested candidates' ability to:

- construct stem and leaf diagram, box and whisker plot';
- determine the mean, inter-quartile range, mean and 10% trimmed mean of a discrete data set;
- describe the shape of the distribution of data from a box and whisker plot'.

In Part (a), most candidates were able to construct the stem-and-leaf diagram successfully. However, many omitted a key which was essential for the interpretation of the data – stem of "2" and a leaf of '1' (2| 1) could just as easily represent '21' as '210 or '2, 1'.'.

In Part (b)(i), many candidates incorrectly determined the median of the data set of 30 values to be the 15th value, treating the data as being continuous. Of those candidates who correctly identified the median as being at the 15½th position, some were then unable to identify its value, the mean of the 15th and 16th values.

In Part (b)(ii), most candidates were not very clear on how to determine the values of the lower and upper quartiles, relying on the formulae 1/4 (n+1) and ¾ (n+1) which do no hold for all data. Candidates could have located the quartiles by first removing the median of the data set from consideration, then finding the respective medians of the lower and upper halves of the remaining values, at the 8th position of each half that consist of 15 values.

When the mean was calculated in Part (b) (iii), there were a few arithmetic errors in totaling the values.

There was some mis-conception of the correct procedure for finding the 10% trimmed mean in Part (b) (iv). Since 10% of 30 is 3 and hence the lowest 3 numbers (21, 22 and 23) and highest 3 numbers (51, 56 and 63) of the ranked data set are to be discarded, the mean of the resulting 24 values is 34.

In Part (c)(i), most candidates exhibited competence in drawing the box-and-whisker diagram, where the lowest value and highest value are represented by 'whiskers' and the lower quartile, median and upper quartile by a segmented 'box' relative to a linear scale.

In Part (c)(ii), most candidates were able to describe the shape of the distribution as being skewed, but some were unable to identify it as being positively skewed. In a positively-skewed distribution, it is as if the values were pulled towards the upper tail of the distribution, so that the whisker connected to the highest value (that is, on the 'positive' side) tended to be longer.

Question 2

This question tested candidates' ability to:

- distinguish among the sampling methods such as, simple random, stratified random, systematic random and quota;
- distinguish between a
  - (i) population and sample
  - (ii) census and sample survey
  - (iii) parameter and statistic.
- apply sampling methods.

This question was attempted by most candidates and was generally well done. There were, however, some areas of concern.

- Candidates, in general, did not relate the disadvantage in Part (a)(v) to the scenario given in the question.
- The majority of candidates ignored the instruction in Part (c) of the question to justify the use of the sample survey in the given example.
- Many candidates used the geographical definition rather than the statistical definition of a population.
- Many did not relate a parameter to a population.

MODULE 2

Question 3

This question tested candidates' ability to solve problems involving probability.

Several candidates were able to score full marks in this question. However, this question was generally not well done by the majority of the candidates.

Part (a)(i) was generally well done, but some candidates were unable to interpret the term "not".

In Part (a)(ii), the majority of candidates were able to interpret the term "or" successfully but used the wrong formula.

Part (a)(iii) was poorly done as many candidates misunderstood the meaning of the term "and".

In Part (a)(iv), the majority of candidates scored at least 2 marks. Many got the order of conditional probability correct in the formula, but showed that they reversed the order in the calculation in the survey.

Part (b)(i) was generally well done.

In Parts (b(ii) and (iii), many candidates did not consider, that sampling was done without replacement in the survey.

Part (b)(iii) was generally poorly done.

Question 4

This question tested candidates' ability to:

- solve problems involving probabilities of the normal distribution using z-scores;
- identify and use the binomial distribution as a model of data.

In Part (a), most candidates were knowledgeable of the standardizing procedures and were able to answer questions accurately. Only a few had difficulty in obtaining a value for $P(Z > -0.04)$, incorrectly stating that $P(Z > -0.04) = 1 - \Phi(.04)$.

In Part (b), very few candidates were able to calculate the value of k. Many were unable to set out solutions clearly in the form of an equation.

In Part (c)(i), most candidates were aware of the binomial approximation but few were unable to apply it correctly.

In Part (c)(ii), the majority of the candidates were able to score full marks.

MODULE 3

Question 5

This question tested candidates' ability to:

- calculate confidence interval for a proportion from a large sample ($n \geq 30$);
- formulate the null and alternative hypotheses;
- justify the use of a normal distribution approximation of a binomial proportion;
- state the critical regions for a given test;
- use an appropriate continuity correction to calculate the test statistic;
- state conclusions drawn from hypothesis testing.

In Part (a), the majority of candidates demonstrated very limited knowledge of sample proportions, many resorting to sample means. Many could not correctly identify the critical value of $z0.015$ and were unable to use the z-table approximately.

In Part (b)(i), majority of candidates were able to state the null and alternative hypothesis. However, some candidates used the sample mean instead of the proportion.

In Part (b)(ii), many candidates did not know the conditions necessary to convert from a discrete model (binomial) to a continuous model (normal distribution).

In Part (b)(iii), many candidates failed to calculate and to state the critical region for the test statistic.

In Part (b)(iv), this question was poorly answered by majority of the candidates. Candidates had little knowledge of calculating a test statistic and using the appropriate continuity correction.

In Part (b)(v), many candidates could not distinguish the acceptance and rejection regions, and often gave explanations that were not valid.

Question 6

This question tested candidates' ability to:

- plot a scatter diagram, calculate and interpret the product-moment correlations co-efficient r, for given data;
- calculate $(\bar{x}, \bar{y})$ and plot these co-ordinates on scatter diagram;
- derive the equation of the regression line y on x in the form $y = a + bx$ and plot this regression line on scatter diagram passing through $(\bar{x}, \bar{y})$;
- make estimation using appropriate regressions line.

Part (a) question was fairly well done, although some candidates did not use the appropriate scale given.

In Part (b)(i), many candidates scored full marks. Only a few substituted into the formulae incorrectly, thus obtaining an incorrect value for r.

In Part (b)(ii), the majority of candidates were able to give a valid interpretation for r.

Part (c)(i), was well done by candidates.

In Part (c)(ii), very few candidates did not indicate the point $(\bar{x}, \bar{y})$ on the graph.

In Part (d)(i), the majority of candidates were able to derive the equation of the regression line y on x in the form $y = a + bx$.

In Part (d)(ii), many candidates did not draw a valid line. They drew a line simply passing through $(\bar{x}, \bar{y})$.

In Part (e), most candidates read the required value from the graph instead of substituting into the equation obtained.

In Part (f), few candidates were able to give a good explanation for this question.

In Part (g), most candidates could not give a valid reason as to why they would use x on y.

UNIT 2

PAPER 02

MATHEMATICAL APPLICATIONS

GENERAL COMMENTS

MODULE 1

Question 1

This question tested candidates' ability to:

- formulate a linear programming model in two variables from real world data, stating its objective function and constraints;
- determine the feasible region for a given linear programming problem;
- determine a unique solution of a linear programming problems.

This question was well done by most candidates.

In Part (a), candidates were able to write down the required inequalities.

In Part (b), candidates were able to draw the graph to illustrate the feasible region and test the vertices of the region. However, a few of them did not use the correct scales on the graph.

Answers

1(a) n: no. of newspaper ads
t: no. of television ads

Maximise $2n + 5t$

Subject to $1500n + 5000t \leq 50\ 000$

$$n < 2t$$
$$\underline{1500n} \quad \leq \underline{30\ 000}$$
$$\underline{5000t} \geq \underline{25\ 000}$$
$$n \geq 0 \ (t \geq 0)$$
$$n,\ t \text{ integers}$$

At (2, 3): $P = 2x + 7y = 25$
(2, 2): $P = 18$
(2.4, 1): $P = 11.8$
(4, 1): $P = 15$

$P = 11.8$ when $x = 2.4$, $y = 1$

Question 2

This question tested candidates' ability to:

- use the Hungarian algorithm to determine the part of the race each runner should be assigned in order to minimize the total race time;
- use the activity network algorithm to construct an activity network diagram in a real-world situation;
- calculate from a given activity network diagram
  - the earliest start time and latest start time
  - the critical path
  - the latest finish time for a given activity.

This question was reasonably well done. In Part (a), most candidates knew how to reduce the rows, then the columns and did so successfully. After this, they were expected to draw straight lines vertically and horizontally to cover the 0's. Since there were only three such lines and four were required for this algorithm, the process could not be terminated. In fact, candidates were expected to identify the minimum of the uncovered elements (in this case 1), and then reduce all uncovered elements by this value, adding this value (1) to all elements intersected by two lines, leaving as is, all elements covered by one line. After this process, vertical and horizontal straight lines were drawn to cover the 0's in the matrix. As these lines turned out to be four, the Hungarian algorithm was terminated and the runners were assigned to the Part of the race that would minimum the total race time.

In Part (b), candidates were required to **first** construct an activity network algorithm and then use it to construct the activity network. However, a number of candidates tried to construct the activity network without first constructing the activity network algorithm. This caused them to lose the marks that were to be awarded for constructing the activity network algorithm. In addition, they drew the network with the order of the activities incorrect.

In Part (c), most candidates were able to get most of the marks for calculating the earliest and latest start times and hence the critical path and the latest finish time for activity E.

Answers

(a)

$$\begin{pmatrix} 48 & 46 & 50 & 44 \\ 49 & 45 & 46 & 49 \\ 47 & 46 & 48 & 44 \\ 51 & 48 & 47 & 45 \end{pmatrix} \qquad \begin{matrix} \text{Row Min.} \\ 44 \\ 45 \\ 44 \\ 45 \end{matrix}$$

$$\begin{pmatrix} 4 & 2 & 6 & 0 \\ 4 & 0 & 1 & 4 \\ 3 & 2 & 4 & 0 \\ & & & \end{pmatrix}$$

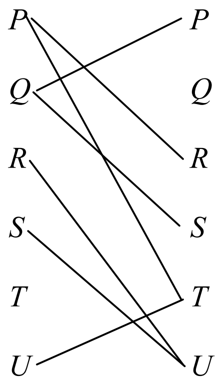
	6	3	2	0
Col. Min.	3	0	1	0

$$\begin{pmatrix} 1 & 2 & 5 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 2 & 3 & 0 \\ 3 & 3 & 1 & 0 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} 1 & 2 & 5 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 2 & 3 & 0 \\ 3 & 3 & 1 & 0 \end{pmatrix}$$

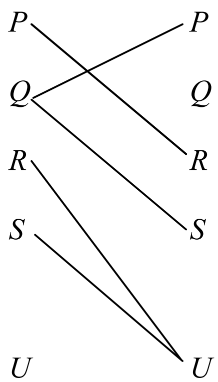
$$\begin{pmatrix} 0 & 1 & 4 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 2 & 3 & 1 \\ 2 & 2 & 0 & 0 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} 1 & 1 & 4 & 0 \\ 2 & 0 & 0 & 5 \\ 0 & 1 & 2 & 0 \\ 3 & 2 & 0 & 0 \end{pmatrix}$$

$W \leftrightarrow 4^{th}$ leg
$X \leftrightarrow 2^{nd}$ leg
$Y \leftrightarrow 1^{st}$ leg
$Z \leftrightarrow 3^{rd}$ leg

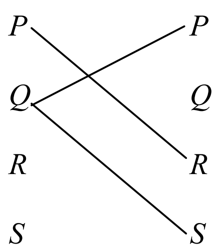
(b) Order of Vertices in Network



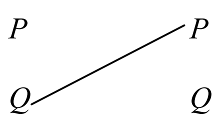
T



U



R, S



P

Q

(c) (i)

	Earliest Start Time	Latest Start Time
A	0	0
B	6	7
C	6	6
D	15	15
E	21	22
F	21	21
G	34	34

(ii) a) Start – A – C – D – F – G – Finish
b) 22 + 12 = 34

MODULE 2

Question 3

This question tested candidates' ability to:

- calculate expected frequencies for a binomial distribution of given parameters;
- perform a χ^2 goodness-of-fit test at the 5 per cent significance level to determine whether the given data may be modeled by a binomial distribution of given parameters.

Overall, this question was well done.

In Part (a), candidates were able to calculate the expected frequencies. However, a few candidates did not follow the instruction to use 1 decimal place in their calculations. Quite a few candidates calculated the final expected frequency, rather than adding the previous expected frequencies and subtracting from 100 (ensuring that the sum of the expected frequencies is 100).

In Part (b), a number of candidates did not state the hypotheses and did not state the conditions to reject the null hypothesis. Also, only a few candidates made a detailed and valid conclusion based on their results.

Answers

Expected values

48.2 38.6 11.6 1.5 0.1

Accepted H_0 at 5 per cent significance level and conclude that data may be modeled by a binomial distribution with parameters $n = 4$ and $p = \frac{1}{6}$

Question 4

This question tested candidates' ability to:

- obtain the value of the constant *a;*
- calculate $P(X < 3)$
- calculate the expected value $E(X)$ and variance $Var\ (X)$
- determine the cumulative distribution function $F(x) = P(X = x)$
- determine the median of X
- calculate $E(3X + 2)$ and $Var\ (3X + 2)$.

The performance on this question was very good. Most candidates recognized that the question involves integration since the distribution was continuous. However, a small number of candidates treated the distribution as a discrete distribution.

In Parts (a) to (c), most candidates integrated successfully and received full marks.

In Part (d), only one candidate got the full marks. Most candidates wrote $F(x) = \frac{x^2}{24} - \frac{1}{24}$,

$1 \leq x \leq 5$, which was the main Part of the solution. The full solution should have been:

$$F(x) = \begin{cases} 0 & , x \leq 1 \\ \frac{x^2}{24} - \frac{1}{24}, & 1 \leq x \leq 5. \\ 1 & , x \geq 5 \end{cases}$$

In Part (e), most candidates recognized that the median could be found by integrating the probability density function and setting the integral equal to ½. Other candidates correctly used $F(m) = ½$ to calculate the median.

In Part (f), most candidates knew how to find the expectation and variance and of a linear combination of variables.

Answers

(a) $a = \frac{1}{12}$ (b) $\frac{1}{3}$ (c) $\frac{31}{9}, \frac{92}{81}$ (d) $F(x) = \begin{cases} 0 & , x \le 1 \\ \frac{x^2}{24} - \frac{1}{24} & , 1 \le x \le 5 \\ 1 & , x \ge 5 \end{cases}$

(e) $\sqrt{13}$ (f) $\frac{37}{3}, \frac{92}{9}$

MODULE 3

Question 5

This question tested candidates' ability to determine the:

- kinetic energy lost in a collision;
- impulse exerted by one body A on another body B after collision.

In the first (a) (i), many candidates were able to use the data in this question to find the required velocity after impact. These then went on to find the kinetic energy before and after impact and hence the kinetic energy lost in the collision.

The responses obtained indicated that candidates were familiar with the concept: impulse equal change in momentum. However, some were unable to use the principle of conservation of linear momentum to calculate the required velocity. A number of candidates confused the Law of Conservation of Momentum with the Law of Conservation of Energy.

In Part (b) many candidates incorrectly used the linear equations of motion $v = u + at$ and $v = \frac{s}{t}$ instead of $\frac{dr}{dt}$ or $a = \frac{dv}{dt}$ Those candidates who used the correct equations were able to find the magnitudes of the velocity, acceleration, momentum and force in the vector form ai + bj. Of course those who used the former equations were able to obtain the required solution.
A common error seen was in the differentiation of 4

Answers

(a)(i) $\frac{28}{9}J$ (ii) $\frac{14}{3}$

(b)(i) $(16\cos^2 t + 25\sin^2 t)^{½}$ (ii) $(16\sin^2 t + 25\cos^2 t)^{½}$

(iii) $(144\cos^2 t + 225\sin^2 t)^{½}$ (iv) $(144\sin^2 t + 225\cos^2 t)^{½}$

Question 6

This question tested candidates' ability to:

- - State Newton's second law of motion
  - obtain the greatest height reached by a Particle which is projected vertically upwards.

- Obtain the values of unknown constants for forces acting along the sides of a trapezium when the system is in equilibrium, using the fact that the vector sum of its forces is zero or the sum of the components in any direction is zero.

Candidates were generally able to show the resultant force $mg + mkv^2$ acting on the Particle. The difficulty arose in writing the acceleration as $v\frac{dv}{ds}$. Those who used $v\frac{dv}{ds}$ as the expression for acceleration, were unable to perform the required integration to find s.

Few candidates were able to resolve the forces in two perpendicular directions and use $x = 0$ and $y = 0$.

Answers

(b) $p = \frac{7}{3}$ $q = 2$.

INTERNAL ASSESSMENT

UNIT 1

Again this year, the overall presentation and quality of the samples submitted were satisfactory. Generally, candidates chose topics that were suitable to their level, and were relevant to the objectives of the syllabus. There were a few candidates who employed techniques that went beyond the level expected.

Project Title:

Some project titles did not clearly indicate what the project was about. Candidates had problems with making the title relate to the project, for example, "internal versus external exams". There is need to have school teachers clarify this a bit more with candidates!

Purpose of Project:

Variables were not clearly defined. In some cases no variables were stated.

Where there were investigations to be done, the purpose of the project was not stated. There was a clear lack of responses to guide the project to a suitable conclusion.

Method of data Collection:

In many cases there was no sampling. Candidates used a fair amount of secondary data. Candidates must state how they collected the data in addition to stating the data.

Presentation of Data:

In general the presentation of data was well done. Use of more sophisticated A' level standard methods, like box and whisker plots, stem and leaf, plots is required.

Statistical Knowledge/Analysis of Data:

There was inappropriate use of concepts, for example, linear regression was attempted without calculation of the final equation of the line and the correlation coefficient was used with simplistic methods. In some cases calculations were done that were unrelated to the statement of task. For example, correlation coefficients were calculated without any justification.

Discussion of Finding/Conclusion:

This section was often ignored or candidates were not prepared enough to relate the results to the purpose of the project.

Communication of Information:

In only a minority of cases did candidates loose marks here. However, it is recommended that candidates familiarize themselves with the technical terms of statistics to produce a higher level of analysis and to state conclusions more efficiently.

List of References:

In some cases candidates failed to state title, author, reference numbers or dates of publication.

UNIT 2

PAPER 02

MATHEMATICAL APPLICATIONS

INTERNAL ASSESSMENT

Generally the topics chosen were suitable for candidates at this level. Again this year, most of the projects submitted by candidates were from the Discrete Mathematics and Probability and Distribution sections. These projects were appropriate and were given adequate treatment by candidates. There was an understanding of what was required in the assessment but a number of candidates who tried to incorporate Mathematics from two or more modules including Module 1 experienced difficulty in doing so. It was noted that the majority of the topics chosen were based on the distribution of a product within small communities and "get rich quick" schemes.

It appeared that the 'hands-on' approach through which candidates were afforded the opportunity to apply their Mathematics in real life situations served them well. Candidates appeared to have developed an overall appreciation of the mathematical concepts used in their projects and many used them with confidence. A few candidates were even able to do more than what was expected of them.

Teachers' marking of the projects showed that they understood the criteria used and are well aware of the type of assessment used. There was generally close agreement between the marks awarded by the teachers and those by the CXC Moderator.

RECOMMENDATIONS

It is recommended that candidates practice more problems involving the use of the normal distribution and questions from the Mechanics module. Exercises involving the use of algebraic manipulation are strongly recommended. It is necessary for teachers to complete the entire syllabus so that candidates can answer the question set adequately.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2009

APPLIED MATHEMATICS

APPLIED MATHEMATICS

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2009

INTRODUCTION

The revised Applied Mathematics syllabus was examined this year for the second time. The revised Applied Mathematics syllabus is a two-Unit course comprising three papers. Paper 01, multiple choice items, and Paper 02, essay questions, were examined externally, while Paper 03 was examined internally by the class teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each Unit were 30 per cent, 50 per cent and 20 per cent respectively.

Unit 1: Statistical Analysis, consists of Collecting and Describing Data, Managing Uncertainty and Analysing and Interpreting Data.

Unit 2: Mathematical Applications, consists of Discrete Mathematics, Probability and Distributions and Particle Mechanics.

GENERAL COMMENTS

For Unit 1, three hundred and seventy candidates wrote the 2009 paper and four wrote the Alternative to SBA paper, namely Paper 03/2. For Unit 2, 165 candidates wrote the 2009 paper and two wrote the Alternative to SBA paper, namely Paper 03/2.

Approximately 85 per cent of the candidates registered for the Unit 1 Statistical Analysis, and 93 per cent of the candidates registered for the Unit 2, Mathematical Applications, obtained acceptable grades, Grades I – V. The standard of work seen from most of the candidates in this examination was generally good.

In Unit 1, candidates appeared to be well prepared in Describing and Collecting Data and in Managing Uncertainty while Analysing and Interpreting Data posed problems for many candidates.

In Unit 2, again this year, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions and generally answered the questions well, with most candidates attempting all of the questions in these sections. There were a number of candidates who appeared to be well prepared in all three modules. Once again, however, Particle Mechanics was not generally well answered. Overall, there was a large number of areas of strength displayed by many candidates. As in previous years, candidates need to pay more attention to their algebraic manipulation.

UNIT 1

Statistical Analysis

Paper 01

The performance on the 45 multiple choice items on Paper 01 produced a mean of 46 out of 90 with scores ranging between zero and 88.

DETAILED COMMENTS

Paper 02

Module 1

<u>Question1</u>

This question tested candidates' ability to:

(a) Distinguish between qualitative and quantitative data, discrete and continuous data.

(b) (i) Explain how a random sample can be selected using the method of random numbers.

(ii) Select a random sample using a table of random numbers.

(c) (i) Distinguish between a population and a sample.

(ii) (iii) State the population of interest in a given survey and identify sampling techniques used in a given situation.

(iv) Perform calculations involving stratified random sampling.

(v) Calculate sector angles of a pie chart from given categories.

This question was attempted by approximately 99 per cent of the candidates, of whom 70 per cent gave satisfactory responses.

Part (a) of this question was very well done, with only a few candidates failing to identify age as a continuous random variable.

Part (b) seemed to have posed a great deal of difficulty for most candidates as they were unable to explain clearly how to obtain a sample of 10 accounts using the method of random numbers. In fact, many candidates tried to explain how to obtain a sample of 10 accounts using the lottery method of sampling.

In Part (c) (i), some candidates gave a geographical definition rather than a statistical definition for population. However, the majority were able to relate the sample to the population.

In Part (c) (ii), candidates correctly stated the population of interest for this survey as the students at the school, while in Part (c) (iii) candidates correctly identified the sampling technique as stratified random sampling.

In addition, in Part (c) (iv) candidates correctly calculated the number of students in the sample from the fourth year group as 9 using stratified random sampling.

Those who attempted Part (c) (v) correctly calculated the angle for each response category using. $\frac{\textit{given value}}{50} \times 360^{\circ}$

Question 2

This question tested candidates' ability to:

(a) Assess the appropriateness of the kind of chart to best illustrate statistical data.

(b) (i) (ii) Construct a frequency distribution table and bar charts.

(c) (i) Assess a given situation in which sampling techniques are more appropriate and why.

(ii) Perform calculations involving stratified random sampling.

(d) (i) (ii) Calculate the mean, median, mode and interquartile range for ungrouped data.

(iii) Construct a box and whisker diagram.

(iv) Interpret the shape of the distribution in terms of skewness.

Part (a) was exceptionally well done. Only a few candidates who failed to follow the instructions lost marks.

Part (b) (i) was exceptionally well done, with approximately 99 per cent of the candidates being able to construct the frequency distribution table from the given data.

In Part (b) (ii), a few candidates illustrated the frequency distribution from Part (b) (i) as a histogram rather than a bar chart.

In Part (c), some candidates had difficulty explaining why a stratified sampling method might be a better technique for choosing the sample rather than using a simple random sampling method. Nevertheless, these candidates were able to select 25 employees by calculating the number of employees in each of the four stated categories using stratified random sampling.

Part (d) (i) was well done by the majority of candidates, who were able to state the mode and median of the distribution.

In Part (d) (ii), most candidates were able to correctly calculate the mean number of hours that students slept as 7.04, but a few candidates used an incorrect position for the lower and upper quartile and hence obtained an incorrect value for the inter-quartile range.

In Part (d) (iii), most candidates were able to draw an accurate box and whisker diagram, with the exception of about 3 per cent of the candidates who drew incorrect whiskers and used incorrect quartiles.

In Part (d) (iv), many candidates correctly identified the shape of the distribution as positively skewed, while the others incorrectly identified the shape of the distribution as either symmetrical or negatively skewed.

Module 2

Question 3

This question tested candidates' ability to:

(a) Calculate $P(A \cup B)$, $P(A \cap B)$ and P(A|B).

(b) Construct and use a probability distribution table for discrete random variables to obtain probabilities.

(c) Construct a tree diagram.

This question was generally well done.

In Part (a), most candidates correctly defined *P(A|B)* = $\frac{P(A \cap B}{P(B)}$) and $P(A \cup B) =$ $P(A) + P(B) - P(A \cap B)$ and were able to obtain the correct response of 0.5 for each of the above. A few candidates defined the P(A|B) and $P(A \cup B)$ incorrectly and so were unable to get the correct result.

Part (b) was generally well done, with candidates being able to calculate $P(G \cap H)$ as 0.28 and $P(G \cup H)$ as 0.82. There were those candidates who did not use that fact that *G* and *H* are independent events and so experienced great difficulty when trying to calculate $P(G \cap H)$ and $P(G \cup H)$.

In Part (c) (i), most candidates were able to illustrate the given information on a well-labelled tree diagram. However, a few candidates placed incorrect labels and probabilities on the branches of the tree diagram.

Part (c) (ii) was generally well done, with candidates being able to use the tree diagram to calculate probabilities, but a few candidates failed to use the information provided by the tree diagram.

Part (c) (iii) was generally well done by the candidates who performed well in the previous parts. Of the 95 per cent of the candidates who attempted this question, 70 per cent gave satisfactory responses.

Part (d) was generally well done by most candidates. The majority of candidates were able to calculate the value of k as 0.2. For (d) (ii), some candidates applied a different approach to find the expectation but still arrived at the correct answer 6. For part (d) (iii), a few candidates were unable to find the cumulative probabilities, 0.4 and 0.3, to give 0.7 as the required solution.

Question 4

This question tested candidates' understanding of the

(a) Binomial distribution –

(i) - (iv) the conditions for which discrete data can be modelled by the binomial distribution,

(b) (i) - (ii) the notation for the binomial distribution and its probabilities, the expected value and the variance of a binomial distribution and to use the binomial distribution to calculate probabilities.

(c) The Normal distribution -

Determine probabilities from tabulated values of the standard normal distribution $Z \sim N(0, 1)$;

Standardising a value of the normal distribution

(d) Solve problems involving probabilities of the normal distribution using z-scores.

Generally, this question was fairly well done by most candidates, and did not appear to be too difficult.

In Part (a), a few candidates did not recognize the binomial model.
Part (b) (i) was well done by the majority of candidates. Many of them were able to use E(X) = np, though some candidates further expressed the answer as a whole number.

For Part b (ii), though many candidates knew the binomial formula for finding probability, many of them confused the probability values of p and $1 - p$. About 60 per cent of the candidates who attempted this part did it correctly. However, many candidates did not correctly interpret the phrase ***"at most 2"*** **.**

In Part (c), there were indications that candidates could read the value from the table, but they experienced problems writing the steps taken to get to the solution. In Part (d), candidates generally had problems writing the standardization correctly as $\frac{x-\mu}{\sigma}$. Many candidates wrote $\mu - x$ instead of $x - \mu$, and many candidates also used the variance, $\sigma^2 = 16$, instead of the standard deviation, $\sigma = 4$ in the standardizing formula. Most candidates were able to convert $P(Z > 1.25)$ to $1 - P(Z < 1.25)$, and also to use φ correctly, $\varphi(-1.5) = 1 - \varphi(1.5)$. However, many candidates did not convert the probability to a percentage as required in the question.

Module 3

Question 5

This question tested the candidates' understanding of:

(a) confidence intervals - description and how to construct

(b) unbiased estimates of parameters

(c) using the normal distribution as an approximation to the binomial.

Generally, this question was very poorly done. Candidates had problems stating definitions or reasons, and explaining concepts.

In Part (a), candidates were asked to give an explanation of the term 95 per cent confidence interval in the context of the population mean. Though most candidates knew that this had something to do with an interval, they could not adequately state anything about the interval. Furthermore, many candidates did not say that the interval contained μ.

In Part (b) (i), less than 50 per cent of the candidates attempted this question, and many of them did not use the correct formula.

Most of the candidates who attempted Part (b) (ii) used the standard deviation of the sample, rather that the unbiased estimate that they were asked to calculate in Part (b)(i).

For Part (b) (iii), most of the candidates were able to write ``*increase the sample size*``, but many of them could not give a second method. Some candidates also confused *confidence level* with *significance level.* A few candidates were able to state that ``*increasing the sample size will be better*``, but not many of them could give a reason for this.

Part (b) (iv) was done well by those candidates who attempted this question.

Very few candidates attempted this Part (c) of the question. Many were unable to calculate the expected interval as $0.1 \times 60 = 6$.

In Part (d) (i), most of the candidates correctly stated that the Normal distribution would be used, but many of them could not give the parameters.

In Part (d) (ii), many candidates in calculating the probability did not use the correct standard error.

Many of them incorrectly standardized using $z = \frac{\mu - \bar{x}}{\sigma}$ instead of $z = \frac{\bar{x} - \mu}{\sigma}$.

Question 6

This question tested the candidates' understanding of:

(a) hypothesis testing using the chi squared distribution
(b) the application of the chi squared table

In Part (a) (i), many candidates interchanged the null and the alternative hypotheses.

In Part (a) (ii), candidates were knowledgeable of the procedures required to use the chi squared test to complete a contingency table by calculating expected values. In Part (a) (iii), many candidates were able to write $(r - 1)(c - 1)$ as the formula for finding the degrees of freedom, but some added rather than multiplied the quantities, or they substituted incorrect values for r and c. Many candidates wrote the critical value, rather than the critical region. Many candidates looked up the table value incorrectly, using the lower 5 per cent value rather than the upper 5 per cent value.

In Part (a) (iv), most candidates attempted to write a conclusion for the test. Using their null hypothesis, they were able to make a valid decision as to reject, or fail to reject the null hypothesis; however, many of them could not state a valid conclusion for the test.

In Part (b), candidates were required to generate the equation of a regression line. However, not many candidates attempted it, but for those who did, it was very well done yielding the correct answer $y = 2 + x$.

UNIT 2

Mathematical Applications

Paper 01

The performance on the forty-five multiple choice items on Paper 01 produced a mean of 56 out of 90 with scores ranging between 0 and 88.

DETAILED COMMENTS

Paper 02

Module 1

Question 1

This question tested candidates' ability to:

(a) Use the activity network in drawing a network model to model a real-world problem.

(b) Calculate the earliest start time, latest start time and float time.

(c) Identify the critical path in an activity network

This question was reasonably well done by most candidates, as 45 per cent of them scored between 20 and 25 marks (at least 80 per cent).

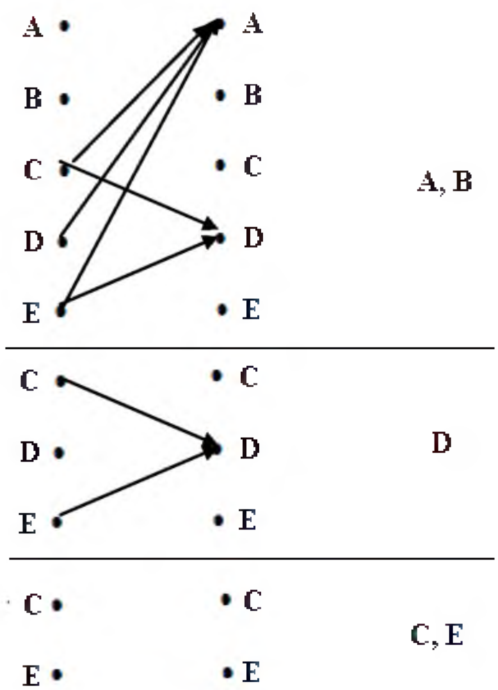
In Part (a), only a few candidates started out with an activity network algorithm. A number of candidates inserted additional edges and left out the start and finish nodes from the activity network.

In Part (b), many candidates did not calculate the earliest and latest start times correctly and so, their float times were incorrect also.

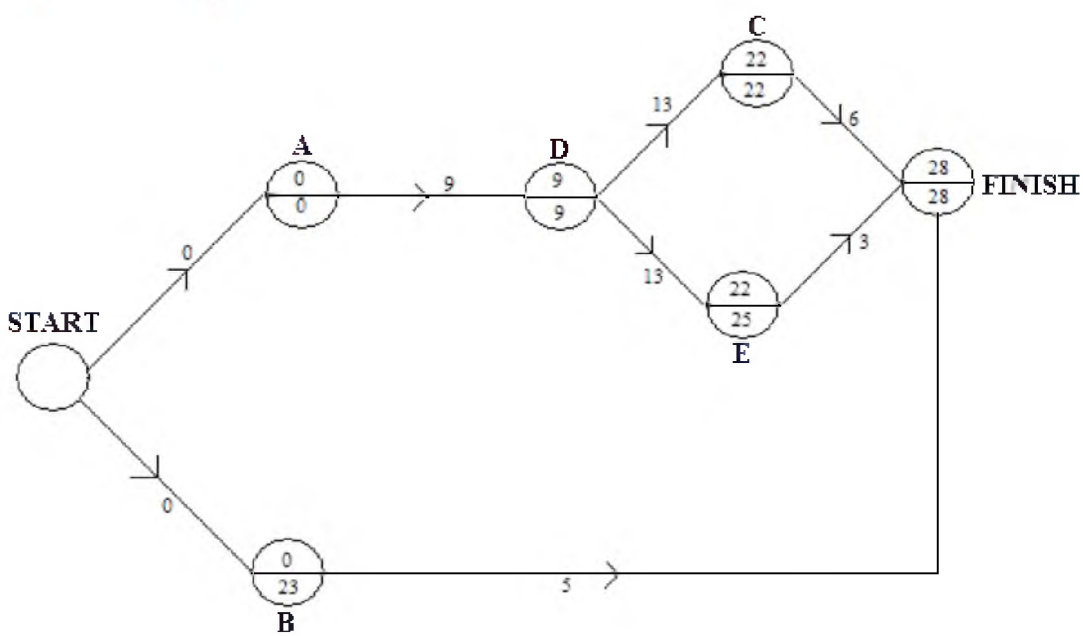
In Part (c), most candidates were able identify a critical path, although some answers were based on incorrect data from Part (b) and were therefore incorrect. Most of the candidates calculated the minimum completion time correctly.

Answers

(a) Activity Network Algorithm



Activity Network



(b)

Activity	Earliest Start Time	Latest Start Time	Float
A	0	0	0
B	0	23	23
C	22	22	0
D	9	9	0
E	22	25	3

(c) (i) Critical path: Start, A, D, C, Finish

(ii) Minimum completion time = 28 days

Question 2

This question tested candidates' ability to:

(a) Formulate simple propositions.

(b) (i – iii) Formulate compound propositions that involve conjunction, disjunctions and negations.

State the converse, inverse and contrapositive of implications of propositions.

(c) Use truth tables to:

i. Determine whether a proposition is a tautology or a contradiction.

ii. Establish the truth values of converse, inverse and contrapositive of propositions.

iii. Determine if propositions are equivalent.

(d) Identify the vertices and sequence of edges that make up a path.

(e) Determine the degree of a vertex.

(f) and (g) Identify the critical path in an activity network.

This question was well done.

In Part (a), a number of candidates confused the conditional with the bi-conditional.

In Part (b), most candidates confused the converse with the inverse and the contrapositive.

Part (c) was generally well done as most candidates were able to recognize a contradiction.

In Part (d), very few candidates were able to list all the correct paths.

Part (e) was generally well done as most candidates were able to state the degree of the vertex D correctly. A small number of students confused the degree of the vertex with the measurement of an angle (for example, 60^0).

Part (f) was well done. Some candidates, however, drew switching circuits instead of logic gates.

Part (g) was well done. A few candidates, however, confused the “AND” and the “OR” gates. Some incorrectly showed more than two inputs. The majority of the candidates were able to conclude correctly whether the circuits were equivalent.

Answers

(a) Tom works hard if and only if he is successful.

(b) (i) The inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$

(ii) The converse of $p \Rightarrow q$ is $q \Rightarrow p$

(iii) The contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

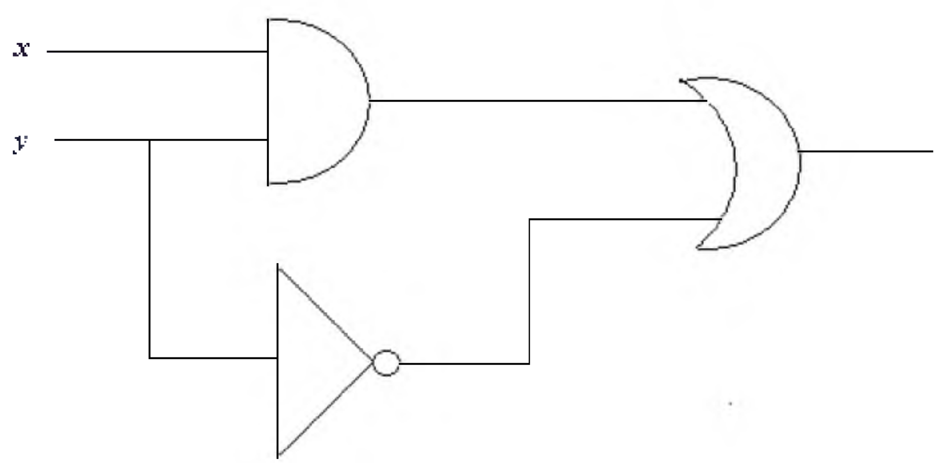
(a)

a	b	~b	$a \Rightarrow b$	$a \wedge (a \Rightarrow b)$	$[a \wedge (a \Rightarrow b)] \wedge \sim b$
T	T	F	T	T	F
T	F	T	F	F	F
F	T	F	T	F	F
F	F	T	T	F	F

The proposition is a contradiction.

(b) Paths: AE, ADE, ADCE, ABDE, ABCE, ABCDE, ADBCE, ABDCE

(c) The degree of vertex D is 4



(d) (i) $\sim(a \vee b)$ or $\overline{a+b}$

(ii) $\sim a \wedge \sim b$ or $\bar{a}.\bar{b}$

a	b	$a \vee b$	$\sim(a \vee b)$
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	1

~a	~b	$\sim a \wedge \sim b$
0	0	0
0	1	0
1	0	0
1	1	1

The circuits in (i) and (ii) are equivalent.

Module 2

Question 3

This question tested candidates' ability to:

(a) (i) Find the number of different arrangements using each letter once.

(ii) Find the probability that an arrangement starts with a consonant and then the consonants and vowels are arranged alternately.

(b) (i) (ii) Calculate and use probabilities associated with conditional, independent or mutually exclusive events.

(c) (i) Use the result $P(a < X \leq b) = \int_a^b f(x)\,dx = F(b) - F(a)$.

(ii) (iii) Calculate the expected value and the variance.

Overall, this question was reasonably well done.

Part (a) (i) was well done by candidates, while Part (a) (ii) presented difficulty for only a few candidates who gave the incorrect result of $4! + 3!$.

In Part (b), the majority of the candidates were able to obtain the correct solution without displaying a tree diagram. Most of the errors in Part (b) (ii) were the result of errors carried forward from Part (b) (i).

In Part (c), the majority of the candidates showed that they understood that they were expected to use integration since the random variable was continuous and not discrete. However, a few candidates incorrectly treated the random variable as discrete. In Part (c)(iii), most candidates expanded $(2-q)^3 = 6$ to get a cubic equation that they were unable to solve (one candidate actually used the

Newton-Raphson method to find an approximation to the root of the equation), rather than finding the cube root of the equation $(2-q)^3 = 6$ and so obtain the value for *q*.

Answers

(a) (i) number of arrangements = 7! = 5040

(ii) number of arrangements = $4! \times 3! = 144$

Probability = $\dfrac{144}{5040} = \dfrac{1}{35}$

(b) (i) P(L) = $P(F \cap L) + P(G \cap L) + P(T \cap L) = 0.665$

(ii) P(T|L) = $\dfrac{P(T \cap L)}{P(L)} = \dfrac{0.045}{0.0665} = 0.677$

(c) (i) P(X > 1) = $\dfrac{1}{8}$

(ii) E(X) = ½, Var(X) = $\dfrac{3}{20}$

(iii) $q = 0.18$

<u>Question 4</u>

This question tested candidates' ability to:

(a) Model practical situations in which the discrete, uniform, binomial, geometric or Poisson distributions are suitable.

Apply the formulae:

(i) $P(X = x) = {}^nC_r p^x q^{n-x}$

(ii) $P(X = x) = \dfrac{\lambda e^{-\lambda}}{x!}$

to calculate probabilities of discrete binomial and Poisson distributions respectively.

(b) Use the formulae for E(X) and Var(X).

(c) Use the Poisson distribution as an approximation to the binomial distribution, where appropriate.

This question was generally well done.

In Part (a), most students recognized that the distribution was binomial and were able to get the majority of the marks for Parts (a) (i) and (ii). However, a few candidates were able to model the situation using $Y \sim Bin\,(5, 0.541)$ and went further to find P (Y = 3).

In Part (b), a few candidates recognized that the Poisson distribution was to be applied. Some, however, used $X \sim Po\,(0.4)$ or $X \sim Po\,(0.04)$ incorrectly instead of $X \sim Po\,(4)$.

In Part (c), very few candidates recognized that the Poisson distribution should have been used to approximate the binomial in this case.

Answers

(b) (i)

a) $P(X = 8) = 0.237$

b) $P(8 \le X \le 10 = 0.541$

(ii) Mean = E(X) = np =7.8

Standard deviation = $\sqrt{npq} = 1.65$

(iii) $Y \sim Bin(5, 0.541)$

$P(Y = 3) = {}^5C_3(0.541)^3(0.459)^2 = 0.334$

(c) $X \sim Po(4)$

$P(X \ge 1) = 1 - P(X = 0) = 1 - 0.135 = 0.865$

(d) $X \sim Bin(100, 0.04)$

Use Poisson approximation since n = 100 > 30 and np = 4 < 5

$P(X \le 3) = 0.433$

Module 3

Question 5

This question tested candidates' ability to:

(i) perform calculations involving a body moving with constant velocity up a plane

(ii) apply the equation F = ma to situations where the body is moving in a horizontal plane

(iii) apply the equation P = Fv

(iv) calculate the maximum speed under the application of variable forces.

In Part (a), there were a few correct responses from those candidates who did not include a force diagram. The majority lost marks because they had:

(i) wrong signs for the weight component and the resistive forces

(ii) missing tractive force

(iii) use of mass (750 kg) as the weight instead of 7500 N (mass × g).

The correct solution was 35000 N.

In Part 5 (b) (i), very few candidates realized that the force found in Part (a) was to be reduced by 650 N on the horizontal ground, ignoring any further reduction (i.e. weight component). The correct solution was acceleration 1 ms^{-2}.

In Part 5 (b) (ii), some candidates realized that at the maximum speed there was no acceleration which led to the correct solution V = 32.3 ms^{-1}.

Question 6

This question tested candidates' ability to:

(a) Determine the velocity and displacement of a particle with variable acceleration.

(b) Apply their knowledge of vectors to show that there was no acceleration along Ox, to find the time when the velocity was perpendicular to the acceleration and to find the distance of the particle from O when t = 3.

This question was poorly done. Forty-eight per cent of the candidates scored between 0 and 4 marks.

In Part (a), most of the candidates realized that they were required to integrate to find expressions for the velocity and the displacement. However, a few tried to use the equations of motion to solve the problem, not realizing that this could not be done since the acceleration was not constant.

The answer to part (b), followed from their solutions to (a). There were a few who got the correct answers, but were unable to explain what the results meant.

In Part (c)(i), only a few candidates were able to differentiate twice, thereby obtaining the coefficient of **" "*i*"** and stating that this indicated that there was no acceleration along Ox.

The candidates who attempted Part (c) (ii) displayed a knowledge of $a.b = 0$ for perpendicularity of vectors a and b, and solved the problem. However, no candidate observed that from (a)(i), the acceleration is along Oy, therefore the velocity is along Ox, hence : $4t^3 - 32 = 0 \Rightarrow t = 2$.

Part (c) (iii) was well done, as candidates were able to substitute t = 3 into $r = 8ti + \left(t^4 - 32t\right)j$, hence finding the required distance.

UNIT 1

Statistical Analysis

Internal Assessment

After careful evaluation of the Internal Assessment samples, it was found that:

1. Some project titles were too long or were not relevant to the course. Project titles must be concise and relevant.
2. Some projects had no title, which was a pity, since this is perhaps the easiest mark to score in the project.
3. Too often, questionnaires and long lists of data were included in the presentation of data. This is unnecessary and should be appropriately placed in the appendices.

4. Diagrams such as pie charts were frequently not properly labelled in the presentation of data. This impacted on the marks scored by candidates in this section.
5. In most cases, the purpose was stated but no variables were identified. Candidates, in general, seemed to be confused as to the meaning of the word “variable”.
6. In many cases, the method of data collection was too simplistic. The method was stated in many instances, but was not described in detail.
7. A small percentage of the teachers (approximately 5 per cent) misunderstood the process of recording the marks on the AMAT 1-3 forms (i.e. the form containing the data on the five samples).
8. The references were omitted in about 10 per cent of the Internal Assessment samples submitted. Far too many candidates did not use an up-to-date and consistent convention in the reference.

UNIT 2

Mathematical Applications

Internal Assessment

1. Some of the topics chosen for study by the candidates were not relevant to Applied Mathematics.
2. A very small minority of teachers misunderstood the process of recording the marks on the AMAT 23 forms.
3. Some diagrams were not labelled in the projects.
4. A small minority of candidates did not follow the stipulated format.
5. Generally, the analyses were comprehensively done, but there were a few occasions when the analysis was not relevant.
6. In many instances, the task was not clearly stated. This section should not have been placed in the introduction and should be more concise. Unnecessary details should not be included here.
7. Method of data collection was in some cases too simplistic or non-existent. This section needs to be clearly described.
8. The Hungarian algorithm was misused in many cases or improperly applied.
9. Many candidates who incorporated logic gates into their projects did so inappropriately.
10. Many candidates lost two marks allocated to “insights into the nature and resolution of problems encountered in the tasks”.

Recommendations

It was evident from the work produced in Unit 1 that candidates were well prepared in Describing and Collecting Data and in Managing Uncertainty while in Unit 2 they were well prepared in Discrete Mathematics and Probability and Distributions. Candidates are still experiencing problems with the Mechanics Module. Candidates have to spend more time solving problems from the Mechanics Module as well as developing the ability to explain concepts. In addition, they need to pay special attention to their algebraic manipulation. It would be helpful if the coverage of the syllabus could be completed in time to allow candidates to spend more time solving problems from all of the Modules.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2010

APPLIED MATHEMATICS

GENERAL COMMENTS

The revised Applied Mathematics syllabus was examined this year for the third time. This is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple choice items, and Paper 02, consisting of 6 essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each unit were 30 per cent, 50 per cent and 20 per cent, respectively.

Unit 1, Statistical Analysis, tested (1) Collecting and Describing Data; (2) Managing Uncertainty and (3) Analysing and Interpreting Data. Unit 2, Mathematical Applications, tested (1) Discrete Mathematics, (2) Probability and Probability Distribution and (3) Particle Mechanics.

For Unit 1, 456 candidates wrote Papers 01 and 02 and 11 wrote the Alternative to the Internal Assessment paper, Paper 03/2. For Unit 2, 197 candidates wrote Papers 01 and 02 and three candidates wrote Paper 03/2.

Approximately 80 per cent of the candidates registered for Unit 1, Statistical Analysis, and 83 per cent registered for Unit 2, Mathematical Applications, obtained acceptable grades, Grades I–V. The standard of work seen from most of the candidates in the examinations was satisfactory.

In Unit 1, candidates appeared to be well prepared in Describing and Collecting Data and in Managing Uncertainty, while Analysing and Interpreting Data continued to be a challenge for many.

In Unit 2, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions. Particle Mechanics again seemed to be a challenge for many candidates. Generally, candidates are still having difficulty with algebraic manipulations.

DETAILED COMMENTS

UNIT 1

Paper 01–Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of 56 out of 90 with scores ranging between 0 and 88.

Paper 02–Essay

Module 1

<u>Question 1</u>

This question tested candidates' ability to

(a) distinguish between qualitative and quantitative data, and between discrete and continuous data

(b) distinguish between a parameter and a statistic

(c) distinguish between a sample and a population

(d) identify simple random sampling, systematic sampling, stratified sampling, quota sampling and cluster sampling

(e) determine class boundaries, class widths, and calculate frequency densities

(f) state some advantages of grouping data.

The question was attempted by all candidates, with about 75 per cent satisfactory responses. In Part (a), candidates performed exceptionally well.

In Part (b), about 60 per cent of the candidates did well. The other 40 per cent knew the definition of parameter or a statistic but did not know how to relate them to the context of the question given.

Part (c) was generally well done by most candidates; a few candidates had difficulty differentiating between a sample and a population.

Answers (i) sample (ii) population

In Part (d), about one-third of the candidates scored full marks. The other candidates showed some measure of confusion as to which technique to use. Responses included techniques such as convenience instead of quota, purposive, snowball or multistage instead of systematic.

Answers (i) cluster (ii) stratified (iii) systematic (iv) quota (v) simple random

Part (e) (i) and (ii) were well done by most of the candidates. A few candidates gave the boundaries as 24−29, while others gave no indication of how they got their answers.

Part (e) (iii) was poorly done by the majority of the candidates. Candidates did not understand the concept of frequency density.

In Part (e) (iv), many candidates knew that the data was grouped but still had difficulty estimating the number of children who took more than 60 minutes to complete the assignment. Very few candidates were able to correctly calculate the answer.

Part (v) was poorly done by most candidates. Many gave some notable responses such as ‘normally distributed’, ‘does not take exact values’ and ‘not accurate’.

Answers (i) 24.5, 29.5 (ii) 10 (iii) 3 (iv) 8 (v) loss of individual data values

Question 2

This question tested candidates’ ability to

(a) (i) construct and use a stem and leaf diagram to display data
(ii) give one disadvantage of using a stem and leaf diagram

(b) determine the median and mode from ungrouped data

(c) calculate the mean from ungrouped data

(d) outline an advantage of using the mean from raw data

(e) calculate the trimmed mean from given data

(f) interpret and describe the shape of a distribution in terms of ‘skewness’

(g) describe the shape of the distribution

This question was attempted by most of the candidates who gave satisfactory responses.

Part (a) (i) was exceptionally well done except for a few candidates who omitted the key. Part (a) (ii) was not well done. Only a few candidates were able to correctly state one advantage of using the stem and leaf diagram to display data.

In Part (b) (i), candidates performed exceptionally well except for a few who calculated the position of the median but were unable to give the final answer. For Part (b) (ii), the majority of the candidates were able to determine the modal age.

In Part (c), all the candidates knew the formula for calculating the mean but a few made errors in summing the given values. Part (d) seemed to have posed a great difficulty for most candidates although some knew the answer but had problems expressing the solutions correctly.

In Part (e), many candidates showed competency in calculating the 8 per cent trimmed mean which was $\frac{8}{100}$ x 25 = 2, but had difficulty calculating the mean using $\frac{\sum x}{n}$ which was $\frac{542}{21}$. Few candidates knew that two values should be subtracted from both sides of the data. Many candidates used n as 23 rather than 21.

For Part (f) (i), half of the candidates had difficulty determining the quartiles. They knew the formula but did not calculate the true value. Many gave the position of the term as the answer.

Part (f) (ii) was exceptionally well done. Candidates were able to calculate the semi-interquartile range of the ages based on their responses from Part (f) (i). A few candidates had the wrong formula and therefore used $\frac{Q_3 + Q_1}{2}$ instead of $\frac{Q_3 - Q_1}{2}$.

Part (g), was fairly well done. The majority of candidates correctly described the shape of the distribution as positively skewed.

Answers *(a) (i) all data values are maintained* *(b) (i) 24* *(b) (ii) 19*

(b) (iii) 27.08 *(d) effected by extreme values or outliers* *(e) 25.84*

(f) (i) 20, 32 *(f) (ii) 6* *(g) positively skewed.*

Module 2

Question 3

This question tested candidates' ability to

(a) calculate:

(i) the probability of an event A, where P(A) represents the number of possible outcomes of A divided by the total number of outcomes

(ii) the probability of two events occurring as $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

(iii) the probability of an intersection of two events A and B i.e. $P(A \cap B)$

(iv) the conditional probability of two events A and B i.e. $P(A|B) = \frac{P(A \cap B)}{P(B)}$

(b) use the property that P(A') = 1 – P(A) where P(A') is the probability that event A does not occur.

(c) calculate, using an appropriate method, the different combinations of patties chosen by patrons at a restaurant.

Part (a) (i) was attempted by approximately 98 per cent of the candidates, with about 65 per cent giving a satisfactory response. Candidates inappropriately applied independent events in their solution.

Part (a) (ii) was attempted by most candidates, with approximately 75 per cent giving a satisfactory response. Some candidates interpreted P(A|B) as $\frac{P(A)}{P(B)}$

For Part (a) (iii), the majority of candidates were unable to identify the region for $P(A' \cap B')$.

Part (b) (i) was generally very well done. However, a few candidates, in stating the probability as a fraction, used an incorrect denominator.

In Part (b) (ii), some of the candidates represented the data in a Venn diagram but incorrectly calculated the intersection. Other candidates incorrectly calculated $P(A \cap B)$ as P(A) x P(B) rather than using $P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

For Part (b) (iii), approximately 50 per cent of the candidates did not recognize that this question was an event without replacement. Part (c) (i) was very well done. The 30 per cent who did (c) (ii) incorrectly did not cube the respective probabilities.

In Part (c) (iii), many candidates were unable to identify all six combinations of the three types of patties. For Part (c) (iv), some candidates did not recognize the use of the conditional probability. Some of them interpreted P(*chicken and the same type*) to be P(*chicken*) x P(*same type*).

Answers *(a) (i) 0.5* *(ii) 0.694* *(iii) 0.14* *(b) (i) 8/15* *(ii) 3/15* *(iii) 3/7*
(c) (i) 0.091 *(ii) 0.142* *(iii) 0.189* *(iv) 0.642*

Question 4

This question tested candidates' ability to

(a) (i) calculate the expected value of independent events
(ii) use the Binomial to calculate probability of independent events
(iii) interpret and calculate probability of independent events

(b) (i) use the Binomial distribution to find *n*
(ii) use the Binomial distribution to find expected value
(iii) use the Binomial distribution to calculate probability

(c) (i) decide when to use the Normal distribution as an approximation of a Binomial distribution
(ii) calculate probabilities using the Normal distribution..

Many candidates scored fairly well on this question, with the most common score being 18 marks out of 25. However, many candidates did not follow instructions as it pertained to stating the answer to three significant figures. Several candidates rounded-off prematurely, and this affected the accuracy of their final answer.

Part (a) (i) was generally well done, with most candidates getting the correct solution of 16.5 days. Surprisingly many candidates did not know the number of days in the month of September (values ranging from 28 to 32 were used). Candidates who obtained no marks on this part of the question demonstrated a clear lack of understanding of discrete probability.

In Part (a) (ii), most candidates correctly identified the use of the Binomial. However, many candidates incorrectly substituted the values for p and q into the formula and therefore were unable to obtain the answer of 0.239.

In Part (a) (iii), many candidates could not interpret the probability of independent events and were unable to obtain the answer of 0.166. Part (b) (i) was very well done by those candidates who attempted it. Part (b) (ii) was generally also well done as most candidates were able to obtain the answer of 10.5.

Part (b) (iii) was generally well done by most candidates who knew that the use of the Binomial was required. However, not many of them obtained the answer of 0.206 as the values for p and q were incorrectly substituted.

In Part (c) (i), most candidates received partial credit as they did not state both conditions. A large sample size was interpreted by most candidates as $n > 30$; however, many of them did not realize that the value of p needed to be close to 0.5. Many candidates answered with $np > 5$ and $nq > 5$ as two separate conditions when these should have been stated together as one condition. Some candidates also gave $npq > 5$ which had to be recognized since it is stated as such in the syllabus.

For Part (c) (ii), many candidates used the Binomial distribution although an approximation with the Normal distribution was required. Many candidates who used the Binomial failed to include $P(x = 0)$ in the solution, and of those who used the Normal distribution, the continuity correction was incorrectly used.

Answers *(a) (i) 16.5 (ii) 0.239 (iii) 0.166 (b) (ii) 10.5 (iii) 0.206 (c) (ii) 0.416*

Module 3

Question 5

This question tested candidates' ability to

(a) identify the distribution of the sample mean
(b) calculate unbiased estimates of the population mean and standard deviation
(c) interpret a confidence interval
(d) construct and use a confidence interval.

In Part (a), many candidates were able to identify the distribution of the sample mean as the Normal distribution but had difficulty stating its parameters.

For Part (b) (i), most candidates failed to identify the sample mean as the middle of the given confidence interval. Many candidates set up the equations for both ends of the interval and attempted to solve them as simultaneous equations but did not complete them due to poor algebraic skills. As a result, many candidates did not obtain the answer of 0.575.

In Part (b) (ii), candidates were able to obtain the answer of 80. Most substituted whatever value was obtained in the previous part into one of the equations for the confidence interval and proceeded to solve for n. This part also indicated that some candidates have weak algebra skills. Most candidates recognized the need to round-off the answer for the discrete variable.

Part (c) was fairly well answered and most candidates obtained the answer of 2. Those getting this part incorrect attempted to construct the confidence interval rather than interpret the information that was given.

Part (d) (i) (a) was well done by the majority of the candidates who obtained the correct answer of 0.99 but failed to write the solution in ‘thousands of dollars’. The most common error was simply dividing the values given for ‘sum of all *x'* by ‘sum of all values of *x'* rather than using the given sample size, $\frac{\sum x^2}{\sum x}$ rather than $\frac{\sum x}{n}$.

Most candidates attempting Part d (i) (b) applied the formula incorrectly, however, for Part d (ii), most candidates knew the formula but were unable to obtain the correct z-value of 1.881.

Answers (b) (i) 0.575 (ii) 80

<u>Question 6</u>

This question tested candidates’ ability to

(a) evaluate the t-test statistic

(b) determine the appropriate number of degrees of freedom from a given data set

(c) determine the t-values from the t-distribution table

(d) apply a hypothesis test for a population mean using a small sample ($n < 30$) drawn from a Normal population with an unknown variance

(e) determine the critical value from the chi squared (χ^2) tables and hence determine the critical region for the test

(f) apply a χ^2 test for independence in a 2 x 3 contingency table.

In Part (a) (i), many candidates interchanged the null and alternative hypotheses. Some candidates incorrectly stated the null hypothesis as H_0: $\mu \geq 150$. For Part (a) (ii), most candidates lost a mark because they did not calculate the expected number of hardback cover textbooks to the nearest whole number. In Part (a) (iii), most candidates were unable to identify the correct rejection region, $\chi^2 > 5.991$. Other candidates gave the critical value rather than the rejection region. Most candidates who attempted Part (a) (iv) were able to state whether the null hypothesis was rejected or accepted, but could not interpret this with a valid conclusion.

For Part (b) (i), most candidates stated the conditions for the use of a t-test rather than the assumption. Part (b) (ii) was fairly well done; however, some candidates used the sample mean $\bar{x}$ instead of the population mean μ and others did not state a mean. In Part (b) (iii), many candidates were unable to identify the correct rejection region. Some candidates did not state the answer as a region but as a value.

Most candidates who did Part (b) (iv) incorrectly used the test value as $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ instead of

$t = \frac{\bar{x} - \mu}{\sigma / \sqrt{n - 1}}$.

Many of the candidates who attempted Part (b) (v) were able to identify the acceptance region of the null hypothesis, but did not state a clear conclusion as was required.

Answers (a) (ii) 7, 56 (iii) $\chi^2 > 5.991$ *(iv) reject* H_0 *; there is no association between the type of book and its cover.*

(b) (i) normal distribution (ii) H_0: $\mu = 150$, H_1: $\mu > 150$ *(iii)* $t > 1.761$ *(iv) accept* H_0: $\mu = 150$*, there is no evidence of an increase in the mean mass of the packages.*

UNIT 2

Paper 01 – Multiple Choice

The performance on the forty-five multiple choice items on Paper 01 produced a mean of 62 out of 90 with scores ranging between 0 and 88.

Paper 02 – Essays

<u>Question 1</u>

This question tested candidates' ability to

(a) derive and graph linear inequalities in two variables

(b) determine the solution set that satisfies a set of linear inequalities in two variables

(c) determine the feasible region of a linear programming problem

(d) determine the optimal solution of a linear programming problem

All the candidates attempted this question but only about 60 per cent performed satisfactorily with scores between 18 and 25 marks.

Part (a) was poorly done. All the candidates who attempted this section knew that they should substitute values into the function $P = x + 2y$. However, most candidates did not use the significant points in the region to find the optimal solution. A few of the candidates misread the scales on the graph.

Part (b) was fairly well done. Most candidates knew how to find the row minimum. However, some candidates maximized instead of minimize. A few candidates utilized other methods such as solving the equations simultaneously rather than the required Hungarian method to find the solution, Candidates also used a version of the Hungarian method that required that the minimum uncovered element be doubled and added to the elements that were covered twice.

In Part (c), most candidates were able to list the three paths, only a few did not start at A and did not finish at A.

Answers

(a) *Using* $P = x + 2y$ *and the points (5.7, 9), (17, 7) and (20, 4.4)*
The maximum value of P is 31 and it occurs when $x = 17$ *and* $y = 7$.

(b)(i) *A is assigned to task 1*
B is assigned to task 4
C is assigned to task 5
D is assigned to task 2
E is assigned to task 3

(b)(ii) *The total minimum time taken by the five persons = 50 + 50 + 50 + 50 + 49 = 249 minutes.*
Any part that starts at A and ends at A. For example: ABCDA, ABEA, ABEDA, AECDA, ABCEA, etc.

Question 2

This question tested candidates' ability to

(a) represent a Boolean expression by a switching or logic circuit

(b) use switching and logic circuits to model real-world situations

(c) use the laws of Boolean algebra to simplify Boolean expressions

(d) use truth tables to determine if propositions are equivalent

(e) derive a Boolean expression from a given switching or logic circuit.

This question was attempted by all the candidates. However, only 70 per cent performed satisfactory.

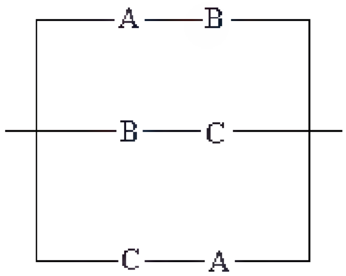
Part (a) was poorly done. Some candidates confused the switching and logic circuits. A few candidates used truth tables to assist them in drawing the switching circuit, but there were some who only did the truth tables, while some of them also used a combination of switching and logic circuits.

Part (b) was well done by most candidates. Only a small number of candidates did not complete the truth table. In Part (c), most candidates knew that they should use gates, but some of them did not combine the gates correctly.

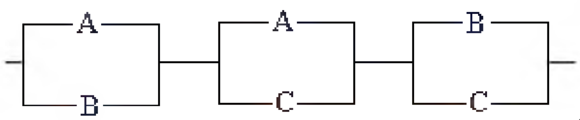
About 90 per cent of the candidates attempted Part (d). Candidates knew how to calculate the float and the earliest start time but some of them did not know how to find the latest start times. Most candidates knew that they needed to use the float to determine the critical path. However, there were a few candidates whose float did not have anything to do with their critical path.

Answers

(a) Tom works hard if and only if he is successful.



OR



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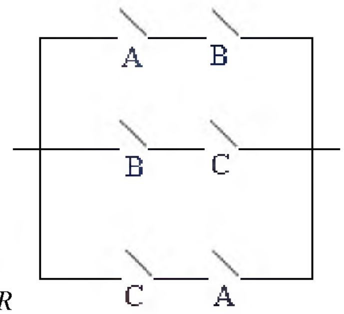
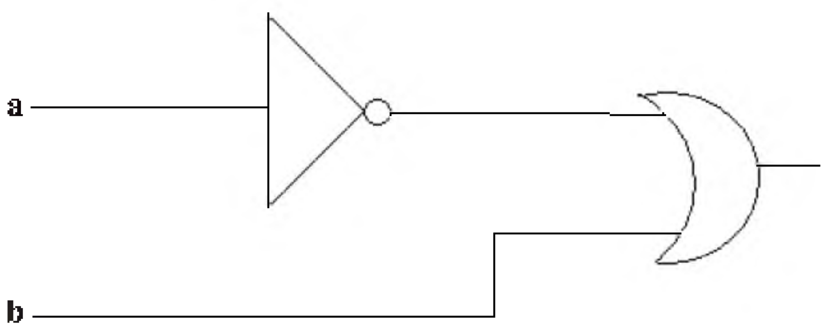


(b)

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \wedge \sim q$	$(\sim p \vee \sim q) \Rightarrow (p \wedge \sim q)$
T	*T*	*F*	*F*	*F*	*F*	*T*
T	*F*	*F*	*T*	*T*	*T*	*T*
F	*T*	*T*	*F*	*T*	*F*	*F*
F	*F*	*T*	*T*	*T*	*F*	*F*

(c)



(d) (i)

Activity	***Earliest Start Time***	***Latest Start Time***	***Float***
A	0	0	0
B	6	6	0
C	16	16	0
D	6	17	11
E	18	19	1
F	18	18	0
G	22	22	0

(ii) *The critical path is: Start, A, B, C, F, G, Finish*

Question 3

This question tested the candidates' ability to

(a) use the cumulative distributive function $F(x) = P(X \leq x)$ to calculate

(i) the constant, k

(ii) the probability between two values

(b) calculate the expected value, variance, median and other quartiles

(c) calculate the probability density function, $f(x)$, from the cumulative distributive function, $F(x)$

The question was fairly well done.

In Part (a), most candidates used integration, instead of just substituting the value for $F(6) = 1$ to arrive at the correct solution.

For Part (a) (ii), most candidates related Part (ii) to Part (i) by using integration. They recognized that they were dealing with the boundary value and hence had to subtract.

Candidates who attempted Part (a) (iii) knew that the median value was 0.5, but did not use the correct formula to find its value. Similarly for Part (a) (iv), many candidates knew that the lower quartile value was 0.25, but did not use the correct formula to find its value.

In Part (b), most candidates recognized that they had to differentiate in order to get a probability density function. However, many of them differentiated the wrong function, and hence got the wrong graph.

For Part (b) (ii), most candidates knew the formulas for $E(X)$ and the $Var(X)$, but used incorrect functions. A few candidates did not subtract the $[E(X)]^2$ when finding $Var(X)$.

In Part (b) (iii), some candidates treated $Var(X)$ and $E(X)$ as being discrete and used the formula $E(X) = \sum xP(X = x)$ rather than $E(X) = \int_{x_1}^{x_2} xf(x)dx$.

Answers

(a) (i) 1/3 (ii) ½ (iii) 4.5 (iv) 3.75 (b) (ii) 9/2 (iii) 3/4

Question 4

This question tested candidates' ability to

(i) solve problems involving probabilities of the Normal distribution using z-scores

(ii) carry out a Chi-square (χ^2) goodness-of-fit test with appropriate number of degrees of freedom. (The situation in this question was modelled by the uniform distribution.)

The question was fairly well done by most candidates.

In Part (a) (i), most candidates were able to get the standardization of the normal distribution, but many could not follow a logical order of steps to get the correct answer. Also, some candidates used a premature approximation of the z value, but they were unable to follow through the solution.

Part (a) (ii) was well done by most candidates and they were able to get at least four out of the five marks.

In Part (b), a few candidates incorrectly used the table as a contingency table. Most candidates were able to get Part (b) (i) correct, with few candidates missing the key terms *independent* and *dependent* in both hypotheses.

The majority of the candidates got Part (b) (ii) correct, but a few candidates incorrectly interpreted the table.

In Part (b) (iii), the majority of candidates read the table incorrectly. Many of them calculated the degrees of freedom correctly, but some used the wrong formula for calculating the value of χ^2. Most candidates had problems writing the region and wrote the critical value instead, that is, instead of writing $\chi^2 > 9.447$, they wrote $\chi^2 = 9.447$. Many candidates did not clearly state their conclusion. They were able to reject the null hypothesis, but could not give a concluding statement such as *time taken is not dependent on distance traveled*.

Answers *(a) (i) 0.241 (ii) 0.204 (b) H_0: the number of employees who work overtime is independent of the distance of their home from the workplace. H_1: the number of employees who work overtime is dependent on the distance of their home from the workplace. (ii) 20 (iii) $\chi^2 = 11.30$*

Question 5

This question tested candidates' ability to use equations of motion for a body moving in a straight line with constant acceleration and to find the velocity of a particle at a given distance from a starting point, given distances and times.

In Part (a), candidates recognized the need to apply an equation of motion for a body moving under the given conditions. However, many candidates encountered difficulty selecting the correct equation to enable them to solve the problem. Many candidates did not appreciate the significance of *constant acceleration* and stated that *distance = speed multiplied by time*.

In Part (b), the problems experienced by candidates were similar to those in Part (a).

Answers

(a) (i) acceleration: 2 ms^{-2}; *(ii) time = 9 secs.*
(b) (i) acceleration: 0.25 ms^{-2}; *(ii) velocity = 5.3 ms^{-1}.*

Question 6

This question tested the candidates' ability to

(a) calculate

(i) the tension in a string

(ii) the angle of inclination of a string to the vertical, given a particle hanging from a fixed point by an elastic string and drawn sideways by a horizontal pull on the particle, with the system in equilibrium.

(b) find the magnitude of

  (i) the acute angle of inclination of the string with the vertical

  (ii) the pull, when the string is about to break given the value of the strain needed to break the string.

(c) Calculate the

  (i) magnitude of a**i** + b**j**

  (ii) angle of inclination of a**i** + b**j** to *Ox* given a particle in equilibrium under the action of forces 5**i**, 3**i** + 6**j**, and a**i** + b**j**.

In Part (a), candidates were able to resolve in two perpendicular directions and to use these two equations to calculate (i) the tension in the string and (ii) the angle of inclination.

Answers *(i) Tension = 10N* *(ii)* *36.9 degrees.*

In Part (b), candidates had little difficulty solving the problem. They were able to resolve in two directions and thereby determined the required values.

Answers *(i) 57.8 degrees* *(ii)* *12.7 N*

For Part (c) (i), candidates recognized that for equilibrium, the vector sum of the forces must be equal to 0. From this the values of a and b were calculated. They were then able to calculate the magnitude as $\sqrt{(a^2 + b^2)} = \sqrt{(8^2 + 6^2)} = 10$

In Part (c) (ii) candidates obtained the value of their angle as 36.9 degrees. However, only a small percentage of the candidates gave the correct answer of (180 + 36.9) degrees as the angle of inclination to Ox. This required the measure of the angle in an anticlockwise direction from the positive direction of Ox.

Paper 03/A – Internal Assessment

UNIT 1

After evaluating the internal assessment projects, it was found that

1. project titles for approximately half of the candidates were too vague and not properly worded. In some projects the titles were missing.

2. in many projects the method of data collection was not sufficiently detailed.

3. candidates included analysis of data in the presentation of data; for example, contingency tables and correlation tables were presented as data. Some of the tables and figures were appropriate for the data.

4. in many projects the symbols used in charts and diagrams were not defined.

5. too often questionnaires and long tests of data were included in the presentation of data. This is unnecessary and should be appropriately placed in the appendices.

6. candidates generally failed to link their findings to the purpose of their projects. In many instances, new information was found in the discussion of findings or the conclusion section which did not relate to the variables under consideration in the project.

UNIT 2

The following relate to internal assessment projects for Unit 2.

1. Generally, candidates demonstrated a high degree of mastery in the mathematical principles pertaining to the syllabus. In most cases, the mathematical analyses were relevant and carried out with few flaws.

2. There was evidence of originality and creativity.

3. Projects were appropriately applied to real-world problems and situations.

4. Over 90 per cent of the candidates were able to effectively communicate information in a logical way using correct grammar and mathematical language.

Some areas of concern were:

1. About 20 per cent of the candidates ignored the stipulated format for the presentation of the project.

2. The statement of the task was not explicit enough in about 30 per cent of the projects.

3. Some of the tables used were not clear; other tables were presented without headings and without reasons for their use.

4. Some candidates presented more data than was need for their analysis.

Recommendations

Candidates have to spend more time solving problems from the Mechanics Module as well as developing the ability to explain concepts.

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

REPORT ON CANDIDATES' WORK IN THE ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2011

APPLIED MATHEMATICS

INTRODUCTION

The revised Applied Mathematics syllabus was examined this year for the third time. This is a two-unit course comprising four papers. Paper 01, consisting of 45 multiple-choice items, Paper 02, consisting of six essay questions and Paper 032 (written by private candidates),made up of three questions, were examined externally while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions from Papers 01, 02 and 032 to each Unit were 30 per cent, 50 per cent and 20 per cent respectively.

Unit 1: Statistical Analysis, tested (i) Collecting and Describing Data; (ii) Managing Uncertainty and (iii) Analysing and Interpreting Data. Unit 2: Mathematical Applications, tested (i) Discrete Mathematics, (ii) Probability and Probability Distributions, and (iii) Particle Mechanics.

GENERAL COMMENTS

In 2011, 580 candidates wrote the Unit 1 examination with and 11 wrote the Alternative to the School-Based paper, Paper 032. For Unit 2, 217 candidates wrote the examination three candidates wrote, Paper 032.

In Unit 1, candidates appeared to be well prepared in Describing and Collecting Data and somewhat in Managing Uncertainty, while Analysing and Interpreting Data seemed to be a challenge for many of them.

In Unit 2, candidates appeared to be well prepared in Discrete Mathematics and Probability and Probability Distributions. Particle Mechanics again seemed to be a challenge for many candidates.

Generally, candidates are still having difficulty with the algebraic manipulations.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of 55 out of 90 with some scores ranging between 12 and 90.

Paper 02 – Essays

Question 1

This question tested candidates' ability to

(a) distinguish between qualitative and quantitative data, and between discrete and continuous data

(b) distinguish between a parameter and a statistic

(c) distinguish among the sampling methods: simple random, stratified, systematic, quota and cluster

(d) outline the differences between simple random and stratified sampling

(e) use systematic sampling and stratified sampling to obtain a sample.

All candidates attempted the question. Eighty per cent of them performed exceptionally well on this question.

In Part (a) candidates performed exceptionally well.

Answers: (i) A, D, E (ii) B, C (iii) A, E (iv) D (v) B, C

Part (b) was well done by the majority of the candidates.

In Part (b) (i) many candidates were able to recognize different sampling methods. However, other terms were used by candidates that were not specified in the syllabus, for example, snow balling and multi-staging method.

Answers: Simple random, systematic, quota, stratified, cluster

In Part (b) (ii) about two-thirds of the candidates, despite the lack of the necessary parameters needed to answer the question, efficiently, were still able to answer satisfactorily.

Answer: n=15

Part (b) (iii) was done exceptionally well by most candidates.

Answer: 8

Part (b) (iv) was poorly answered. Less than one-third of the candidates produced complete answers. The answers were too simplistic and candidates failed to make a proper comparison between the given terms – stratified and simple random sampling.

Answers: In simple random, every member of the population has an equal chance of selection while in stratified the members have equal chance of selection only in strata.

OR

Stratified is more representative of population, while in simple random one stratum may be over or under represented.

Part (c) was done exceptionally well.

Answer: Parameter is a numerical measure calculated from population and statistic is a numerical measure calculated from sample data.

<u>Question 2</u>

This question tested candidates' ability to

(a) construct and use a stem and leaf diagram to display data

(b) calculate the trimmed mean from given data

(c) determine the quartile

(d) calculate the inter−quartile range

This question was attempted by all candidates and the responses were satisfactory.

Part (a) (i) was done satisfactorily. Candidates understood the concept of median, but some of them used the middle value from the *x*-axis.

Answer: 25

In Part (a) (ii), some candidates also used the upper quartile reading and lower quartile reading from the *x*-axis to find the inter-quartile range.

Answer: 9 minutes

In Part (a) (iii), most candidates were able to recognize that 26 persons waited 20 minutes. These candidates were able to follow through to produce the correct answer. However, some candidates read off the value as 23 and were only awarded one mark for subtraction.

Answer: 74 persons waited 20 minutes or more at the clinic.

Candidates generally understood how to construct a stem and leaf diagram as required for Part (b) (i) (a), but had problems with split classes and therefore produced incorrect diagrams.

Answer:

0	*4 4*
0	*7 8 8 9 9*
1	*0 0 0 1 2 2 3 3 3 4 4*
1	*6 6 7 8*
2	*0*
2	*5*
3	*0*
	1

key: 1|6 means 16

In Part (b) (i) (b), 98 per cent of the candidates were able to arrive at the correct mode.

Answers: 10 and 13

In Part (b) (i) (c), 65 per cent of the candidates were able to find the position of the median, but were unable to state the median.

Answer: median is 12

Eighty-five per cent of the candidates were able to calculate the position of the upper quartile and lower quartile in Part (b) (i) (d), however, they subtracted the positions instead of the values.

Answer: Interquartile range = 7

In Part (b) (i) (e), most candidates were able to calculate the 8 per cent trimmed mean, which was $\frac{8}{100} \times 25 = 2$, but had difficulty calculating the mean using $\frac{\sum x}{n} = \frac{260}{21}$. Most candidates knew that two values should be subtracted from both sides of the data, however some of them used n as 23 rather than 21.

Answer: 8% trimmed mean $\rightarrow \frac{8}{100} \times 25 = 2$ *(discard top 2 and bottom 2 numbers)*

8% trimmed mean $= \frac{260}{21} = 12.4$

In Part (b) (i) (f), 90 per cent of the candidates were able to describe the shape of the distribution.

Answer: positively skewed

In Part (b) (ii), the majority of the candidates were able to identify that the mean and range were the two measures which were affected by outliers.

Question 3

This question tested candidates' ability to:

(a) calculate the probabilities $P(x = a)$, $P(x < a)$ where $x \sim Bin(n, p)$

(b) calculate the mean of a binomial distribution

(c) use the normal distribution as an approximation to the binomial distribution where appropriate $np > 5$ and $npq > 5$ and apply a continuity correction.

(d) (i) use a cumulative distribution function table

(ii) compute probabilities

(iii) calculate the expected value of $E(X)$ and $Var(X)$ of a discrete random variable X

Part (a) (i) was attempted by 90 per cent of the candidates, with about 80 per cent giving a satisfactory response. Although many candidates recognized the application of the binomial many of them encountered difficulties using the parameters p and q and the respective indices.

In Part (a) (ii), many candidates were able to recognise that $P(X \geq 4) = 1 - P(X < 4)$ or

$1 - P(X \leq 3)$. However, a few candidates were able to interpret it as $1 - P(X \leq 4)$ and

$1 - P(X < 3)$

Part (b) was generally well done.

In Part (c), many candidates were able to use the normal approximation

$X \sim N(19, 15.2)$ and standardized correctly $P\left(\frac{13.5 - 19}{\sqrt{15.2}} < Z < \frac{21.5 - 19}{\sqrt{15.2}}\right)$

However, some candidates were unable to calculate $\emptyset(-1.411)$ in order to arrive at the correct solution. They encountered difficulties using the standard normal curve and subtracting the required table obtained from 1. This information later assisted them in obtaining $\emptyset(-1.411) = (1 - 0.9208)$.

In Part (d) (i), many candidates did not attempt to construct the probability distribution table; instead they used the cumulative distribution function for the given table. However, 90 per cent of the candidates who attempted to construct the probability distribution table were successful in their attempt and gave the correct answers.

In Part (d), most candidates were able to recognise that the probability of $x>3$ is equivalent to $P(x = 4) + P(x = 5) + P(x = 6)$ or $1 - P(x \leq 3)$. A few candidates did not interpret the $P(x \leq 3)$ correctly and therefore obtained the wrong solution. One example of the incorrect solutions was $P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$.

In Part (d) (iii), many candidates incorrectly used the cumulative distribution function to calculate the value of E(*X*) and Var(*X*). However, approximately 75 per cent of them were able to calculate E(*X*) using the correct method. Additionally a few candidates were unable to recall the correct formula to calculate the Var(*X*). Other candidates interpreted the question incorrectly by constructing a frequency table to calculate E(*X*) and Var(*X*) of the distribution.

Answers

(a) (i) 0.302 (ii) 0.121 (b) 2 (c) 0.660 (d) (i) 0.01, 0.22, 0.41, 0.22, 0.14
(d) (ii) 0.77 (d) (iii) 4.26, 0.972

Question 4

This question tested candidates' ability to:

(a) (i, ii, iv) calculate $P(A \cap B)$ and $P(A \cup B)$

(iii) calculate the conditioned probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$

(v) state whether A and B are independent events, identify independent events

(b) Construct a tree diagram and use the laws of probability to solve problems involving the diagram.

This question was attempted by more than 95 per cent of the candidates; however, only forty per cent of them attained greater than 50 per cent of the marks in the final score.

In Part (a) (i), candidates assumed that the events A and B were independent events and demonstrated a lack of proficiency of conditional probability. Even though candidates encountered difficulties, some were able to deduce and use

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

In Part (a) (ii), most candidates were able to state the formula for $P(A \cup B)$ and therefore use it to obtain the solution.

In Part (a) (iii), candidates demonstrated understanding of conditional probability since approximately 70 per cent of them were able to arrive at the correct solution.

In Part (a) (iv), candidates demonstrated a lack of understanding of $P(\bar{A} \cap B)$ and therefore were unable to give the solution.

In Part (a) (v), the majority of candidates were able to identify that A and B are not independent. However, some candidates were unable to state that

$$P(A).P(B) = 0.6 \times 0.4 = 0.24, \; P(A \cap B) = 0.08$$

In Part (b) (i), most candidates demonstrated a comprehensive understanding of the probability tree and were able to label all the probabilities in their prospective places.

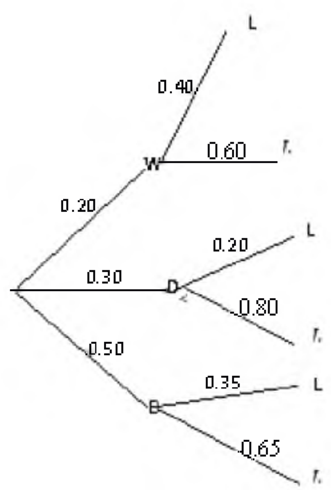
In Part (b) (ii) (a), candidates demonstrated a lack of knowledge in utilizing the tree diagram and deducing a formula. This resulted in candidates' inability to provide a favourable response.

In Part (b) (ii) (b), candidates demonstrated a lack of understanding of $P(B/\bar{L}) = \dfrac{P(B \cap \bar{L})}{P(\bar{L})}$ and were therefore unable to obtain $\dfrac{0.5 \ x \ 0.6}{1 - 0.315} = 0.474$ correct to three significant figures.

Answers

(a) (i) 0.08 (ii) 0.92 (iii) 0.133 (iv) 0.32
(v) $P(A \cap B) = P(A).P(B)$, then A and B are not independent.

(b)(i)



(ii) a) 0.315 b) 0.474

<u>Question 5</u>

This question tested candidates' ability to

(a) draw scatter diagrams to represent bivariate data

(b) calculate the mean of x and y i.e . $\bar{x}$ and $\bar{y}$ and plot on a scatter diagram

(c) calculate equation and draw the regression line y and x passing through $\bar{x}$, $\bar{y}$ on a scatter diagram

(d) calculate and interpret the value of r, the product mean correlation coefficient

(e) and (f) make an estimation using the appropriate regression line.

This question was exceptionally well done.

For Part (a), many candidates were able to plot the scatter diagram accurately. However, candidates *must* make every effort to use materials provided in the exam. Candidates opting to use their own scale inadvertently complicated the question and may have lost marks due to inaccuracies in scale.

In Part (b), calculation of $\bar{x}$ and $\bar{y}$ was very well done. Some candidates failed to realize that the summary statistics had been provided and calculated required statistics from raw data. Most candidates were successful in plotting $(\bar{x}, \bar{y})$.

For Part (c), the majority of candidates were able to identify the equation y on x as being $y = a + bx$ and most candidates were able to substitute correctly into the equation. Inconsistencies in approximated values were the major reason for loss of marks. Candidates must read the instructions of the paper and give answers to *three significant figures unless otherwise specified.* Many candidates failed to draw the regression line on their graph.

Many candidates showed competency in stating the formulae for finding r and substituting accurately into that equation for Part (*d*). The main reason for loss of marks was the incorrect substitution of $n = 20$. Though many candidates were successful in finding $r = 0.965$, the majority of candidates were unable to interpret this value as strong positive correlation. Just stating positive correlation was insufficient to obtain the mark assigned.

Part (e) was generally well done as most candidates were able to correctly identify the required equation and substitute $x = 45$ accurately to get $y = 43.5 \approx 44$ years.

Part (f) was well done, however, approximately 40 per cent of candidates were unable to state that the regression line obtained was unreliable as the required value, $x = 45$, was outside the range of values given in the table.

Answers:

(b) $\bar{x} = 28.6$ and $\bar{y} = 28.8$
(c) $y = 3.15 + 0.897x$
(d) $r = 0.965$ which implies *strong positive* correlation
(e) $y = 43.5 \approx 44\,years$
(f) This value is not reliable as $x = 45$ lies outside the range of values given for x.

Question 6

This question tested candidates' ability to

(a) apply the central limit theorem where n ≥ 30 and use the fact that Var $\bar{X} = \frac{\sigma^2}{n}$ where $\bar{X}$ is the sample mean, μ the population mean and n the sample size.

(b) determine the degree of freedom for a given test and level of significance

(c) (i) formulate a null hypothesis H_0 and an alternate H_1
(ii) calculate the expected frequencies, E, when the null hypothesis is true
(iii) (a) determine the appropriate number of degrees
(b) determine the critical region for a given test
(c) determine the test statistic
(iv) give a valid conclusion of the test

Part (a) was well done by the majority of the candidates who recognized the need to use the central limit theorem.

For Part (b) most candidates correctly obtained the degrees of freedom. There were a few candidates who were unable to determine the level of significance. They mistook the *p* value = 0.95 to be the level of significance.

In Part (c)(i), about 50 per cent of the candidates were not sure how to state the null and alternate hypotheses. Most of these candidates then had inaccurate conclusions.

Part (c)(ii) was generally well done. Most candidates calculated the expected values correctly. There were a few candidates who did not state the formula and gave answers to more than one decimal place or rounded off their answers to the nearest whole number. This resulted in the total sample size being more than 300.

Candidates need to be reminded that final answers should give the same result as the observed values.

More that 90 per cent of the candidates correctly calculated the degrees of freedom correctly.

In Part (c)(iii)(b), about 40 per cent of the candidates did not read the critical value correctly which resulted in their critical region being incorrect. The value of the test statistic, Part (c)(iii) c) was also incorrectly, calculated mainly because candidates' expected values were incorrect.

Most candidates were able to state the conclusion clearly, for Part (c)(iii)(iv). However, their reasons were unclear for the most part. This is an area that needs attention.

Answers

(a) $\bar{X} \approx N\left(100, \frac{16}{50}\right)$ (b) 4 d.f. ; 5% s.f.

(c) H_0: no association between gender and response of school teachers
$\mathbf{H}_1$: there is an association between gender and response

Observed values:

	In favour	against	No opinion	total
Male	102	62	11	175
female	73	45	7	125
total	175	107	18	300

(iii) a) 2; b) c.r. $\chi^2 > 5.991$ c) $\chi^2_{calc} = 10.995$

(iv) reject H_0

Paper 03/A -Internal Assessment

The internal assessment for Applied Mathematics tested candidates' ability to apply the theories learnt in the syllabus to real−world situations.

After evaluating the internal assessment projects, it was found that:

1. Innovation and creativity were lacking in over 50 per cent of the samples as candidates often chose topics that were used by fellow classmates.

2. Project titles were often vague.

3. Some candidates did not list the variables in the purpose section.

4. Some candidates misused the sampling technique in the method of data collection section. For example, in comparisons of gender, simple random sampling was used when stratified random sampling was the more appropriate technique to use.

5. In many instances, charts, graphs and diagrams were given which were not relevant to the purpose of the project.

6. Questionnaires and long tables of data were unnecessarily included in the presentation of data in about 20 per cent of the samples. These should be appended at the end of the projects.

Areas of Strengths and Weaknesses Identified

Strengths

1. Recalling basic definitions, for example, a sample, a population
2. Drawing a stem and leaf diagram
3. Reading χ^2 and t−tables
4. Standardizing values using the normal distribution
5. Recognizing when the binomial distribution is to be used

Weaknesses

1. Candidates were unable to obtain a sample mean given the end points of a confidence interval
2. Candidates were unable to define a critical region given the critical value
3. Candidates incorrectly interchanged the H_0 and the H_1 for the χ^2 test

UNIT 2

Paper 01 – Multiple Choice

Performance on the forty-five multiple-choice items on Paper 01 produced a mean of 61 out of 90 with scores ranging between 18 and 90.

Paper 02 – Essays

Question 1

This question tested the candidates' ability to:

(a) formulate conditional propositions

(b) derive a Boolean expression from a given logic circuit,

(c) represent a Boolean expression by a switching circuit.

(d) establish the truth value of

- negation of simple propositions,
- compound propositions that involve conjunctions, disjunctions and negations,
- conditional and bi conditional propositions,
- use truth tables to determine if propositions are equivalent

(e) use the laws of Boolean Algebra (de Morgan's Law) to simplify Boolean expressions

All candidates attempted this question but only about 40 per cent attained a score greater than 20.

Part (a) was fairly well done. All the candidates who attempted this section successfully filled out the truth table. However, a few candidates did not comment on the equivalence of p and $(\sim p \vee \sim q) \Rightarrow (p \wedge \sim q)$.

Part (b) was fairly well done. Most candidates were able to write the Boolean expression for the logic circuit whilst some used the incorrect logic notation (+ and .) for (**or** and **and**). Candidates should ensure that the correct notations are used.

Part (c) was satisfactorily done. Candidates were unable to distinguish between a switching and a logic circuit.

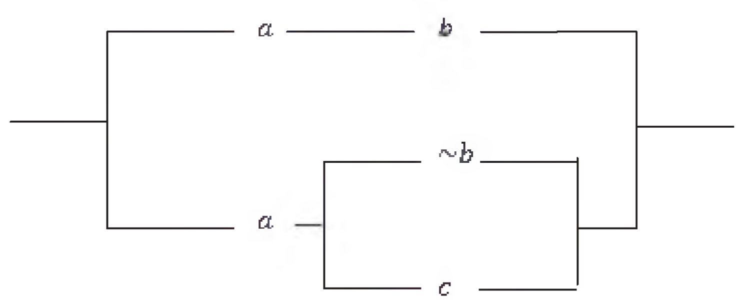
Part (d) was very well done. Candidates were able to convert statements to a Boolean expression.

Part (e) was poorly done. Candidates were unable to apply the laws of Boolean algebra to prove the identity.

Answers:

(b) $[p \wedge (q \vee \sim r)] \vee r$

(c)



(d) (i) $p \Longrightarrow q$ (ii) $\sim q \Longrightarrow \sim p$

Question 2

This question tested candidates' ability to:

(a) solve a minimization assignment problem (4×4) by the Hungarian Algorithm

(b)(i) use the activity network algorithm to draw a network diagram to model a real-world problem

(ii) calculate the earliest start time, latest start time and float time

(iii) identify the critical path in an activity network

This question was well done. Candidates attempting Part (a) were knowledgeable of the Hungarian algorithm, however, some candidates did not apply the concept of stopping the algorithm when the number of lines of shading is equal to the dimension of the matrix.

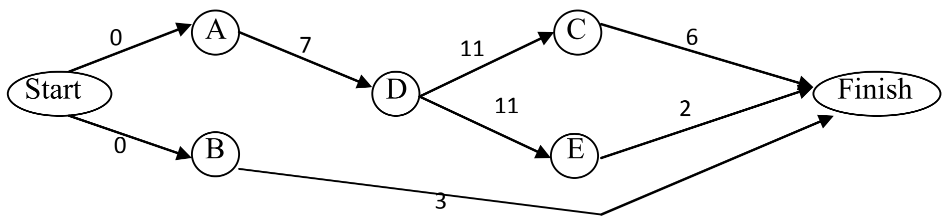
In Part (b) (i), some candidates drew two unnecessary edges (AE and AC) since D was preceded by A. The labelling of the duration and activity in the network were not done correctly by some candidates. Candidates need to be reminded that the START and FINISH nodes should be included in the network diagram.

In Part (b) (ii), candidates encountered difficulty in calculating the latest start time and identifying the order of the critical path.

Answers:

(a) A – 1, B – 4, C – 3, D – 2.

(b) (i)



(ii)

(iii)

Activity	Earliest Start Time	Latest Start Time	Float
A	0	0	0
B	0	21	21
C	18	18	0
D	7	7	0
E	18	22	4

(iii)(a) Start – A – D – C – Finish (b) 24 hours

<u>Question 3</u>

This question tested candidates' ability to

(a) calculate the number of selections of n distinct objects taken r at a time

(b) calculate the number of ordered arrangements on n objects taken r at a time

(c) identify probability models, the geometric distribution

(d) calculate the E(X) for the geometric distribution

Part (a) (i) was well done except for a few candidates who did not divide by the 3. Some candidates treated all "M's" individually. Some of them were not sure and did both.

Part (a) (ii) was partially done by the majority of candidates. The most common error was either not multiplying by ‘3’ or not dividing by ‘3!’. Several candidates again treated each “M” individually. A few candidates, instead of multiplying by ‘3’, multiplied by ‘3!’.

Part (a) (iii) was well done. The most common error was doing ‘5! x 4!’.

Part (b) was well done except for some candidates who did “${}^{n}C_{r} + {}^{m}C_{s}$” instead of “${}^{n}C_{r} \times {}^{m}C_{s}$.

Part (c) (i) was poorly done since candidates did not recognize the distribution as ‘Geometric’ and/or did not use the appropriate parameter.

Part (c) (ii) (a) was fairly well done.

In Part (b), candidates interpreted the question wrongly and solved for ‘greater than 3’ and not ‘greater than 2’.

In Part (c), many candidates interpreted the question wrongly and solved for ‘less than or equal to 4’ instead of ‘less than or equal to 3’.

Part (d) was well done.

Answers:

(a) (i) 840 (ii) 360 (iii) 120

(b) 209

(c) (i) X ~ Geo(1/3) (ii) a) 2/9 (b) 4/9 (c) 19/27 (d) 3

Question 4

The question tested candidates’ ability to:

(a) calculate probabilities using the Poisson distribution

(b) calculate probabilities using the binomial distribution

(c) calculate probabilities using appropriate counting techniques

Part (a) (i) was fairly well done. Most errors were due to candidates using an incorrect or incomplete formula. ‘γ^{x}’ many times treated as ‘γx’.

In Part (a) (ii), several candidates were unable to correctly interpret the inequality.

In Part (a) (iii), candidates failed to use the appropriate parameter for fortnight and continued using the parameter given for weekly average.

In Part (b), most candidates identified the correct distribution and parameters. Few candidates failed to find 2/3 of 12 and proceeded to solve using the value of X as 2/3 instead of using a value of 8. Again, a few candidates were unable to interpret the inequality correctly. Most candidates were aware of how to use the Binomial formula.

In Part (c) (i), most candidates obtained full marks. The few candidates who did not, treated the problem as though the marbles were being replaced.

Part (c) (ii) was poorly done since candidates struggled to obtain the correct amount of arrangements that there were thus resulting in loss of marks.

Part (c) (iii) posed a challenge to candidates since many of them failed to obtain the correct amount of arrangements and their corresponding probabilities.

Answers

(a) (i) 0.0498 (ii) 0.353 (iii) 0.161

(b) 0.8885

(c) (i) 1/21 (ii) 2/7 (iii) 4/9

Question 5

This question tested candidates' ability to

(a) apply Newton's laws of motion to
- a particle moving along an incline plane with constant acceleration,
- a system of two connected particles
- resolve forces on an inclined plane.

(b) solve problems involving power

This question was not well done since only 20 per cent of the candidates attained a mark greater than 20. Most candidates attempted Part (a). Some candidates did not recognize that $\sin^{-1}\left(\frac{1}{4.4}\right)$ was the angle.

In Part (a) (i) (a), some candidates did not include the normal reaction on the inclined plane.

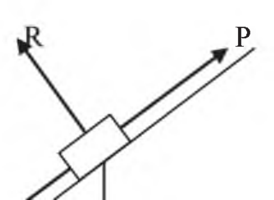
For Part (a) (ii) (b), some candidates did not convert the speed to ms^{-1}.

In Part (a) (ii) (c), candidates were familiar with the equation to compute Power but used the initial velocity instead of the final velocity.

Part (b) was fairly well done.

Answers:

(a) (i)



200N

θ 1100g N

(ii) a) $0.5\ ms^{-2}$ b) $3250\ N$ c) 65kW

(b) (i) $\frac{10}{9}ms^{-2}$ (ii) $\frac{400}{9}N$

Question 6

This question tested candidates' ability to:

(a) draw and use velocity – time graphs

(b) calculate the work done by a constant force.

This question was attempted by most candidates with only ten per cent attaining marks greater than 20.

In Part (a) (i), most candidates were able to interpret the given information to produce the graph.

For Part (a) (ii), three methods were used to answer this question but the majority of the candidates were unsure of what was required. Candidates should be consistent with the units $(minutes \rightarrow hrs/s)$.

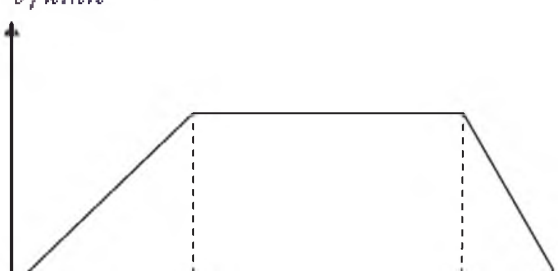
In Part (b) (i), candidates were familiar with the formula to compute work done but some used the incorrect component of the force.

For Part (b) (ii), candidates were unable to resolve the forces on the inclined plane. In addition, candidates were unable to apply the formula:

Work done = Work done against gravity + work done against frictional force

Answers:

(a) (i) v/kmh^{-1}



v

0.5km 1.5 km 0.3 km

O $\frac{1}{10}$ t/h

(ii) $961\ kmh^{-2}$

(b) (i) $276\ J$ (ii) $239N$

UNIT 2

Paper 03/A – Internal Assessment

1. Statement of Task

   - Needs to be more specific
   - Must include a definition of the variables

2. Method of Data Collection

   - Most candidates did not understand what it means to collect data

3. Evaluation

   - Approximately ten per cent of candidates did not state a conclusion.
   - Conclusions need to be more precise and related to the analysis and statement of task

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2012

APPLIED MATHEMATICS

GENERAL COMMENTS

The revised Applied Mathematics syllabus was examined this year for the fourth time. This is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple choice items, and Paper 02, consisting of six essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each unit were 30 per cent, 50 per cent and 20 per cent, respectively.

Unit 1, Statistical Analysis, tested Collecting and Describing Data, Managing Uncertainty and Analysing and Interpreting Data.

Unit 2, Mathematical Applications, tested Discrete Mathematics, Probability and Probability Distributions and Particle Mechanics.

In 2012, the number of candidates writing the examinations for Units 1 and 2 were 672 and 237, respectively.

Approximately 80 per cent of the candidates registered for Unit 1, Statistical Analysis, and 83 per cent of the candidates registered for Unit 2, Mathematical Applications, obtained acceptable grades, I–V. The standard of work seen from most of the candidates in this examination was satisfactory.

In Unit 1, candidates appeared to be well prepared in Describing and Collecting Data and somewhat in Managing Uncertainty, while Analysing and Interpreting Data seemed to be a challenge for many candidates.

In Unit 2, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions. Particle Mechanics again seemed to be a challenge for many candidates.

Generally, candidates are still having difficulty with algebraic manipulations.

Overall, for Paper 031, the School-Based Assessment (SBA), students demonstrated a high degree of mastery in the mathematical principles pertaining to the syllabus. In most cases, the mathematical analyses were relevant and carried out with few flaws.

There was evidence of originality and creativity and projects were appropriately applied to real-world problems and situations. Over 90 per cent of the students were able to effectively communicate information in a logical way using correct grammar and mathematical language.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Performance on the 45 multiple choice items on Paper 01 produced a mean score of 57.15, standard deviation of 18.15 and scores ranging from 14 to 90.

Paper 02 – Essay

Module 1: Collecting and Describing Data

<u>Question 1</u>

This tested the candidates' ability to

- distinguish between population and a sample, a census and a sample survey, a parameter and a statistic
- define a sample frame
- distinguish between random and non-random samples
- identify different sampling methods
- calculate the number of items in a sample
- construct a stem and leaf diagram
- calculate the inter-quartile range
- determine the median from a set of data.

Though this question was generally well done, many candidates could not identify cluster sampling. Most candidates were able to identify systematic sampling.

Candidates had difficulty explaining the advantages of using different methods of sampling.

Most candidates were able to calculate the number of workers in the sample using option B.

For Part (b), many candidates could not give a definition for a sampling frame. For Part (c), the stem and leaf diagram was generally well done, but then candidates could not state the advantage of using the stem and leaf diagram.

In determining the median, candidates were able to state the location of the median, but did not state the median.

Most candidates knew how to find the IQR, but many of them read the values for Q_1 and Q_3 incorrectly.

Answers

(a) (i) A: Cluster sampling, non-random

B: Stratified sampling, random

C: Systematic sampling, random

(ii) Not all workers will have a chance of being selected

(iii) Possibility of bias

(iv) 16 workers.

(b) (i) Sampling frame is a set of elements from the population from which the sample is drawn

(ii) Households may not have a telephone

(ii) Only persons listening to the programme will call

(c) (i)

Stem	Leaf
4	0 2 7
5	3 6 7 8 8 9 9
6	0 1 2 3 4 6 7 9
7	1 4 4 5
8	1 2 2

key: 7|4 = 74

(ii) All data values are preserved

(iii) Median = 62; IQR = 15

Question 2

This question tested candidates' ability to

- identify a population, census, sample, statistic, parameter
- determine the mode and median from a frequency distribution
- calculate the mean and the standard deviation from a frequency distribution
- use cumulative frequency curves.

Calculation of the standard deviation was a problem since many candidates could not correctly quote the formula for standard deviation.

Candidates also had problems identifying the normal distribution. Candidates used descriptions such as 'symmetric to left and to the right', 'normally symmetric', 'left skewed normally'.

Whereas candidates could read values from the graph, using the graph to calculate values proved to be a challenge.

Answers

(a) Population, census, sample, statistics, parameter

(b) (i) Mode = 3, Median = 3

(ii) Mean = 3; SD = 1.48

(iii) The distribution is normal

(c) 350; 35; 7.14 per cent IQR = 14; x = 36.5 minutes

Module 2

Question 3

This question tested candidates' ability to

- calculate the probability of (i) the union of two events: $P(P \cup Q)$ and (ii) the conditional probability of two events $P(P|Q)$
- identify (i) independent and (ii) mutually exclusive events
- solve probability problems
- use a probability distribution table to obtain probabilities.

Parts (a) (i) a) was attempted by approximately 96 per cent of the candidates, with about 98 per cent giving a satisfactory response.

Part (a) (i) b) was attempted by approximately 96 per cent of candidates, with about 85 per cent giving a satisfactory response. Some candidates gave the incorrect definition for $P(R|Q) = \frac{P(R \cap Q)}{P(R)} \, or \, \frac{P(R)}{P(Q)} \, or \, \frac{P(Q)}{P(R)}$.

Parts (a) (ii) a) and b) were attempted by approximately 96 per cent of candidates, with about 70 per cent giving a satisfactory response. Many candidates were unable to state their answers in clear mathematical terminology (jargon) and provide correct reasons to support their answers.

Parts (b) (i) a) and b) were attempted by approximately 90 per cent of candidates, with less than 50 per cent providing satisfactory responses. Many candidates were unable to distinguish between Parts (a) and (b). This indicated that candidates lack proper understanding of the concept $P(F' \cap E')$ and $P(E\,only\,or\,F\,only)$.

Part (b) (i) c) was attempted by approximately 90 per cent of candidates, with more than 80 per cent providing satisfactory responses. However, many candidates did not approximate correctly to three significant figures.

For Part (b) (ii), most candidates were unable to identify the concept of combined probability; as a result, less than 30 per cent of candidates who attempted this part provided satisfactory responses.

Parts (c) (i) and (ii) were generally well done, with approximately 96 per cent of candidates attempting and 98 per cent giving satisfactory responses. However, a few candidates were unable to identify and use the correct formula for the mean; they attempted to divide the mean by the total number of orders.

Answers

(a) (i) 0.88; 0.2 (ii) Not independent; Not mutually exclusive

(b) (i) 0.07; 0.65; 0.483 (ii) 0.21

(c) (i) 0.45 (ii) 6.48

<u>Question 4</u>

This question tested candidates' ability to

- identify a binomial distribution and state its parameters
- calculate probability using the binomial distribution
- use the normal distribution to calculate probabilities.

Most candidates attempted Part (a) (i); however, the parameters in some cases were either interchanged or incorrect. Some candidates were unable to state the distribution as binomial, using normal notation instead.

In Part (a) (ii) a), most candidates attempted to use the formula; however, values used were incorrectly substituted. In some cases, candidates incorrectly stated the formula $P(X = r) = C_r^n\, p^r q^{n-r}$.

For Part (a) (ii) b), several candidates failed to use the property $P(X \geq 1) = 1 - P(X < 1)$ or $1 - P(X = 0)$. In many instances $1 - P(X = 1)$ or $1 - [P(X = 0) + P(X = 1)]$ was used.

The majority of candidates attempted Part (a) (iii) but did not do the entire question. They were able to correctly use the formula for $E[X]$.

For Part (b) (i), most candidates had a general understanding of standardizing; however, some used the wrong values at times. Candidates who demonstrated an understanding of this concept rounded off their values too early; this had an effect on their final answers. Some also used continuity correction which was unnecessary. The remaining set of candidates was unable to arrive at the correct answer because of standardizing using the variance rather than standard deviation or finding the root of 14.5 in a few cases. Also, some candidates did not know to subtract their value from 1. Another common mistake was misreading the values from the table or adding values incorrectly; for instance, for $\phi(1.379) = 0.9147 + 0.0014$ the expression 0.9147 + 0.14 was frequently seen.

In Part (b) (ii), some candidates were able to formulate the statement $P(X > c) = 0.20$ and standardize; however, others read the table for $\phi(0.2)$ or even if they arrived at the correct conclusion $\phi^{-1}(0.8)$, they could not read the tables successfully. Candidates were able to derive the equation $c = (0.842 \times 14.5) + 630$, but in the other cases $c = 630 - (0.842 \times 14.5)$ was seen.

Most candidates failed to accurately complete Part (b) (iii). Again, continuity correction was commonly seen and many candidates were able to standardize but as in Part (b) (i), candidates rounded off too early so again table values were incorrect. Also, candidates failed to demonstrate an understanding of solving probabilities using the normal distribution. For instance, $P(-1.379 < Z < 1.379) = 2(1 - \phi(1.379))$ was frequently observed.

Very few candidates had difficulty understanding what was required for Part (b) (iv). The answer calculated in Part (b) (iii) was correctly used to solve this part of the question.

Answers

(a) Bin (12, 0.45); 0.0923; 0.999; 18

(b) 0.0839; 642.2; 0.8332; 54.

Module 3

Question 5

The question tested candidates' ability to

- state the mean, variance and distribution of the mean, $\bar{x}$ of the sample
- calculate the probabilities that the sample mean is less than a given value
- calculate a 98 per cent confidence interval for the mean length
- calculate an unbiased estimate for the standard deviation
- state the condition necessary for a valid use of a t-test to test the hypothesis about the mean of x
- state the null hypothesis H_0 and an alternative hypothesis H_1
- apply a one-tailed test
- determine the critical value from tables for a given test and level of significance
- identify the critical region for a given test
- state a valid conclusion of the given test.

In Part (a) (i), many candidates were able to correctly identify the normal distribution and state the mean. However, they had difficulty identifying the variance.

In Part (a) (ii), many candidates used the incorrect value $\frac{\delta^2}{\sqrt{n}}$ rather than $\frac{\delta}{\sqrt{n}}$. However, most of them were able to read off the table values as well as apply $1-\phi(z)$ to calculate the probabilities.

In Part (b), many candidates used a one-tailed test as opposed to the two-tailed test. Many candidates also used the variance instead of the standard deviation. As a result, many candidates did not obtain the interval (75.73, 76.07).

In Part (c) (i), many candidates were able to write a correct formula. However, depending on the formula that they used, they did not use the $\frac{n}{n-1}$ factor. However, most candidates found the square root of their answer.

Part (c) (ii) was well answered and most candidates were able to identify two conditions necessary for the t-test; many candidates did not state that the distribution must be normal.

For Part (c) (iii) a), most candidates were able to set up the hypothesis correctly but neglected to test for μ, for example, $H_0 : x = 3.5$ and $H_1 : x < 3.5$. Many candidates used statements rather than symbols, but did not state that it was the mean battery life that was being tested. Several candidates used a two-tailed test.

In Part (c) (iii) b), many candidates used their unbiased estimation from Part (c) (i) and obtained an incorrect t_{calc}. Several candidates used $\frac{\hat{\sigma}^2}{\sqrt{8}}$ in the denominator. Several candidates obtained a positive value for t_{calc} because they used (3.5 - 3.4).

In Part (c) (iii), although many candidates were able to identify the correct degrees of freedom and read the correct table value of 1.895, they did not identify the critical region.
In Part (c) (iii), most candidates were able to state whether the null hypothesis was rejected or accepted, but could not interpret this with a valid conclusion.

Answers

(a) $\bar{X} \approx N\left(420, \dfrac{12.7^2}{49}\right)$; 0.0491

(b) (75.73, 76.07)

(c) (i) 0.2; (ii) Small sample size, variance unknown, normal distribution (iii) Accept H_0: $\mu = 3.5$; the advertisement is not overstating μ

Question 6

This question tested candidates' ability to

- formulate a null hypothesis H_0, and an alternative hypothesis H_1
- relate the level of significance of the probability of rejecting H_0 given that H_0 is true
- determine the critical values from tables for a given test and level of significance
- determine the degrees of freedom
- determine probabilities from χ^2- tables
- apply a χ^2 test for independence in a contingency table
- draw scatter diagrams to represent bivariate data
- give a practical interpretation of the regression coefficient
- draw the regression line of y on x passing through $(\bar{x}, \bar{y})$ on a scatter diagram
- make estimations using the appropriate regression line.

This question was attempted by 95 per cent of the candidates. Eighty per cent of them performed exceptionally well on this question.

For Part (a) (i), most candidates gave appropriate responses for the hypothesis. Few of the candidates however misrepresented the null and the alternative hypothesis.

In Part (a) (ii), most candidates demonstrated the ability to calculate the degree of freedom using the correct formula. Some candidates included the totals in their rows and columns during their calculations.

In Part (a) (iii), candidates demonstrated their understanding of the critical region using either a graphical or calculated approach. Some candidates, after identifying the critical value, did not convert this to a critical region.

In Part (a) (iv), the majority of candidates was able to complete the table effectively. For Part (a) (v), most candidates were able to give an appropriate conclusion using their results from Part (a) (i) and (ii).

In Part (b) (i), the majority of candidates demonstrated their ability to plot six or more points correctly. For Part (b) (ii) a), only a minority of candidates were able to demonstrate their understanding of the *x* coefficient and its relationship to the rainfall at the two stations. In Part (b) (ii) b), most candidates were able to state the formula for mean and use it correctly.

In Part (c), candidates were able to plot the mean on their graph but a significantly large number of candidates were unable to draw the regression line correctly.

In Part (d), candidates demonstrated their knowledge to find *y* given *x* either graphically or by calculations.

Answers

(a) $\chi^2 \geq 5.991$, (v) reject H_0; opinion is dependent of sex.

(b) (ii) For every 1 cm increase of rainfall at Station A there will be a 0.8 cm increase at Station B

(iii) Mean rainfall at A = 4.3; Mean rainfall at B = 3.87

(c) y = 3.98

Paper 031 – School-Based Assessment (SBA)

Statistical Analysis

Approximately 80 per cent of students were able to score marks for project title and purpose. If they did lose marks, it was because they failed to clearly state the variables used in the study. Students are reminded to be concise in the statement of purpose. In some cases, the use of statistical theory was limited by students' choice of topic. Students also displayed a lack of creativity in topic selection.

A few students failed to secure the two marks allotted for data collection. In some cases the method selected was not appropriate and often when appropriate, students failed to adequately describe their choice. Most were able to describe the general method and select an appropriate method but failed to describe its application adequately.

Students were able to get the three marks allotted to data presentation and demonstrated adequate expertise in the use of statistical language, jargon and symbols. Marks were lost because tables and charts were not labelled and in some cases were ambiguous.

Most students demonstrated fair statistical knowledge. Minor inaccuracies of statistical concepts were identified and often overlooked by teachers. Teachers need to pay special attention to the calculation of averages as well as the expected values for the chi-squared distribution. In most cases, there was a logical flow of data and this was extended to the discussion.

Approximately half of the students failed to relate their findings to the purpose of the project, although all the calculations would have been completed. In many cases, this section was too wordy. Students need to be concise in their conclusion. The conclusion must also be clearly stated and must be related to the purpose of the project.

Students who refer to websites must make an effort to follow the accepted convention in a consistent manner. Students must also make an effort to select reliable websites. Most books listed were properly referenced. Students must endeavour to use multiple references.

Students' choice of font style and size made projects very difficult for reading. They are urged to follow conventional formatting used in academic papers. (Times New Roman, font size 12, double spacing, etc.)

Areas of Strengths and Weakness

Strengths

- Recalling of basic definitions, for example, a sample, a population
- Drawing a stem and leaf diagram
- Reading values from the normal distribution, the chi square and the t-tables
- Standardizing the normal distribution
- Recognizing when the binomial distribution is to be used.

Weaknesses

- Using independent events inappropriately
- Calculating unbiased estimate of the standard deviation
- Being unable to obtain a sample mean given the end points of a confidence interval
- Being unable to define a critical region given the critical value
- Interchanging the H_0 and the H_1 for the χ^2 test
- Determining when to use the t-test rather than the Z test
- Failing to use three significant figures when giving answers.

Paper 032 – Alternative to School-Based Assessment

Scripts for ten candidates were marked and the overall performance was satisfactory.

Question 1

This question tested candidates' ability to

- explain why sampling is necessary
- determine what constitutes a population
- identify sampling methods
- distinguish between qualitative and quantitative data
- construct a histogram from a grouped frequency distribution
- determine the mode and median from grouped frequency distribution
- calculate the mean and trimmed mean from a sample.

Marks for this question ranged from 7 to 17 with half of the candidates earning fewer than ten marks and the other half earning ten or more marks. Completing the histogram and calculating the mean were generally well done. Although candidates could easily identify the modal and median class, they had difficulty stating the actual mode and median. Most candidates had no idea what *trimmed mean* meant as they simply found ten per cent of the mean calculated in the previous section of the question.

Answers

1. (a) (i) Impossible to identify the entire population

(ii) The ice-cream lovers in the island

(iii) Cluster

(iv) Qualitative

(b) (i) Individual data values are lost

(iii) Mode = 54.5 (iv) Median = 54.5

(c) (i) Mean = 218.47 (ii) Trimmed mean = 191.72

Question 2

This question tested candidates' ability to

- list elements of a probability space
- identify elements of an event
- calculate the probability of event
- recognise the conditions for using the binomial distribution
- use the binomial distribution to calculate probability
- use the normal distributions as an approximation for the binomial distribution.

For this question, marks ranged from 0 to 20. Four candidates performed poorly, obtaining either zero or two marks and one candidate failed to respond to this question. The other five candidates performed well with scores ranging from 14 to the maximum possible marks obtainable, 20. Candidates who performed poorly obviously had no knowledge of this topic. Candidates who did well generally had difficulty with Part (b) (ii) – using an appropriate approximation for finding $P(x \geq 340)$.

Answers

(a) (i) 0.049 (ii) 0.347 (iii) 0.189

(b) (i) 0.240 (ii) 0.0032

Question 3

This question tested candidates' ability to

- calculate unbiased estimates for the mean and the variance
- formulate null and alternative hypotheses
- evaluate the t-test statistic
- apply the t-distribution in a hypothesis test of the mean and determine the validity of the t-test
- use the regression line to estimate values of y
- calculate and interpret the product moment correlation coefficient.

Marks awarded for this question ranged from 1 to 20 with one candidate not providing a response. Three candidates performed very well with scores of 18 or more, while all other candidates got awarded scores of ten or fewer than ten. From the three questions in the paper, this question had the poorest performance. The best performance was seen in Parts (a) (i) a) and (b) (i). Candidates failed to differentiate between working with a sample and working with a population. Although candidates had the formula and values for Part (b), these made errors in substitution and simplification.

Answers

(a) (i) Mean = 13.33 ; SD = 5.35

(ii) Accept H_0

(iii) Distribution must be normal

(b) (i) y = 48.35 (ii) r = 0.51; moderate, positive

The questions in this paper were of appropriate difficulty; adequate time was given to candidates to complete the examination and the marks were appropriately allotted to the various items. It is evident that some candidates were prepared for this paper while others were unprepared and therefore performed poorly.

UNIT 2

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean score of 58.42, standard deviation of 17.40 and scores ranging from 21 to 90.

Paper 02 – Essay

Question 1

This question tested candidates' ability to

- derive and graph linear inequalities in two variables
- determine whether a selected trial point satisfies a given inequality
- determine the solution set that satisfies a set of linear inequalities in two variables
- determine the feasible region of a linear programming
- identify the objective function and constraints of a linear programming problem
- determine a unique optimal solution (where it exists) of a linear programming problem
- formulate linear programming models in two variables from real-world data.

This question was attempted by the majority of candidates with about 60 per cent of them providing satisfactory responses and 20 per cent getting a mark over 19.

In Part (a), few candidates achieved full marks as they were unable to define the variables x and y as the number of backhoes of Type *A* and Type *B* respectively. Candidates were also unable to reproduce the correct linear programming model with some candidates forgetting to include the non-negative constraints, as well as to 'maximize' the objective function.

In Part (b), many candidates were unable to represent the equations of the straight lines on the Cartesian plane accurately. They were, however, able to shade the feasible region based on the lines that were drawn.

Part (c) posed problems for candidates as they read the coordinates of the vertices inaccurately, (0, 15) instead of (0, 16). Also one of the vertices $(16\frac{2}{3}, 8\frac{1}{3})$ did not contain non-negative integers. Candidates approximated this vertex to (17, 8) which was found to lie outside the feasible region. Many candidates were, however, able to attain the correct number of each type of backhoe to give a maximum weight of the soil to be removed and the maximum weight of the soil.

Answers

(a) (i) Maximize: $P = 40x + 60y$ where x is the number of backhoes of Type A and y is the number of backhoes of Type B

(ii) $50x + 20y \leq 1000$; $2x + 8y \leq 128$; $x + y \leq 25$; $x \geq 0$; $y \geq 0$

(c) (i) 12 backhoes of Type A and 13 backhoes of Type B

(ii) The maximum weight of soil removed in one day = 1260 tonnes

Question 2

This question tested candidates' ability to

- formulate simple propositions
- determine if propositions are equivalent using truth tables
- use the laws of Boolean algebra to simplify Boolean expressions
- represent a Boolean expression by a switching or logic circuit.

Part (a) (i) was well done by the candidates. Most candidates obtained full marks. Part (a) (ii) was also well done by the majority of candidates.

For Part (b), many candidates found it difficult to determine the truth value. Candidates used truth tables to determine the answer. However, this method was not required. Candidates need to spend more time with this type of proof.

For Part (c) (i), 80 per cent of the candidates were able to assign propositional statements. However, only about 60 per cent were able to express the statement in symbolic form. Problems encountered were missing brackets and incorrectly placed brackets.

For Part (c) (ii), candidates encountered problems simplifying logic symbolic statements and did not change the OR to AND.

Candidates understood the concept of "NOT (NOT p)".

For Part (d) (i), the majority of candidates obtained the correct solution. Many candidates recognized the solution by carefully observing the diagram.

For Part (d) (ii), about 80 per cent of the candidates were able to construct the truth table from Part d (i) above. Simple mistakes were made in obtaining "$\sim B \vee C$". Hence, this affected their final result.

For Part (e), most candidates were able to draw the three logic gates. Weaker candidates did not realize that the brackets should have been worked first in order to obtain the final correct solution.

Answers

(a) (i)

p	q	r	$q \wedge r$	$p \Rightarrow (q \wedge r)$	$p \Rightarrow r$	$p \Rightarrow q$	$(p \Rightarrow r) \wedge (p \Rightarrow q)$
T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	F
T	T	F	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

(ii)

p	q	$p \vee q$	$(p \vee q) \Rightarrow p$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	T

(b) $2+3=5 \Rightarrow \sim \{(2+2=4) \wedge (4+4=8)\}$

$T \Rightarrow \sim \{T \wedge T\}$
$T \Rightarrow \sim T$
$T \Rightarrow F$

Therefore, the statement is false

(c) (i) p: It is hot

q: It is sunny

$\sim(\sim p \vee q)$

(ii) $\sim(\sim p \vee q) = \sim\sim p \wedge \sim q = p \wedge \sim q$

(d) (i) $A \wedge (\sim B \vee C)$

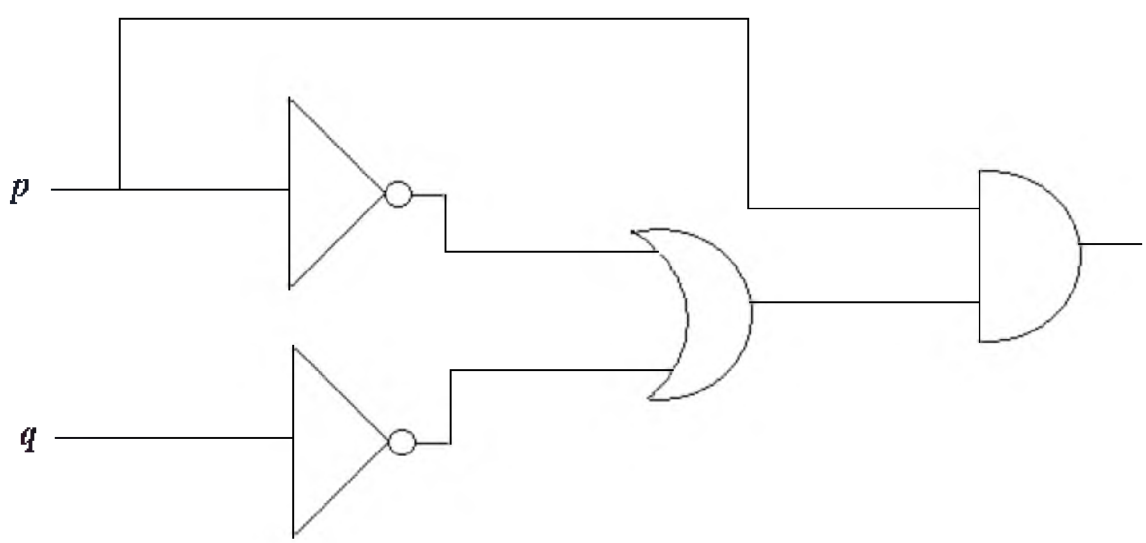
(ii)

A	B	C	$\sim B$	$\sim B \vee C$	$A \wedge (\sim B \vee C)$
1	1	1	0	1	1
1	1	0	0	0	0
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	0	1	0
0	0	1	1	1	0
0	0	0	1	1	0

The circuit is on when:

- A is on, B is off and C is off
- A is on, B is off and C is on
- A is on, B is on and C is on

(e)



Module 2

Question 3

This question tested candidates' ability to

- apply the properties of the probability density function *f* of a continuous random variable *X*
- calculate the expected value of a continuous function
- calculate $P(X \geq 1)$
- use the cumulative distribution function
- solve problems using the normal distribution with a continuity correction.

The question was generally well answered with over 65 per cent of candidates being awarded a mark of 16 or more (out of 25 possible marks). Of these candidates, a little less than half scored 20 marks or more.

Common errors made by several candidates were:

- Poor handling of algebraic manipulation of equations obtained after integration in Part (a) (i).
- Improperly interpreting the interval for Part (a) (i) by treating the variable as discrete rather than continuous
- Failing to use the given function as a cumulative distribution function although this was stated in Part (b),.
- In Part (c) (ii), candidates failed to take the inverse of ϕ when required to solve the equation $\frac{x-16}{3} = \phi^{-1}(0.6732)$. Most candidates did not follow the instruction to give the answer exactly or to three significant figures.

Answers

(a) (i) $a = -3/64$, $b = 1/4$

(ii) $P(X \geq 1) = 57/64$

(b) (i) $26/125$ (ii) 3.97

(c) (i) 0.0668 (ii) 17.3

Question 4

This question tested candidates' ability to

- formulate and use the probability function $f(x) = P(X = x)$
- apply the formula for probability using the Binomial distribution.

Nearly half of the candidates writing this paper were awarded scores higher than 15 out of the 25 possible marks. Close to 30 per cent of candidates did very well, obtaining scores of 20 or more. A small number of candidates did not respond to this question and 35 per cent obtained a score of ten or fewer than ten.

The question was set at an appropriate level of difficulty; however, some common errors were:

- Failing to take into account all the probabilities required by the question, especially $P(X = 0)$.
- Inappropriately making an approximation using the normal or Poisson distributions where the binomial was the appropriate distribution to use.
- Making computational errors when calculating probabilities in Part (a) (iii).
- Using an incomplete formula for calculating the variance.

The best responses were those given for Part (a) (ii).

Answers

(a) (ii) $\{4, 5, 6, 7, 8\}$

(iii)

x	4	5	6	7	8
$P(X = x)$	$1/21$	$2/7$	$1/3$	$2/7$	$1/21$

(iv) 6

(v) 0.976

(b) (i) ${}^{20}C_4(0.04)^4(0.96)^{16}$ (ii) 0.238

Module 3

Question 5

This question tested the candidates' ability to

- calculate the coefficient of friction between an object and a surface
- calculate the parallel force required to pull an object down an inclined plane at a constant speed
- calculate the total power output of the motion
- verify the horizontal range of a projectile and then determine the maximum range and the time of flight for a shell fired from a gun.

In Part (a) (i), candidates knew the formula $F = \mu R$, but applied it incorrectly.

Some candidates drew the 600 N force parallel to the horizontal instead of parallel to the plane. Candidates need to read the questions carefully.

Part (a) (ii) presented a challenge to candidates. Some of them used the 600 N from Part (a) (i). Candidates also had difficulty resolving forces parallel to the plane correctly.

Teachers need to remind students to draw force diagrams so they can get a visual representation of what they are doing.

Part (b) was poorly done. Candidates did not recall that in the equation $W = Fd$, the force needed to be found in the direction of the motion.

In Part (c), candidates were required to know when to use $v = u + at \; or \; s = ut + \frac{1}{2}at^2$. If they set $v = 0$ in $v = u + at$, then they would get the time to reach the maximum height. Also, If they set $s = 0$ in $s = ut + \frac{1}{2}at^2$, then the non-zero value of t is the time of flight.

In conclusion, candidates need more independent practice in problem solving.

Answers

(a) (i) 0.517 (ii) 31.2 N

(c) (ii) 45°

(iii) a) 12250 m b) 49.5 s

Question 6

This tested candidates' ability to calculate the

- negative acceleration (deceleration) and substitute into an appropriate equation of motion to find the velocity
- velocity immediately after impact using the law of conservation of momentum
- kinetic energy
- constant force acting on the object
- braking force using $F = ma$ and $v^2 = u^2 + 2as$.

In Part (a) (i), candidates incorrectly implemented the conservation of momentum. Some did not negate the acceleration.

For Part (a) (ii), candidates incorrectly substituted the given velocity 6 ms^{-1}.

In Part (b), many candidates used the velocity from Part (a) to find the change in kinetic energy instead of finding the final velocity using the equation $v = u + at$ t and using it to find the answer.

In Part (c), some candidates used the braking force as 3850 N instead of 3150 N. Some forgot to negate the acceleration. The majority of candidates got an answer of 487 m as a result of applying integration.

Answers

(a) (i) Velocity = 3.464 ms^{-1} (ii) Velocity = 5.83 ms^{-1}

(b) (i) Change in kinetic energy = 3360 J (ii) The force = 24 N

(c) Minimum length of the runway = 595 m

Paper 031 – School-Based Assessment (SBA)

Mathematical Applications

Some students failed to identify the relevant variables as well as mathematical concepts to be used in the statement of task and did not clearly state the method(s) being used, or provide an adequate description of its use.

In the section Mathematical Knowledge and Analysis, most students were able to successfully carry out simple mathematical processes and correctly interpret the resulting data.

The evaluation section was poorly done. Approximately 75 per cent of students failed to identify limitations and problems encountered during their study. Students were unable to articulate how to rectify the problems that were encountered during their study.

On the other hand, students were able to obtain full marks in the communication of information section. Many of them were able to articulate and present their findings in a logical manner, and in most cases, using proper grammar.

Overall, teachers did a fairly good job in guiding students in topic selection, and in the modelling of various experiments. However, attention must be paid to the assignment of marks in the rubric provided. *Teachers are reminded that fractional marks are not to be awarded to students.*

Allocation of Marks

- Approximately 84 per cent of students were able to secure marks in the 41–60 range.
- Approximately 60 per cent of all students sampled scored excellent grades in the SBA (51–60).
- Approximately five per cent of students scored under 31 marks out of a possible 60 marks.

Areas of Concern

- About 20 per cent of students ignored the stipulated format for the presentation of the project.
- The statement of the task was not explicit enough in about 30 per cent of the projects.
- Some students analysed the data before the data was even collected. As a result, some tables were not clear, and many tables were presented without headings and without reasons for their use.
- Some students presented more data than was needed for their analysis.

Areas of Strength and Weakness

<u>Strengths</u>

- Construction of the truth table.
- Construction of a logic gate.
- Identifying a feasible region.
- Producing a linear programming model.
- Working with resolving forces vertically and horizontally.

Weaknesses

- The majority of students did not write their answers to three significant figures.
- Few students used diagrams for the mechanics questions.
- Motion in two directions, for example, *forces perpendicular to each other.*
- Using the law of conservation of momentum too early to solve the problem.
- Too often students omitted to identify variables.
- Plotting straight lines.
- Not recognizing that they should use integer values: too many used decimal values
- Too often students did not identify logic assignments, for example, *p: it is hot*

Paper 032 – Alternative to School-Based Assessment

The seven candidates doing this paper performed poorly. Only one candidate performed well. Understanding of the basic concepts was weak.

Module 1

Question 1

This question tested candidates' ability to

- construct an activity network diagram
- calculate earliest start time, latest start time and float time
- identify the critical path in an activity network.

Candidates failed to demonstrate any understanding and synthesis of the graphical representation of the project flow network and how to compute the values (earliest start time, latest start time, float time) needed to determine the critical path and minimum completion time.

Module 2

Question 2

This question tested candidates' ability to

- solve problems involving probabilities of the normal distribution
- calculate expected values
- use the formula for a geometric distribution.

Candidates demonstrated a lack of understanding of basic probability concepts. Candidates had an idea of the need to standardize but were unable to correctly carry out the desired operation.

Answers

(a) (i) 0.252 (ii) 0.161 (iii) 80

(b) 0.913

(c) 0.36

Module 3

Question 3

This question tested candidates' ability to

- resolve forces, on particles, in mutually perpendicular directions
- calculate and use displacement, velocity, acceleration and time in simple equations
- apply Newton's laws of motion
- apply rates of change for velocity and acceleration.

Algebraic manipulation of the mechanics problem in Part (c), including the appropriate use of integration, was extremely weak. Generally, candidates failed to demonstrate a grasp of basic concepts in force, work and displacement. Most candidates used the incorrect function (sine) instead of the correct trigonometric function (cosine) when resolving the resulting force. Candidates mostly failed to identify the appropriate formula to use in computing for time in Part (b). Candidates did not recognize the use of differentiation of the velocity to generate the equation to find the time.

Answers

(a) W = 5280 J

(b) t = 35.7 s

(c) k = 0.1; $a = 10\ e^{-0.2s}$.

Overall, the difficulty level of the questions was reasonable and appropriate for this level. The time provided for solutions was adequate. The conceptual grounding of the candidates (except one) attempting this paper was very weak.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2013

APPLIED MATHEMATICS

GENERAL COMMENTS

The revised Applied Mathematics syllabus was examined in 2013 for the fifth time. This is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple choice questions, and Paper 02, consisting of six essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions from Papers 01, 02 and 031 to each unit were 30 per cent, 50 per cent and 20 per cent respectively.

Unit 1, Statistical Analysis, tested (1) Collecting and Describing Data; (2) Managing Uncertainty and (3) Analysing and Interpreting Data.

Unit 2, Mathematical Applications, tested (1) Discrete Mathematics; (2) Probability and Probability Distributions; and (3) Particle Mechanics.

For Unit 1, 652 candidates wrote the 2013 paper and 12 wrote the Alternative to the Internal Assessment paper, Paper 032. For Unit 2, 313 candidates wrote the 2013 paper and 10 candidates wrote the Alternative to the Internal Assessment Paper, Paper 032.

Generally, candidates are still having difficulty with (i) algebraic manipulations and (ii) problem solving in the mechanics module.

DETAILED COMMENTS

UNIT 1

Paper 01─Multiple Choice

Performance on the 45 multiple choice questions on this paper produced a mean of approximately 62.5 out of 90, standard deviation of 17.9, with scores ranging from 18 to 90.

Paper 02─Essay

Module 1: Collecting and Describing Data

<u>Question 1</u>

This question tested candidates' ability to

- identify whether variables were qualitative or quantitative
- distinguish between population, sample, census or sample survey
- use random sampling numbers provided in the statistical tables to obtain a random sample of a given size.

Parts (a), (b) and (c) were done exceptionally well by most candidates. However, it was common for candidates to also classify qualitative data as either discrete or continuous. Although the solutions were provided in the exam script, spelling errors were common. Part (d) was fairly well done by most candidates. However, several candidates were unable to express themselves clearly. This part required that candidates provide two reasons why it may be necessary to conduct a sample survey rather than use a census.

Similarly, for Part (e) (i), explanations provided by candidates were not clear. Most did not provide a complete explanation of how to use the random table. Although a two-digit random number was provided several explanations included a four-digit table or using the random table to assign numbers which would then be drawn from a hat to select the sample of 15.

Approximately half of the candidates scored full marks for Part (e) (iii). Many provided some parts of the process and generally failed to provide an appropriate range for assigning numbers or not indicating that the start position is a random point.

For Part (e) (i), any response that indicated that non-overlapping groups are required. Alternatives could include that the groups were not well defined, members of each group were not unique or that the groups were not mutually exclusive.

Question 2

This question tested candidates' ability to

- determine the mode class, mean and standard deviation from a frequency distribution
- determine the median and interquartile range from a stem and leaf diagram.

For Part (a) (i), the majority of candidates performed exceptionally well. Some candidates obtained at least one mark for identifying the modal class but failed to identify the boundaries. Part (a) (ii) was poorly answered. Most candidates did not recognize that 67 was the midpoint of the interval and therefore needed to estimate the frequency by taking one-half of the total frequency for the interval. Many failed to add the frequency of the higher interval. The most common response was to take the sum of both intervals. Most candidates obtained at least one mark for identifying the total number of students (20). The correct response to this part was: $\frac{\frac{10}{5}+2}{20} x\, 100 = 35\, per\ cent.$

Part (a) (iii) was answered correctly by approximately 60 per cent of the candidates. The most common error was taking the sum of the mid points and dividing by the number or intervals. In some cases, errors in calculations were observed. The correct response to this part was 65.65 inches.

Part (a) (iv) was poorly done as most candidates did not apply the formula correctly. The correct response was 2.593 inches.

Part (b) (i) was completed at a satisfactory level. A correct response would have been that all data values are retained. Acceptable alternatives included stating that the mode or range, are clearly visible or that the exact value of the mean and median can be calculated. Part (b) (ii) a) was correctly answered by nearly all candidates.

Because of the way the data values were distributed some candidates seemed to obtain the correct response but were in fact reading the wrong position of 13 rather than the required position of 13.5.

In Part (b) (ii), the majority of candidates knew they needed to subtract the two quartiles. However, most had problems calculating the exact values of both quartiles. Some candidates identified the position using $\frac{n+1}{2}$ and $\frac{3}{4}(n+1)$which did not provide correct quartiles due to a small sample size. A few candidates found the semi-interquartile - range. Part (b) (ii) (d) required that candidates construct a box-and-whiskers diagram. It was evident that some candidates were not knowledgeable about this type of diagram they proceeded to construct dot plots or bar charts. Although the graph paper provided included the scale, some candidates developed an alternative scale while others used a different graph paper.

For Part (b) (ii) e), most candidates obtained the two marks allocated. The majority of candidates was able to identify that the distribution was skewed but they had problems determining whether the distribution was negatively or positively skewed.

Question 3

This question tested candidates' ability to

- calculate the probability of the intersection of two events, P and Q
- construct a Venn diagram
- solve probability problems.

Part (a) (i) was attempted by 98 per cent of the candidates with about 60 per cent giving a satisfactory response. Some candidates did not use the conditional probability but multiplied $P(R)$ by $P(T)$, for example $P(R) \cdot P(T)$.

Approximately 97per cent of the candidates attempted Part (a) (ii) and responses were satisfactory.

Some candidates either identified the regions on the diagram without calculating the respective probabilities or they calculated the probabilities without identifying them on the Venn diagram.

Part (a) (iii) a) was attempted by approximately 95 per cent of the candidates with about 55 per cent of them successfully completing the question. Some candidates did not take into account that Q and T were independent.

Part (b) was also attempted by 98 per cent of the candidates, with only 40 per cent giving satisfactory responses.

Question 4

This question tested candidates' ability to

- calculate the expected value, variance and probability from a probability distribution
- carry out calculations based on a probability density function.
- identify a binomial distribution and use it to calculate probabilities.

Parts (a) (i) and (ii) were generally well done. For Part (a) (ii) b), the most common mistake was that candidates neglected to square the mean before subtracting.

Part (b) (i) was attempted by approximately 90 per cent of the candidates with about 55 per cent giving satisfactory responses. Some responses failed to acknowledge that the p.d.f. started at 1, hence the length of the rectangle was taken as k and not $k - 1$. Some candidates used the integration method with some success.

For Part (b) (ii), most candidates gave a satisfactory response. Many candidates were able to arrive at the correct answer by simply subtracting $\frac{1}{5}$ from the total area of 1.

For Part (c) (i), some candidates failed to use the binomial distribution. Part (c) (ii) was attempted by almost every candidate with about 97 per cent giving satisfactory responses. Some candidates used the probability that was calculated from Part (c) (i) as the value of p.

In Part (c) (iii), most candidates correctly used the normal approximation to the binomial; however, many either neglected or incorrectly applied the continuity correction. Some candidates used the variance instead of the standard deviation when standardizing. Almost every candidate was able to correctly read off a value from the table, but a few continued to subtract this value from 1.

Question 5

This question required candidates to

- state the distribution of the sample mean
- calculate the probability that the sample mean is greater than a given value
- calculate a 94 per cent confidence interval for the mean
- formulate a null hypothesis and an alternative hypothesis
- carry out a particular test at the 5 per cent level of significance

For Part (a) (i), most candidates were able to obtain two marks by identifying the distribution as normal and the mean as 35. However, many candidates failed to identify the variance correctly which should have been $\frac{100}{8}$. Many candidates did not use the appropriate symbolic form in their statement.

In Part (a) (ii), most candidates were awarded five marks for correct calculation of 0.148. Some candidates were able to calculate the *z*-value but were unable to read the correct value from the table. Approximately 40 per cent of candidates did not use $\sqrt{8}$ in the formula and therefore lost a mark.

For Part (b) (i), the majority of candidates was able to calculate the correct confidence interval of 3.894, 4.506. Many candidates used the limits for a 95 per cent confidence interval.

In Part (b) (ii), although many candidates were able to give the correct solution as 38 packages, others failed to recognize that the data given was discrete and apply this to the calculated value.

Part (c) (i) was well done with candidates providing acceptable responses, which included:

- normal distribution and small sample ($n < 30$) or unknown variance.

Part (c) (ii) was answered well by most candidates. However, the appropriate symbols were not always used. Although the question stated the use of symbols, some candidates provided the answer in words. The correct responses were $H_o: \mu = 10$ and $H_1: \mu < 10$.

Part (c) (iii) posed challenges for most candidates. Several did not use the appropriate degree of freedom ($8 - 1 = 7$). Although they had the hypothesis correctly stated in the preceding sub-part, they proceeded to use a two-tailed test. Many candidates failed to state the conclusion in words relevant to the question and only stated it as 'accept' or 'reject'.

Question 6

This question tested the candidates' ability to utilize regression analysis and chi-square analysis in solving problems.

In Part (a) (i), many candidates did not correctly interpret the value of 0.09 in the equation. Most candidates made reference to the gradient and y-intercept, but were unable to relate the information to the question and offer meaningful interpretations.

Part (a) (ii) and (iv) were well done.

For Part (b) (i), the majority of candidates was able to provide a favourable response. Parts b (ii) to (iv) were done fairly well.

For Part (b) (iii) b), most candidates were able to state or show the correct critical region. However, some candidates calculated an incorrect critical value for the chi-squared test; while others read the value from the table under the 5 per cent column, instead of the 95 per cent column.

In Part 6 (b) (iii) c) there were many variations of values due to rounding errors. In some cases, an incorrect formula or the incorrect usage of the formula was also applied.

Paper 031 – School-Based Assessment (SBA)

Approximately 80 per cent of candidates were able to score full marks for project title and purpose. If they did lose marks, it was because they failed to clearly state the variables used in the study. Candidates are reminded to be concise in the statement of purpose. In some cases the use of statistical theory was limited by the candidates' choice of topic. Candidates also displayed a lack of creativity in topic selection.

Some candidates failed to secure the two marks allotted for data collection. In some cases the method selected was not appropriate and often when appropriate, the candidate failed to adequately describe their choice. Most were able to describe the general method and select an appropriate method but failed to describe its application adequately.

Data presentation and the use of statistical language, jargon and symbols were for the most part satisfactory but candidates lost marks due to tables and charts being unlabeled and in some cases ambiguous.

Most candidates demonstrated a fair statistical knowledge. Minor inaccuracies of statistical concepts were identified and often overlooked by teachers. Teachers need to pay more attention to the calculation of expected values for the Chi-squared distribution. In most cases there was a logical flow of data and this was extended to the discussion.

In some cases, candidates failed to relate their findings to the purpose of the project, although all the calculations would have been completed. In many cases, this section was too wordy.

Candidates need to be concise in their conclusion. The conclusion must also be clearly stated and must be related to the purpose of the project.

Candidates must endeavor to use multiple references, but at the same time, when using websites as a reference they must make an effort to follow the accepted convention in a consistent manner. Candidates must also make an effort to select reliable websites.

Candidates' choice of font style and size made it very difficult for reading. Candidates are urged to follow conventional formatting used in academic papers. (Times New Roman, font size 12, double spacing, etc.)

Many of candidates sampled were able to performed, fairly well in the Unit 1 Internal Assessment.

Paper 3/2 – Alternative to the School Based Assessment

Question 1

This question tested candidates' ability to

- State whether given statements involve the use of interviews or observations
- Construct frequency distribution from raw data
- Construct and use histograms to analyse data
- Outline the relative advantages and disadvantages of using frequency distribution in data analysis
- Determine or calculate mode for grouped data.

Parts (a) (i) and (a) (ii) were fairly well done. Most candidates understood how to distinguish between the usages of interviews and observations.

Parts (b) (i) and (b) (ii) were well done by most candidates.

For Part (c), most candidates demonstrated a high level of competence in distinguishing between the usage of bar charts and histograms.

Part (d) (i) was generally well done. Part (d) (ii) posed the most difficulty for some candidates since they did not use any of the two methods which involved using the histogram or the formula for the mode which is $L + \frac{\Delta_1}{\Delta_2 + \Delta_2} C$.

Answers

(a) (i) Interview ii) Observation
(b) (i) Be more specific about family. How many people live in your immediate household?
(ii) Define young, middle age, old using ranges < 35, 35–50 > 50
(c) (i) Bar chart ii) Histogram
(d) (i)

Range	Frequency
10–19	7
20–29	9
30–39	13
40–49	10
50–59	7
60–69	4

(d) (ii) Individual data values are lost

(d) (iii) Reducing the amount of data
Easier to see at a glance
Clearer idea of distribution of the data
(d) (v) 35

Question 2

This question tested candidates' ability to

- identify and use the concept of probability
- calculate simple probability without replacement using
  - $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
  - Mutually exclusive events $P\frac{A}{B} = \frac{P(A \cap B)}{P(B)}$
- state the assumptions made in modelling data by a binomial distribution
- identify and use the binomial distribution as a model of data where appropriate
- use the notation $X \sim B(n,p)$ where n is the number of independent trails and p is the probability of a successful outcome in each trial.
- calculate the probabilities $P(X = a)$, $P(X > a)$, $P(X \geq a)$ or any combination of these where $X \sim B(n,p)$
- use the normal distribution as a model of data as appropriate
- determine probabilities from tabulated values of the standard normal distribution $Z \sim N(0,1)$
- solve problems involving probabilities of the normal distribution using z-scores.

All candidates answered Part (a) (i) correctly while for Part (a) (ii), some candidates were unable to write down the required probabilities needed to arrive at the correct solution.

For Part (b) (i), most candidates were able to give three assumptions made in modelling data by the binomial distribution. However some statements were not mathematically precise.

The majority of candidates was able to answer Part (b) (ii) question correctly.

Most candidates who attempted Part (c) were able to convert the values to z-scores, use the correct formula for Ø and read the table values correctly. However, some candidates were unable to simplify the Ø formula correctly while others gave the incorrect table values.

Answers

(a) (i) $\frac{1}{4}$

(a) (ii) a) $\frac{1}{22}$

(a) (ii) b) $\frac{21}{44}$

(b) i) Assumptions for binomial:

- n- distinct trials
- independent trials
- 2 outcomes for each trial
- Probability of success the same for each trial

(ii) a) 0.0318

(ii) b) 13.5 or 14

c) 0.7745 or 0.775

<u>Question 3</u>

This question tested candidates' ability to

- calculate the confidence intervals for a population mean or proportion using a large sample (n >30) drawn from a population of known or unknown variance
- evaluate a *t*-test statistic
- determine the appropriate number of degrees freedom for a given data set
- apply a hypothesis test for a population mean using a small sample () drawn from a normal population of unknown variance.
- formulate a null hypothesis and an alternate hypothesis *H*
- apply a one-tailed or two-tailed test appropriately
- determine the critical values from tables for a given test and level of significance.
- identify the critical or rejection region for a given test and level of significance
- evaluate from sample data the test statistic for testing a population mean or proportion

Candidates' scores ranged from 0-19 for this question, with less than 40 per cent of the candidates getting over 12 out of a total of 20 marks.

For Part (a) (i), some candidates used the formula correctly; however; marks were lost for using the incorrect confidence factor and not finding the square root of the sample size. In Part (a) (ii), most candidates used $E(x) = np$ correctly to find the expected value.

Part (b) (i) was well done. However, some candidates failed to state the hypotheses correctly and also used the incorrect test , that is, they used a one-tailed test instead of two-tailed test. In Part (b) (ii), some candidates did not know the criteria for using a t-test and therefore failed to use the t-test in this question.

In Part (b) (iii), most candidates had the correct degree of freedom but failed to write the critical value as a region t>2.365, t<-2.365.

Parts (b) (iv) and (v) were well done by most candidates.

Answers

(a) (i) 95 per cent confidence interval for $U = (23.2, 24.8)$
(ii) Expected value (54)
(b) (i) Hypothesis for the tailed test = $H_0: \mu = 14$, $H_1: \mu \neq 14$
(ii) Correct test to use was the *t*-test
Reasons = Sample size is small and the variance of the population is unknown
(iii) Critical Region = $-2.365 < t < 2.365$
(iv) Calculated Value $T_{cal} = 1.91$ or 1.79
(v) Decision – Accept Ho since T_{cal} is in the acceptance region; that is $1.79 < 2.365$ or $1.91 < 2.365$

UNIT 2

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of approximately 65.3 out of 90, standard deviation of 17.93 and scores ranging from 22 to 90.

Paper 02 - Essay

<u>Question 1</u>

This question tested candidates' ability to

- establish the truth value of the negation of simple propositions and compound propositions that involve conjunctions, disjunctions and negations;
- use truth tables to determine whether a proposition is a tautology or contradiction and if propositions are equivalent;
- use the laws of Boolean algebra to simplify Boolean expressions
- represent a switching circuit or logic circuit. Using a Boolean expression

This question was generally well done, with the majority of candidates obtaining 17 marks and over. Parts (a) and (b) were attempted by all candidates with most candidates getting full marks. De Morgan's law was used incorrectly in Part (c) with candidates obtaining $\sim p \vee \sim q$ instead of $\sim p \wedge \sim q$.

In Part (d), candidates forgot to assign statements to variables, that is, let p represent it is hot and q represent it is sunny. Candidates who wrote equivalent propositions instead of statements lost a mark.

For Part (e) (i), most candidates obtained full marks, whilst Part (e) (ii) was poorly done. Most candidates were unable to simplify the expression using the Absorption law. Also, candidates were unable to differentiate between a switching circuit and logic gates. They must be reminded to identify the laws of Boolean algebra being used. Part (f) was generally well done. There were some candidates, however, who simplified the statements using De Morgan's law and then drew the circuit using logic gates.

Answers

(a)

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

(b) $(p \wedge q) \wedge \sim(p \vee q)$ is a contradiction since the final column of the truth table contains all "F"s.

(c) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

(d) It is not raining or it is not sunny.

(e) (ii) $p \wedge q \wedge (p \vee r) \wedge (q \vee (r \wedge p) \vee s) \equiv p \wedge q$. The corresponding switching

circuit is

Range of Marks	Responses
NR	2
0–4	7
5–7	10
8–11	33
12–14	41
15–16	23
17–18	48
19–25	86

Question 2

This question tested candidates' ability to:

- determine the degree of a vertex
- calculate the earliest, latest starting time and float time
- identify the critical path in an activity
- solve a minimization assignment problem by the Hungarian algorithm.

This question was well done with at least 60 per cent of the candidates obtaining a score between 19 and 25.

For Part (a) (i), many candidates were able to determine the earliest start time of S and X. A wide range of answers were given for the minimum completion time of the project in Part (a) (ii). Part (a) (iii) was well done as candidates were able to correctly work out the latest starting times for X and Q. In Part (iv), many candidates recognized that there was more than one critical path for the activity network. The majority of candidates were able to accurately calculate the float time of T in Part (v). A diagram or table indicating EST, LST and Float times would have assisted candidates in answering this question.

For Part (b), a variety of responses were received such as $45^0, 60^0, 360^0$. This indicated that some candidates did not understand the meaning of *degree of a vertex* and also that the degree of a vertex has no units. Many of them did not realize that a loop has a degree of two.

Part (c) was generally well done. However, candidates failed to perform the final shading to indicate the end of the algorithm. Some candidates repeatedly found row and column minimums indicating that they did not know when to terminate the algorithm. Candidates used a variety of algorithms to get the answer with most obtaining the correct matching and the correct total minimum cost.

Answers:

(i) EST of S and X are 4 and 7 respectively.

(ii) 13 days

(iii) LST of X and Q are 7 and 0 respectively.

(iv) Start-Q-T-X-Finish, Start-R-T-X-Finish, Start-Q-S-X-Finish

(v) 0 (5-5)

(b) Degree of vertices A and B are 4 and 2 respectively

(c) (i) $W_1 - S_2, W_2 - S_1, W_3 - S_3, W_4 - S_4$

(ii) $ 16.00

Range of Marks	Responses
NR	1
0–4	2
5–7	3
8–11	14
12–14	19
15–16	21
17–18	26
19–25	127

Question 3

The question tested candidates' ability to

- calculate and use the expected values and variance of linear combinations of independent random variables
- calculate $P(X = x) = q^{x-1}p$, where $X = \{1, 2, 3 \ldots\}$
- use formula for $E(X)$ and $Var(X)$ where X follows a discrete uniform, binomial geometric or Poisson distribution
- solve problems involving probabilities of the normal distribution using z-scores.

For Parts (a) (i) and (ii), the majority of candidates scored full marks. They displayed a good understanding of the concepts and showed a high level of competency in solving the given problem.

In Part (a) (iii), candidates were asked to calculate the variance. Some candidates did not square the coefficient of the variance or use the '+' sign in calculating variance, resulting in loss of marks.

For Part (b) (i), some candidates used the incorrect inequality sign for example. Candidates were asked to calculate $P(X \geq 3)$, and seemed not to know what to do. Common errors were $1 - P(X \leq 3)$ and $P(X = 1), P(X = 2)$. Some candidates did not recognize that zero should be included in the latter if they are using the alternative method that is $1 - P(X \leq 2)$ which include $\{0,1,2\}$.

In Part (b) (ii), candidates demonstrated mastery of content as full marks were awarded to the majority of candidates. In Part (b) (iii), candidates were asked to calculate $P(X = 5)$. Some candidates used the incorrect formula for example, pq^r and pq^{r-1}, instead of q^4p which is the correct formula.

In Part (c), very few candidates used the continuous correction method instead of the normal distribution. Candidates need to be taught when to use the continuity of correction method the Poisson method/normal distribution for approximation.

Answers

(a)(i) 12	(ii) -11	(iii) 48
(b)(i) $\frac{4}{9}$	(ii) 3	(iii) $\frac{16}{243}$ or 0.658
(c)(i) 0.309 to 3dp	(ii) 0.964 to 3 dp	

Question 4

This question tested candidates' ability to

- formulate and use the probability function $f(x) = P(X = x)$
- apply the formula $P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$ $x = 0,1,2,3,\ldots$
- use the Poisson distribution as an approximation to the binomial distribution
- apply the properties of the probability density function, f, of a continuous random variable:
  i) $F(x) \geq 0$
  ii) $\int_{-\infty}^{+\infty} f(x)dx = 1$

Parts (a) (i) and (ii) were well done by the majority of candidates. Some candidates used the incorrect inequality sign. Part (a) (iii) was poorly done by most candidates as many of them were unable to use the binomial formula correctly.

In Part (b), approximately 25 per cent of the candidates misinterpreted the question and used the binomial formula instead of the Poisson approximation.

Part (c) was well done by the majority of the candidates.

Answers

(a) (i) 0.184 (3 s.f) (ii) 0.567 (3 s.f) (iii) 0.285 (3 s.f)

(b) 0.677 (3 s.f)

(c) (i) $k = \frac{1}{3}$ (ii) $t = \frac{9}{4}$

Question 5

This tested the candidate's ability to

- resolve forces, on particles, in mutually perpendicular situations
- use the appropriate relationship $F = \mu R$ or $F \leq \mu R$ for two bodies in limiting equilibrium;
- solve problems involving concurrent forces in equilibrium;
- apply Newton's Law of Motion
- apply wherever appropriate the following rates of change:

$$v = \frac{dx}{dt},$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx}$$

- Formulate and solve first order differential equation as models of linear motion of a particle when the applied force is proportional to its displacement or its velocity.

This question was poorly done. Those who attempted it scored less than 12 marks.

In Part (a) (i), some candidates had difficulty interpreting the words and creating a diagram. The weight of the rod was incorrectly put at the end, the string was incorrectly drawn and the point R was placed incorrectly. Candidates also forgot the force at the hinge P. They did not draw the coplanar force diagram meeting at one point or in the form of a closed triangle (as if it was in equilibrium). Candidates drew two forces with the correct angle between the two. In Part (a) (ii), only two candidates used Lami's theorem and a few used the moments method correctly.

In Part (b), most candidates drew the diagram correctly; however, the forces were added incorrectly and the resolution of the weight was also done incorrectly. Candidates did not use the resultant force to be ma and some even stated it was equal to zero. Some candidates attempted to use the work–energy theorem but did so incorrectly, stating potential energy plus initial kinetic energy minus final kinetic energy was equal to the work done. Also, those who stated that the work done equals change in kinetic energy eliminated the negative sign because they did not consider the fact that the final force was less than the initial force which implied deceleration.

Part (c) was generally well done. Most candidates separated the variables and integrated, with a few reciprocating both sides and integrating with respect to x. A few candidates used definite integrals which eliminated the substitution to find the integration constant. Those who got this question wrong either integrated incorrectly or did find the constant of integration.

Part (c) (ii), was also well done with the majority of candidates substituting well. Some candidates made the mistake of using the initial velocity as opposed to the final velocity and some of them did not recognize that $\frac{dx}{dt}$ was the velocity. They used $v = u + at$ with $t = \frac{1}{k}\ln\frac{10}{10-kx}$.

Answers:

(a) (ii) $T = 20\sqrt{2}\ N$

Range of Marks	Responses
NR	4
0–4	68
5–7	50
8–11	57
12–14	25
15–16	15
17–18	17
19–25	12

Question 6

This question tested candidates' knowledge of

- acceleration and tension relating to two particles corroded by a light inextensible string which passes over a pulley
- projectiles.

Most candidates attempted this question. However, it was poorly done. Less than 40 per cent of the candidates scored 12 marks or more out of a maximum of 25 marks.

Part (a) (i) was generally well done. For Part (a) (ii), most candidates substituted correctly in the formula and therefore gained marks. In Part (a) (iii), some candidates used the incorrect formula, $v = \frac{d}{t}$ or $d = vt$ is used for constant velocity, instead of $s = ut + \frac{1}{2}at^2$. However, the substitution for their answer from Part (a) (i) was done correctly for the most part.

For Part (b), candidates knew the equation impulse = change in momentum. However, they failed to realize that it was a vector quantity, hence when the ball bounced from the wall the sign would have changed.

Part (c) was poorly done as candidates had problems using the equations of motion to determine the equation of trajectory.

In Part (d), most candidates used the correct formula but it was noted that candidates had problems converting the formula to have only one variable or ratio that is, $y = x\tan x - \frac{1}{2}\frac{gx^2}{v^2\cos^2 x}$. Also, to form the equation into a quadratic and solve it was problematic for most candidates.

Answers

(a) (i) $\frac{2}{7}g\ ms^{-2}$ (ii) $\frac{450}{7}N\ or\ 64.3\ N$ (iii) $51.4\ m$

(b) 18Ns

(c) $y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{v^2\cos^2\alpha}$

(d) $\alpha = 77^0\ or\ 71^0$

Range of Marks	Responses
NR	11
0–4	40
5–7	28
8–11	33
12–14	8
15–16	11
17–18	11
19–25	29

Paper 031 – School-Based Assessment (SBA)

Mathematical Applications

Approximately 40 per cent of the candidates failed to identify the relevant variables as well as mathematical concepts to be used in the statement of task. Additionally, some candidates did not clearly state the method(s) being used, nor provided an adequate description of its use.

The section Mathematical Knowledge and Analysis, was fairly well done as candidates were able to successfully carry out simple mathematical processes and correctly interpret the resulting data.

The evaluation section was however poorly done. Candidates failed to identify limitations and problems encountered during their study. Some candidates were unable to articulate how to rectify the problems that were encountered during their study.

However, many candidates were able to obtain full marks in the communication of information section. Candidates were able to articulate and present their findings in a logical manner, and in most cases, using proper grammar.

Overall, it was evident that students were well guided in the selection of a topic, and in the modeling of various experiments.

General comments

1. Generally, candidates demonstrated a high degree of mastery in the mathematical principles pertaining to the syllabus. In most cases, the mathematical analyses were relevant and carried out with few flaws.
2. There was evidence of originality and creativity.
3. Projects were appropriately applied to real world problems and situations.
4. Over ninety percent of the candidates were able to effectively communicate information in a logical way using correct grammar and mathematical language.

Areas of Concern

1. Some candidates ignored the stipulated format for the presentation of the project.
2. The statement of the task was not explicit enough in some of the projects.
3. Some candidates analysed the data before the data was even collected. The result of this was tables that were not clear, and even some tables that were presented without heading and without reasons for their use.
4. Some candidates presented more data than was need for their analysis.

Areas of Strength and Weakness

Strengths

1. Originality and creativity.
2. Appropriately applied to real world problems and situations.
3. Effectively communicate information in a logical way using correct grammar and mathematical language.

Weaknesses

1. Vague titles.
2. Variables not mentioned or defined.
3. Table headings not stated.
4. Some discussions and conclusion not clearly related to the purpose.
5. References – did not use a recognised format.

Paper 032 Alternative to the School Based Assessment

Module 1

Question 1

This question tested candidates' ability to

- derive and graph linear inequalities in two variables
- determine whether a selected trial point satisfied a given inequality
- determine the solution set that satisfies a set of linear inequalities
- determine the feasible region of a linear programming problem
- formulate, in symbols, compound propositions
- graph terminology: *path*

For Part (a), most of the candidates were able to derive and graph the linear inequalities in two variables. Fewer candidates were able to identify the feasible region. The majority of candidates was unable to use $c = x + 2y$ to calculate the minimum value.

In Part (b), most candidates were able to recall the converse, contrapositive and the inverse.

Part (c), was particularly well done as most candidates understood the question and were able to give the correct solution.

Answers

(a) (i) {(0,12),(3,0)}{(3,0)(0,21)}{(0,3)(9,0)} (ii) C = 6.9 at (2.7, 2.1)
(b) (i) $q \implies p$ (ii) $\sim q \implies \sim p$ (iii) $\sim p \implies \sim q$
(c) AC, ABC, ADC

Question 2

This question tested the candidates' ability to:

- calculate the number of ordered arrangements of n objects taken r at a time, with or without restrictions
- calculate probabilities of events (which may be combined by unions of intersections) using appropriate counting techniques
- carry out a chi-square (χ^2)goodness of fit test, with appropriate number of degrees of freedom

For Part (a), the majority of candidates was able to answer the question correctly. Part (b) was well done as the majority of candidates were able to score full marks.

For Part (c), the majority of the candidates was able to use binomial formula correctly to calculate the expected value.

Answers

a) $\frac{7!}{2!}$
b) $\frac{1}{21}$
c) i) 0.6

(ii) 52.2, 36.8, 9.8, 1.1, 0.1
H_0: data fit binomial distribution with n= 4, p = 0.15
H_1: data do not fit binomial distribution with n= 4, p = 0.15
χ^2_{test}= 0.669
since $x^2_{test} < 3.841$ accept H_0

Question 3

This question tested candidates' ability to

- use vectors to represent forces
- calculate the resultant of two or more coplanar forces
- resolve forces on particles on incline planes
- calculate the work done by a constant force
- solve problems involving kinetic energy and gravitational potential energy
- apply the principle of conservation of energy
- solve problems involving power
- apply the work energy principle in solving problems

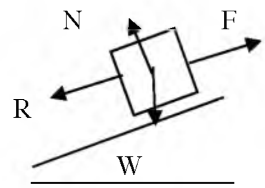
Parts (a) (i) and (a) (ii) were well done by most candidates. For Part (b), most of the candidates were able to recall the converse, contrapositive and the inverse.

Part (c) was answered very poorly, all candidates were unable to solve this question involving kinetic energy and gravitational potential energy. However, two candidates obtained full marks using another method which involved $v^2 = u^2 + 2as$ and $F = ma$.

Answers:

a) i) 6i+12j ii) $\sqrt{180}$

b) i)



ii) 194N

c) 295N

Recommendations

- For the Hungarian algorithm, teachers should teach the different methods for solving the optimization problem.
- Teachers should teach students how to interpret the obtained results such as the regression coefficients.
- Students need to practise more worded problems in the Mechanics section. They do need to draw diagrams to assist them in problem solving.
- It would be good practice for students to use the APA style for writing references in the School-Based Assessment (SBA).

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2014

APPLIED MATHEMATICS

GENERAL COMMENTS

Applied Mathematics syllabus is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple-choice questions and Paper 02, consisting of six essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions to candidates' final score from Papers 01, 02 and 03 to each unit were 30 per cent, 50 per cent and 20 per cent, respectively.

Unit 1, Statistical Analysis, was tested in three modules: (1) Collecting and Describing Data, (2) Managing Uncertainty and (3) Analysing and Interpreting Data.

Unit 2, Mathematical Applications, was tested in three modules: (1) Discrete Mathematics, (2) Probability and Distributions and (3) Particle Mechanics.

Both units were tested in two papers: Paper 01, which consisted of 15 multiple-choice items from each module, and Paper 02, which had two questions from each module. Candidates were also required to complete a project for school-based assessment (SBA) on each unit. Candidates who did not do the SBA wrote an alternative Paper 032, consisting of three questions, one from each module. Each unit was tested at three cognitive levels - Conceptual knowledge, Algorithmic knowledge, and Reasoning. Each question in the Paper 02s was worth 25 marks, while each in the Paper 032s was worth 20 marks.

For Unit 1, 691 candidates wrote the 2014 examination and 14 wrote Paper 032, the Alternative to the School-Based Assessment (SBA). For Unit 2, 321 candidates wrote the 2014 examination and 10 candidates wrote the Paper 032, the SBA.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice questions on this paper had a mean of approximately 60.28 and standard deviation of 16.30, with scores ranging from 16 to 90.

Paper 02 – Essay

Module 1: Collecting and Describing Data

Question 1

The question tested candidates' ability to:

- Distinguish among the following sampling methods – simple random, stratified random and systematic random, and use these techniques to obtain a sample
- Construct and use box-and-whisker plots and cumulative frequency curves (ogives)
- Calculate the standard deviation of ungrouped data

This question was generally well done with approximately 55 per cent of candidates scoring more than 16 out of 25.

Parts (a), (b), (c) and (d) were satisfactorily done, with approximately 78 per cent getting their marks from this section. A few candidates confused the names of the sampling methods, or just did not know the names.

Part (e) seemed to be a challenge for some candidates. In Part (e) (i), some candidates forgot to multiply by 10,000 while others forgot to divide by 6. Part (e) (ii) was poorly done. The few candidates who attempted this part multiplied by 10,000 rather than $\sqrt{10000}$. Some also did not notice that the formula on the sheet is for grouped data.

Part (f) was fairly well done, but some candidates were unable to transition between the cumulative frequency and the class frequency. In Part f (ii), some found 60 per cent of 80 rather than 60 per cent of 50. Others found 60 per cent of 50 but did not interpolate to find the time. The median was also a challenge. Candidates either found half of 80 or used a $(n + 1)$ formula.

The box-and-whisker plot was primarily well handled. Some candidates terminated the lower whiskers at ten rather than zero. Inaccurate calculations of the quartiles caused the 'box' to be incorrect though it was evident that they knew how to construct the diagram.

Solutions:

(a) (i) employees in the children clothing department
(ii) accessories made an average of \$30 000
(b) (i) simple random; (ii) systematic; (iii) stratified
(c) method in (b) iii – stratified
(d) 2
(e) (i) \$81 666.67; (ii) \$421.96 $\approx$ \$422.00
(f) (i) 34; (ii) 53 minutes; (iii) Median = 50 minutes

Question 2

This question tested candidates' ability to:

- Determine the class size and class boundary of the distribution.
- State the disadvantages of presenting data in a grouped frequency distribution.
- Determine the mean, variance and standard deviation of a grouped data.
- Draw a histogram and estimate the mode within the modal class.

For Part (a) (i), most candidates were able to identify the class and the boundaries. In Part (a) (ii), the majority of candidates knew that they had to subtract the two boundaries to obtain the class width. For Part (iii), the majority of candidates was able to explain the disadvantage. However, approximately 10 per cent of the candidates stated the advantage.

For Part (b), the majority of candidates was able to perform the various calculations to obtain the mean, variance and the standard deviation. Many of the candidates were able to use the two formulas that were suggested in the formula sheet. However, 10 per cent of the candidates chose to use other formulas such as the coded method and other variations of the ones given on the formula sheet, for example, $\frac{1}{\Sigma f}\left(\sum f x^2 - \frac{(\Sigma f x)^2}{\Sigma f}\right)$; $s^2 = \frac{\Sigma f (x-\bar{x})^2}{\Sigma f}$.

Approximately 10 per cent of candidates treated the data as if it were a sample, and used $(\sum f - 1)$ to respond to the question.

In Part (c), most candidates were able to draw the graph accurately. Several candidates did not use the graph sheet provided, which made the question difficult as they then had to find the correct scales.

In Part (d) (i), approximately 50 per cent of the candidates were able to estimate the mode using the histogram. Some candidates used the interpolation formula. Many candidates were able to identify the class but were unable to calculate the mode.

For Part (d) (ii), the majority of candidates was able to arrive at six for the number of consultations. However, approximately 40 per cent of the candidates were unable to get the correct solution.

Solutions:

(a) (i) 29.5, 39.5 (ii) 10 (iii) Data values are lost, further statistics must be estimated.
(b) (i) 32.9 min. (ii) 77.4 (iii) 8.8
(c) histogram
(d) (i) mode = 33.5 (ii) 6

Module 2: Managing Uncertainty

Question 3

This question tested candidates' ability to:

- Calculate the probability of events *A*, P (*A*), as the number of outcomes of *A* divided by the number of possible outcomes.
- Use the property that $P(\overline{A}) = 1 - P(A)$, where $P(\overline{A})$ is the probability that A does not occur.
- Calculate the $P(A \cup B)$ *and* $P(A \cap B)$.
- Calculate the conditional probability $P(A/B)$ where $P(A/B) = \frac{P(A \cap B)}{P(B)}$ is the probability that A will occur given that B has already occurred.
- Identify independent and mutually exclusive events.
- Construct and use tree diagrams.
- Use the probability $P(A \cap B) = P(A) \cdot P(B)$ *or* $P(A/B) = P(A)$ where *A* and *B* are independent events.

Part (a) (i) tested candidates' ability to complete the tree diagram. The question was well done. One common mistake made was in candidates writing 1 per cent and 2 per cent as 0.1 and 0.2 resulting in incorrect probabilities on the branches. Another common mistake was candidates adding probabilities along branches rather than multiplying. The majority of candidates did Part (a) (ii) correctly. The main mistake occurred with the placement of the decimal point.

For Part (a) (iii), most candidates recognized that they had to sum the defective probability from each machine. In Part (a) (iv), the majority of candidates recognized that it was a conditional probability. However, in most cases, they used the wrong event in the denominator or used the union rather than the intersection in the numerator. In addition, few candidates calculated P (defective) for the numerator. Part (a) (v) was very poorly done. Some candidates attempted to use the Binomial distribution, but this was not necessary to find a solution. Some other candidates recognized that exactly one means that the first was defective and the second was not defective, but they did not switch to say that the first was not defective and the second was defective.

Part (b) (i) was generally well answered. However, candidates had problems using algebra to simplify the expression and solve the equation. For Part (b) (ii), most candidates recognized it was a conditional probability and answered the question correctly. A few candidates used P (*N*) in the denominator instead of P (*M*). Part (b) (iii) was poorly done. Very few candidates constructed a Venn diagram to answer this question, but those who did, answered the question well.

For Part (c) (i), the majority of candidates correctly identified the events were not mutually exclusive; however, the explanation offered was inadequate. Many candidates confused mutually exclusive and independent events when offering their explanations. In Part (c) (ii), candidates appeared to be guessing and very seldom offered a mathematically sound explanation for the lack of independence.

Solutions:

(a) (i) 0.4, 0.02, 0.98, 0.01, 0.99

(ii) 0.012 (iii) 0.016 (iv) 0.75 (v) 0.0315

(b) (i) 0.45 (ii) 0.33 (iii) 0.4

(c) (i) Not mutually exclusive since $P(M \cap N) \neq 0$

(ii) Not independent since

$P(M \cap N) = 0.2,\ P(M) \cdot P(N) = 0.27 \neq 0.2\ or\ P(N/M) \neq P(N)$

Question 4

This question tested candidates' ability to:

- Identify the conditions of the binomial distribution and use it to calculate probabilities.
- Calculate the expected value, standard deviation and probability from a binomial distribution.
- Use the notation $X \sim Bin\ (n,\ p)$ where n is the number of independent trials and p is the probability of a successful outcome in each trial.
- Calculate the probabilities $P(X = a)$, $P(X > a)$, $P(X < a)$, $P(X \leq a)$, $P(X \geq a)$, or any combination of these where $X \sim Bin(n, p)$.
- Use the normal distribution as a model of data, as appropriate.
- Determine probabilities from tabulated values of the standard normal distribution.
- Solve problems $Z\sim N(0,1)$involving probabilities of normal distribution using Z scores.
- Use normal distribution as an approximation to the binomial distribution where appropriate $(np > 5, npq> 5)$ and apply a continuity correction.

Part (a) was generally well done by most candidates.

In Part (b) (i), most candidates knew the formula for the binomial expansion, however approximately 15 per cent of the candidates used it incorrectly. Candidates used the correct values for p and q but with the incorrect exponents, for example, $P(X=3) = {}^{12}C_3\, p^9 q^3$instead of $P(X=3) = {}^{12}C_3\, p^3 q^9$.

For Part (b) (ii), the majority of the candidates misinterpreted $P(X \geq 2)$, for example,
$1 - P(X=0) + P(X=1) + P(X=2)$ instead of $1 - P(X=0) + P(X=1)$.

A small number of candidates used the alternative method, that is, $P(X=2) + P(X=3) + \ldots + P(X=12)$ but they approximated each probability too early and this resulted in their answers being either truncated or greater than 1.

In Part (c) (i), most candidates observed that it was a normal distribution but some failed to standardize correctly. Other candidates standardized correctly and obtained the correct table value however did not subtract the table value from 1 to give the region specified. Some candidates also failed to obtain the correct table value.

Approximately 98 per cent of the candidates attempted Part (d) (i), which was well done. Part (d) (ii), was also well done as most candidates scored full marks. The minority, however, found the variance (*npq*) but failed to find its square root while others used an incorrect standard deviation formula. A few candidates used the correct formula but took the product of *np* and their *n* value in *npq*.

For example: $np = 200 \times 0.82 = 164$
$npq = 164 \times 0.82 \times 0.18$

instead of : $np = 200 \times 0.82 = 164$
$npq = 164 \times 0.18$

In Part (d) (ii), most candidates did not apply the continuity correction. They also used the standard error instead of the standard deviation when standardizing. A few candidates did not correctly read the tables to obtain the probabilities. Some candidates also subtracted their table value from 1 which was incorrect based on the required region.

Solutions:
(a) independent trials; finite number of trials; probability of success same for each trial; two outcomes only for each trial. (Any three correctly stated)
(b) (i) 0.012 (ii) (0.99968)
(c) 0.0668
(d) (i) 164 (ii) $\sqrt{5.43}$ (iii) 0.9732

Module 3: Analysing and Interpreting Data

Question 5

This question tested candidates' ability to:

- Calculate unbiased estimates for the population mean, proportion or variance.
- Calculate confidence intervals for a population mean or proportion using a large sample ($n \geq 30$) drawn from a population of known or unknown variance.
- Formulate a null hypothesis H_0 and an alternative hypothesis H_1.
- Apply a one-tailed test or a two-tailed test, appropriately.
- Relate the level of significance to the probability of rejecting H_0 given that H_0 is true.
- Determine the critical value from tables for a given test and level of significance.
- Identify the critical or rejection region for a given test and level of significance.
- Evaluate from sample data the test statistic for a given test and level of significance.
- Evaluate a t-test statistic.
- Determine the appropriate number of degrees of freedom for a given data set.
- Determine probabilities from t-distribution tables.
- Apply a hypothesis test for a population mean using a small sample ($n < 30$) drawn from a normal population of unknown variance.

Part (a) (i) a) was well done with most candidates producing the correct response of 7.42 minutes.

For Part (a) (i) b), many candidates found the variance of the set of data as opposed to the unbiased estimator for variance. Several different variations for the formula were used, however only approx. 55 per cent of the responses yielded the correct answer.

In Part (a) (ii) a), most candidates understood the concept of the null and alternative hypothesis. However, many neglected to state the answer using statistical symbols. Additionally, some candidates used x or $\bar{x}$ as opposed to μ, while others stated the incorrect inequalities for the hypotheses.

Part (a) (ii) b) was not done well by most candidates. Most candidates did not realize that a t-test was required. Even those who correctly calculated the degrees of freedom went on to use the wrong distribution.

For Part (a) (ii) c), most of the candidates understood how to calculate the value of the test statistic. However, some confused the order of the numerator, using $\mu - \bar{x}$ (7- 7.42), or used $\hat{\sigma}$ as the denominator, neglecting to use $\frac{\hat{\sigma}}{\sqrt{n}}$.

For Part (a) (ii) d), most candidates were able to come to the correct conclusions based on their solutions from Parts (a) (ii) b) and c). Some candidates, even though they came to the correct conclusion, neglected to also state the decision (fail to reject H_0).

Part (a) (ii) e) was generally well done with approximately 87 per cent of the candidates stating the correct assumption of the distribution being normal, which was already stated in the question itself.

Part (b) was generally poorly done. Only approximately 20 per cent of the candidates were able to achieve full marks for the question. Most candidates did not treat the question as a confidence interval for proportions. However, most candidates did correctly state the correct Z value of 1.96.

Solutions:

5 (a) (i) a) 7.42mins b) 9.54

(ii) a) $H_o: \mu = 7$ $H_1: \mu > 7$

b) $t > 1.796$

c) $t_{test} = 0.471$

d) Accept H_o, mean = 7

e) the distribution is normal

5 (b) (0.217, 0.475)

Question 6

This question tested candidates' ability to:

- Utilize regression analysis to solve application problems.
- Use chi-square analysis in problem solving.
- Develop null and alternative hypothesis test.
- Interpret the chi-square value as it relates to the null or alternative hypothesis.
- Calculate the product-moment correlation coefficient 'r' and interpret the 'r'.
- Calculate the expected frequency.

In Part (a) (i), most candidates were able to develop the correct null and alternative hypotheses and for Part (a) (ii), the majority of candidates was able to calculate the degrees of freedom correctly. Some of the candidates who attempted Part (a) (iii) had problems reading the X^2 values from the table. Many candidates used diagrams to represent their critical region correctly. Part (a) (iv) was well done. Some candidates struggled to write a clear conclusion. However, those who did, wrote excellent conclusions.

For Part b) (i), most candidates were able to calculate the regression equation correctly. For Part (b) (iii), many of the candidates struggled to interpret "*b*" in the regression equation as it relates to the given information. Candidates wrote that "*b*" is the gradient of the regression line y=7.58 + 0.59x. Approximately 25 per cent of the candidates were able to interpret the "*b*" value correctly. Part (b) (iv) was well done by most candidates however, some candidates had a problem interpreting the "*r*" value. Almost 95 per cent of the candidates were able to calculate a value for "*r*" correctly, but only 60 per cent were able to explain that "*r*" was a moderate and positive correlation.

Many candidates only stated that it was a positive correlation.

Solutions:-

(a) (i) H_o : there is no association between predicted grade and actual grade
H_1 : there is an association between predicted grade and actual grade

(ii) (a) degrees of freedom 4 (b) critical region $\chi^2 > 9.488$

(iii) 23.0 (3 significant figures)

(iv) since $\chi^2 = 9.1625 < 9.488$, do not reject H_o
There is no association between the predicted grade and actual grade.

(b) (i) $a = 7.56$, $y = 7.56 + 0.587x$

(ii) 29.3

(iii) an increase of 1 mark on the aptitude test will give an increase of 0.587 on the productivity score.

(iv) $r = 0.442$; there is moderate to weak and positive correlation between the mark on the aptitude test and the productivity score.

Paper 032 – Alternative to the School-Based Assessment (SBA)

Module 1: Collecting and Describing Data

<u>Question 1</u>

This question tested candidates' ability to:

- Distinguish between a population and a sample, a census and a sample survey, a parameter and a statistic.
- Distinguish between random and non-random samples.
- Identify sampling methods.
- Outline advantages and disadvantages of sampling methods.

Most candidates had an idea of how to respond to Parts (a) (i) and (ii). However, in Part (a) (i), some candidates referred to a social population rather than a statistical population. Generally, candidates were unable to distinguish between a parameter and a statistic as required in Part (a) (iii). Candidates did not understand Part (a) (iv).

Part (b) (i) was poorly attempted. Although most candidates attempted Parts (b) (ii) most failed to describe the entire process. Several students used a raffle method rather than simple random.

Part (c) (i) was generally well done. Parts (c) (ii) and (iii) were relatively well done. However, in finding the mean, several candidates used a total of seven rather than 100. The median was poorly done with most candidates failing to take into consideration the given frequencies. Part (c) (v) was done correctly by most candidates.

Solutions:

(a) (i) – (iv) definitions of terms

(b) (i) classes not unique / overlap / students can be in more than one class

(ii) Assign a two-digit number, starting at 00, to each student.
Start at a random point on the table, read, in a stated direction, 2-digit numbers.
Note numbers belonging to students until 20 valid values are obtained.

(c) (i) 100 (ii) 221 (iii) 2.21 (iv) 2 (v) positively skewed

Module 2: Managing Uncertainty

Question 2

This question tested candidates' ability to:

- Use basic concepts of probability.
- Define mutually exclusive events.
- Calculate conditional probabilities.
- Identify and use the Binomial distribution.
- Use the normal distribution to calculate probabilities.

Although most candidates attempted Part (a), only a few got full marks. Some of the outcomes were not presented by candidates in Part (a) (i) and probabilities were not calculated accurately in Parts (a) (ii) and (iii).

Part (b) was well done by most candidates. However, most candidates were unable to correctly use the tables for more than two decimal points.

Parts (c) (i) and (ii) were generally well done.

Solutions:
(a) (i) BBB, BBR, BRB, RBB, BRR, RBR, BRR, RRR
(ii) 0.315 (iii) 0.813
(b) 0.0694
(c) (i) 0.237 (ii) 42

Module 3: Analysing and Interpreting Data

Question 3

This question tested candidates' ability to:

- Use the central limit theorem.
- Calculate confidence interval for a proportion from a large sample.
- Give practical interpretation of regression coefficients.
- Make estimates using the regression line.

For Parts (a) (i) to (iv), a few candidates used the proportion. Most candidates had an idea of what a confidence interval should look like and the corresponding *z*-value for CI of 95 per cent. However, substitution for the required proportion was poorly done indicating unfamiliarity with this type of question.

For Part (b) (i), many candidates failed to provide a complete solution with all of the components. The other two sections were generally well done. Candidates did demonstrate weakness in correctly using the tables for more than two decimal points.

For Part (c), candidates failed to interpret correctly the values with respect to the problem. Most candidates only used the terms *gradient* and *intercept*.

Solutions:

(a) (0.195, 0.302)

(b) (i) $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$. (ii) $\bar{x} \sim N\left(48, \frac{144}{75}\right)$ (iii) 0.0152

(c) (i) The life of the machine when it is new or at time zero.

(ii) For every unit increase in speed that the machine works, its life is reduced by 0.08 hours.

(iii) 3.4 hours

Paper 031 – School-Based Assessment (SBA)

The general presentation and standard of the samples submitted were satisfactory. Students selected topics that were generally suitable for their level, and were appropriate to the objectives of the syllabus. There were a few students who used techniques that went beyond the level expected.

Generally, the sizes of the samples submitted were adequate and marks were entered correctly onto the moderation forms.

<u>Project Title</u>

Approximately 30 per cent of the titles in the samples submitted were too vague. For instance titles like, ‘Free Throws’ or ‘Multiple Choice Test’ are too vague. Some students simply stated a mathematics topic such as ‘Critical Path Analysis’ or ‘Linear Programming’ for their title. Students should endeavour to relate their title to some real life situation. It is imperative that teachers explain this a bit more to students.

<u>Purpose of Project</u>

Variables were not clearly defined in many cases and in some cases, no variables were identified.

<u>Method of data collection</u>

Although this section was well done, students must describe how they collected the data and state which sampling technique is being used.

Presentation of Data

In this section, some of the diagrams and charts presented by students were unrelated to the purpose. Nonetheless, many students demonstrated mastery in the construction of histograms, pie charts, stem and leaf and box plots. Too many students presented long tables of data in this section. Students are reminded to append these at the end of the project in the appendix. In general, the presentation of data was well done.

Statistical Knowledge/Analysis of Data

The majority of students demonstrated a good grasp of statistical concepts in the syllabus and scored well in this area. However, a sizable minority of students did many calculations but gave no explanation for them. Some fundamental things about the chi-square test were of some concern:

- Using 2 x 2 contingency table without applying the Yates correction.
- Not merging cells with frequencies less than 5.

Discussion of Finding/Conclusion

This section was in many cases too verbose and much of it was irrelevant to the purpose of the project. A few students did not relate the findings to the purpose of the project.

Communication of Information

Only a small minority of students lost marks in this section. Students are reminded to proof read their projects before submission.

List of References

Most students were able to score well for this section but usually just one reference was given. Students should endeavour to use multiple references preferably using the APA style.

UNIT 2

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of approximately 63.61 out of 90, standard deviation of 16.51 and scores ranging from to 90.

Paper 02 – Essay

Module 1: Discrete Mathematics

Question 1

This question tested candidates' ability to:

- Determine the converse, inverse and contrapositive of a preposition.
- Establish the truth values of the converse, inverse and contrapositive of a preposition.
- Determine whether a preposition is a tautology or a contradiction.
- Construct the truth table values of compound prepositions that involve conjunctions, disjunctions and negations.
- Determine the truth values of conditional prepositions.
- Derive a Boolean expression from a given switching or logic circuit.
- Represent a Boolean expression by a switching or logic circuit.
- Use the laws of Boolean algebra to simplify Boolean expressions.

In Part (a), some candidates had difficulty in distinguishing between the contrapositive and the inverse.

For Part (b), the majority of candidates demonstrated competence in constructing a truth table but most candidates had difficulty formulating the inverse. Additionally, a significant minority of candidates were unable to provide the correct number of permutations of inputs to properly evaluate the outputs.

For Part (c), most candidates were able to generate the correct outputs and identify and justify the output as a tautology. However, some candidates were unable to provide the correct number of permutations of inputs to properly evaluate the outputs.

Generally, candidates did Part (d) well. However, rather than being consistent with their use of notation, a number of candidates used combinations of alternative Boolean operator notations, for example, $(a \wedge b) + c'$.

For Part (e), candidates were able to adequately answer the question, with the vast majority demonstrating a very good knowledge of the distributive law. However, many candidates were unable to represent B and C in parallel to A. Additionally, some candidates produced logic circuits instead of switching circuits.

Solutions:

(a) $q \rightarrow \sim p$

(b)

p	q	$\sim p$	$\sim p \rightarrow q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

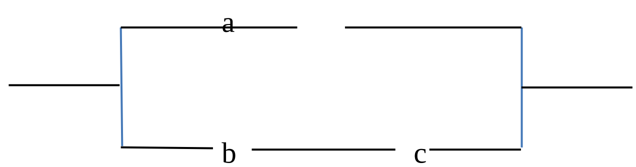
(c) (i)

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \vee (q \rightarrow r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	T	F	T	T	T

(ii) Tautology because all the truth vales are true.

(d) $\sim (a \wedge b) \vee \sim c$

(e) (i)



(ii) $(a \vee b) \wedge (a \vee c)$

<u>Question 2</u>

This question tested candidates' ability to:

- Use the activity network algorithm to draw a network diagram to model a real world problem.
- Calculate the earliest start time, latest start time and float time.
- Identify the critical path in an activity network.
- Derive a Boolean expression from a given switching or logic circuit.
- Establish the truth value of compound prepositions that involve conjunctions, disjunctions and negations.

Overall, the question was very well done with over 90 per cent of candidates obtaining a score between 16 and 25 and almost 40 per cent obtaining a perfect score.

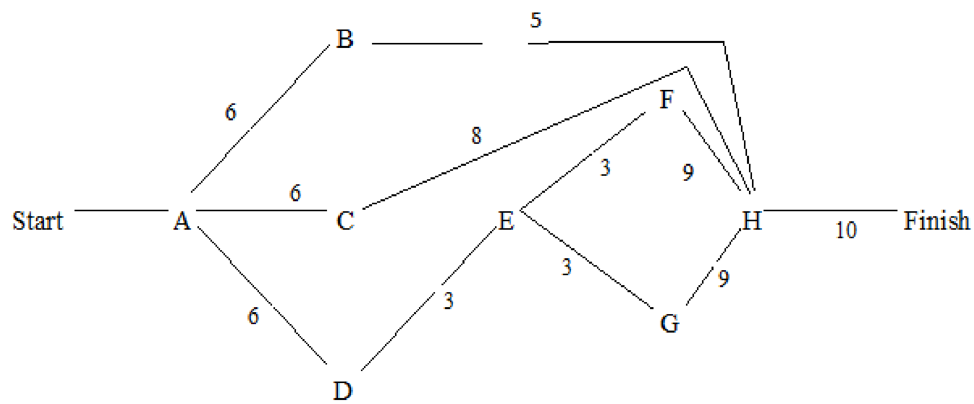
Part (a) (i) was relatively well done by candidates. However there were too many variations in the methodology used to construct the activity networks. The syllabus (Specific Objective (c) 4.) states that "activities will be represented by vertices and the duration of activities by edges".

For Part (a) (ii), most candidates successfully calculated the earliest start times, latest start times and float times from their diagrams.

Part (a) (iii) was very well done, with the majority of candidates identifying critical paths. However, many candidates placed simultaneous events on a single critical path, for example, start-A-D-E-F-G-H-finish, instead of start-A-D-E-F-H-finish and start-A-D-E-G-H-finish.

Part (b) was generally very well done, although a few candidates confused the symbols for disjunction and conjunction.

Solutions:



(a) (i)

(ii)

Activity	EST	LST	FLOAT
A	0	0	0
B	6	16	10
C	6	13	7
D	6	6	0
E	9	9	0
F	12	12	0
G	12	12	0
H	21	21	0

(iii) Start A D E G H Finish

Start A D E F H Finish

(b) (i) $p \wedge (p \vee \sim q)$

(ii)

p	q	$\sim q$	$p \vee \sim q$	$p \wedge (p \vee \sim q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	F
F	F	T	T	F

Module 2: Statistical Analysis

Question 3

This question tested candidates' ability to:

- Calculate the number of selections of n distinct objects taken r at a time.
- Calculate a number ordered arrangements with restrictions.
- Calculate probabilities of events using appropriate counting techniques.
- Calculate probabilities associated with conditional, independent or mutually exclusive events.

For Part (a), most candidates obtained the desired probability using $P(A' \cap B') = 1 - P(A \cup B)$. Marks were lost where candidates treated the events as being mutually exclusive.

For Part (b), candidates who used combinations generally obtained full marks while those using the fractional probability method usually did not recognize that there were three ways to get the solution and so did not multiply by three in Part (a) and similarly by six in Part (b). Many candidates failed to recognize that sampling was without replacement and solved these items as with replacement.

For Part (c), most candidates recognized that the conditional probability was required but the accuracy of calculations was weak. Fewer candidates used the combinations method in this part of the question. Similarly, many candidates failed to recognize sampling without replacement.

For Part (b) (ii), where full marks were not received in this item, candidates obtained partial credit for either obtaining part of the numerator or the denominator correct.

Few candidates achieved more than three marks for this item with many candidates being able to draw the diagram but unable to use the probabilities. The product of five and four factorial was seen more often than the two factorials and recognizing that having two directions required multiplying the probability by two.

Solutions:
(a) 0.34
(b) (i) a) 0.0975 b) 0.0887
(c) (i) 0.0702 (ii) 0.0542 (iii) 11520

Question 4

This question tested candidates' ability to:

- Calculate and use the expected value and variance of linear combinations of independent random variables.
- Model practical situations using a Poisson distribution.
- Solve probability problems involving the normal distribution.

Part (c) (i) was not marked since the paper had a print error that rendered the problem unsolvable. The item should have read $P(X + Y = 3)$ and not $P(X + Y) = 3$. Where attempted, the majority of candidates did well.

For Parts (a) (i) and (ii), the majority of candidates scored at least three of the six marks. Several candidates used the Binomial distribution rather than the Poisson distribution.

For Part (b) (i), most candidates attempted to use the correct formula; however, some had difficulty getting the denominator correct. Nearly all candidates were able to read the table correctly but failed to interpret its use correctly (needing to subtract from one).

Part (b) (ii) was poorly done. Most candidates were unable to correctly manipulate the table value for the negative z-value.

Part (c) was done exceptionally well by most candidates. They were aware of how to evaluate the expected values and variances and in a few cases, after calculating the correct expected value, proceeded to further divide by ten.

For Part (c) (iii) a), the majority of candidates correctly interpreted $E(3x - 2y)$ and correctly substituted their values to evaluate the expression.

Most candidates who attempted Part (c) (iii) b) failed to change the subtraction to addition. A few candidates failed to take the square of the coefficients.

Solutions:

(a) (i)	0.168	(ii)	0.9999 = 1 (3 s.f.)
(b) (i)	0.271	(ii)	0.322
(c) (i)	Ignored due to printed error		
(ii) (a)	1.3 (b) 0.61 (c) 2.05	(d)	1.75
(iii) (a)	-0.2 (b) 12.49		

Module 3: Particle Mechanics

Question 5

This question tested candidates' ability to:

- Distinguish between displacement and distance, velocity and speed.
- Draw and use a displacement time graph.
- Identify forces acting on a body in a given situation.
- Solve problems involving concurrent forces in equilibrium.

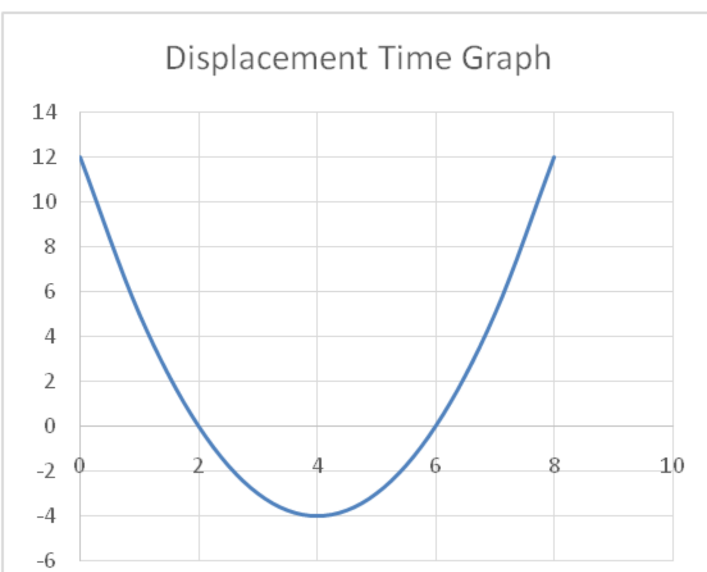
In Part (a), most candidates drew the graph accurately using appropriate scales. Some candidates got confused with distance vs displacement and velocity vs speed

In Part (a) (ii) c), candidates confused displacement = 0 for velocity = 0, that is, the velocity = 0 when the gradient of the displacement time graph is zero.

Part (b) was very poorly done. For Part (b) (i), candidates did not recognize that S was the resultant of the frictional and normal forces, thus this caused some confusion in the remainder of the question. In Part (b) (ii), candidates resolved vertically and horizontally but did not do so properly. The knowledge of trigonometry was not sufficient for this question. An alternative approach (Lami's theorem) would have simplified the question but this was not used by candidates. No candidate got full marks. Part (b) (iii) was poorly done.

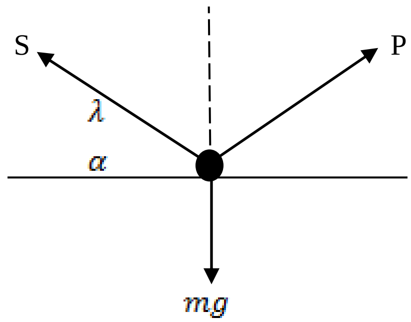
Solutions:

(a) (i)



(ii) a) $17\ m$ b) $-3ms^{-1}$c) $t = 4$

(c) (i)



(ii) $P = mg\sin\lambda;$ $\alpha = \lambda$ (iii) $P = 5m\ N$

Question 6

This question tested candidates' ability to:

- Formulate the equation of trajectory of a projectile.
- Use the equations of motion for a projectile to determine the angle of projection and time of flight.
- Formulate and solve first order differential equations as models of linear motion of a particle.

Part (a) was well done. Candidates generally were able to derive the equation of the trajectory. However, many candidates simply stated the trajectory rather than deriving it as asked in the question. Probably candidates did not understand the meaning of the word 'formulate'.

Part (b) (i) was poorly done. Candidates used the value of $y = 4$ instead of -4 and hence were unable to achieve the required equation. Although Part (b) (ii) was generally well done, many candidates chose to input values into their calculator instead of using the quadratic formula and showing appropriate steps.

In Part (b) (iii), some candidates used the simple formula $d = \frac{s}{t}$, instead of $x = (u\cos\theta)t$ and then proceeded to find *t*. Many candidates also did not approximate their value *t* to the nearest second.

In Part (c), many candidates assumed that the acceleration was constant and therefore used the incorrect formula. Candidates also integrated incorrectly by ignoring the constant of integration which led to obtaining the incorrect answer.

Very few candidates did Part (d) well. Resolution of forces was poorly done. Constant acceleration/force was incorrectly assumed. Many candidates also failed to calculate $\int vdv$.

Solutions:

(a) $y = x\tan\alpha - \frac{gx^2 \sec^2\alpha}{2u^2}$ (b) (ii) $\alpha = 11^0$ or 77^0 (iii) $t = 2s$ (c) $T = \frac{2}{3}s$

(d) $v = 5.04\ ms^{-1}$

Paper 032 – Alternative to the School-Based Assessment (SBA)

<u>Discrete Maths</u>

<u>Question 1</u>

This question tested candidates' ability to:

- Formulate simple propositions.
- Negate simple propositions.
- Construct compound propositions.
- Establish truth values of simple propositions.
- Compound propositions that involve conjunctions, disjunctions and negations.
- Represent a Boolean expression by a switching or logic circuit.
- Convert a maximization problem into a minimization problem.
- Solve a minimization assignment problem by the Hungarian Algorithm.
- Identify vertices and sequence of edges that make up a path.
- Determine the degree of a vertex.

Part (a) was generally well done by most candidates. Using either T and F or 0 and 1, candidates were able to complete the truth table successfully. However, many did not explicitly show or indicate the equivalent columns.

In Part (b), the logic diagram was correctly constructed by most candidates.

For Part (c), most candidates did not show all steps of the Hungarian algorithm with a few going directly to the allocations.

For Part (d), many candidates gave the paths from A to C rather than the requested paths from C to A.

Solutions:

(a) (i) $\sim p \wedge (p \rightarrow q)$

(ii) truth tables indicating equivalent columns

(b) logic gate diagram

(c) from Hungarian algorithm A assigned to IL, B to GX, C to AB, D to YG
(d) (i) CDA, CBA, CBDA, CDBA
(ii) 4

Probability and Distributions

Question 2

This question tested the candidates' ability to:

- Use the cumulative distribution function F(x).
- Use the result $P(a < X < b) = F(b) - F(a)$.
- Calculate expected value and variance from a given distribution.
- Calculate and use the expected value and variance from a linear combination of independent random variables.

For Parts (a) (i) to (v), most candidates were able to correctly calculate the constant in Part (a) (i) however, most of them did not seem to understand what was required for the remaining four parts of this section. In Part (a) (ii), candidates were required to find the area under the curve using integration rather that substituting the given values into the function and then evaluating. Most candidates knew that to find the median they were to use $F(x) = \frac{1}{2}$, but did not know what to do with it. In Part (a) (iv), candidates did not seem to recognize the need to differentiate in order to arrive at the probability density function. Similarly for Part (a) (v), candidates did not recognize the need to use integration to find the required probability.

Part (b) was well done.

Solutions:

(a) (i) $\frac{1}{3}$ (ii) $\frac{1}{3}$ (iii) 4.5

(iv) $f(x) = \begin{Bmatrix} \frac{1}{3} & 3 \leqslant x \leqslant 6 \\ 0 & otherwise \end{Bmatrix}$ (v) $\frac{9}{2}$

(b) (i) $3\frac{1}{2}$ (ii) $12\frac{1}{2}$

Module 3: Particle Mechanics

Question 3

This question tested the candidates' ability to:

- Calculate velocity, acceleration and time using the equations of motion.
- Calculate the work done by a constant force.
- Represent the contact force between two surfaces in terms of its normal and frictional component.
- Resolve forces on particles on an inclined plane.

This question was not well done as candidates had problems with resolving forces. Part (a) was generally well done though some candidates did not convert the speed of the body to ms^{-1}.

In Part (b), many candidates did not use the formula $W = Fs$. One candidate used $P = Fv$ to solve the question. However, the answer was left as the mass and not the weight.

Part (c) was poorly done. Candidates did not resolve forces at the $3m$ mass or the $4m$ mass and hence, could not solve the problem.

Solutions:

(a) (i) $a = -6.25\ ms^{-1}$ (ii) $t = 4s$ (b) $W = 450\ N$ (c) $a = 2.64\ ms^{-1}$

Paper 031 – School-Based Assessment (SBA)

Generally, the topics chosen were suitable for students at this level. The majority of the projects sampled were from the Discrete Mathematics and Probability and Distribution segments of the syllabus. A few interesting projects were seen in mechanics as well. These projects were appropriate and were generally well done by candidates.

Teachers' assessment of the projects was generally satisfactory. There was generally close agreement between the marks awarded by teachers and those by the CXC Moderator.

Statement of Task

The majority of students scored full marks in this section. Approximately 30 per cent of students did not identify the variables. A small minority of students gave irrelevant information in this section and seemed unclear on what was required.

Data Collected

This section was generally well done but in a small number of cases, the data collected was not realistic.

Mathematical knowledge/Analysis

Most students scored well in this area and demonstrated a good grasp of the mathematical principles taught.

Evaluation

No insights into the nature of and resolution of problems encountered in the tasks were seen for most students but most were able to score the three marks for the conclusion.

Communication of Information

This section was generally well done by most students.

STRENGTHS AND WEAKNESSES

Strengths

- Recognition of conditional probability
- Knowing when and how to standardize
- Use of the z-table to find critical values
- Differentiating between distance and displacement
- Ability to calculate '*r*' the product moment correlation co-efficient correctly

Weaknesses

- Confusion about the concepts of independence and mutually exclusive;
- Confidence interval for proportionality — proportion was treated by many as if it was a mean;
- Ignoring the constant of integration
- Resolution of forces in the particle mechanics module
- Deficiency in knowledge of the Lami's Theorem; this method was attempted by a negligible number of students, who proceeded to use it incorrectly.

RECOMMENDATIONS

- Candidates should practice more problems involving the use of the chi-squared test, normal distribution and mechanics.
- Candidates need to have a comprehensive understanding of the various modules, paying special attention to the mechanics module of Unit 2.
- Candidates are still having difficulty with (i) algebraic manipulations and (ii) problem solving especially in the mechanics module. Pre-requisite skills — namely finding the mean and standard deviation for statistics, and trigonometry for Particle Mechanics — need to be reviewed before starting the modules.
- It would appear from the performance of Module 3 in both Units, that these modules are hurriedly done. More time needs to be allocated to these modules.
- Tables and formula sheets that are used in the examination should be the same ones used in the classroom.
- Candidates need to get used to writing their final answers correct to three significant figures. They however, should be cautioned against premature approximation.
- Teachers should instruct students to cite references to inculcate good research skills from early.

Candidates also need to:

- Familiarize themselves more with different sampling methods.
- Know how to calculate the mode from a histogram.
- Use the normal approximation to the binomial distribution.
- Recognize and use the continuity correction accurately.
- Identify when and how to use the t-test.
- Use the equations of motion for a projectile to determine the angle of projection and time of flight.
- Be able to distinguish between logic gates and switching circuits.
- Familiarize themselves with Lami's Theorem.
- Be able to distinguish between logic gates and switching circuits.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2015

APPLIED MATHEMATICS

GENERAL COMMENTS

The Applied Mathematics syllabus is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple-choice questions and Paper 02, consisting of six essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions to candidates' final score from Papers 01, 02 and 03 to each unit were 30 per cent, 50 per cent and 20 per cent, respectively.

Unit 1, Statistical Analysis, was tested in three modules: (1) Collecting and Describing Data, (2) Managing Uncertainty, and (3) Analysing and Interpreting Data.

Unit 2, Mathematical Applications, was tested in three modules: (1) Discrete Mathematics, (2) Probability and Distributions, and (3) Particle Mechanics. Both units were tested in two papers: Paper 01, which consisted of 15 multiple-choice items from each module, and Paper 02, which had two questions from each module. Candidates were also required to complete a project for school-based assessment (SBA) on each unit. Candidates who did not complete the SBA, wrote an alternative Paper 032, consisting of three questions, one from each module.

Each unit was tested at three cognitive levels − conceptual knowledge, algorithmic knowledge and reasoning. Each question on Paper 02 was worth 25 marks, while each on Paper 032 was worth 20 marks. For Unit 1, 691 candidates wrote the 2014 examination and 14 wrote Paper 032, the Alternative to School-Based Assessment (SBA).

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice questions on this paper had a mean of approximately 60.8 and standard deviation of 17.3 with scores ranging from 18 to 90.

Paper 02 – Essay

Module 1: Collecting and Describing Data

<u>Question 1</u>

This question tested candidates’ ability to:

- Distinguish between a population census and a sample survey.
- Distinguish between qualitative, quantitative, discrete and continuous types.
- Define a sample frame.
- Identify different methods of collecting data and their appropriateness.
- Calculate the number and types of items in a sample according to the sampling method used
- Calculate the interquartile range.

Candidates were able to answer Part (a), description of data; Part (b), distinguishing between census and sample survey; Part (c), distinguishing between direct observation and personal survey techniques: and Part (d) (i), calculating the sample needed from an identified stratified sample, reasonably well and scored most of their marks in these areas.

In Part (d) (ii), most candidates were unable to specifically state that in using the stratified random method, there was a ratio or proportionality being used. Part (d) (iii) posed the greatest challenge for many candidates. They were not able to clearly state that the question that was posed actually consisted of two questions, but they did recognize that there were two or more possibilities. Most of those who recognized that there were two questions just rewrote the question that was posed or another ambiguous question. In addition, some candidates who did not recognize that it was two questions were able to write two distinct questions with only a few recognizing that options for the questions were necessary.

Solutions

(a) (i) Qualitative (ii) Quantitative (iii) Discrete (iv) Continuous (v) Continuous

(b) (i) Census – All members of the population are surveyed.

Sample – Some of the members of the population are surveyed.

(ii) Any two of the following:

- Too costly
- Too time consuming
- Inaccessible population
- Destruction methods
- Too large a population

(iii) a) Sample – assuming that more than 15 persons made a purchase

b) Census – choosing all of the 5 persons

(c) (i) Direct observation (ii) Personal survey (iii) Personal survey

(d) (i) Masons = $\frac{45}{125} \times 25 = 9$

(ii) All groups/strata will be sampled proportionally or in ratio to each group

(iii) a) Reason: Two questions were asked.

b) Rewriting: (Circle the response that you think answers the question.)

- How often do you buy lunch at the worksite?
  Never Two days or less Three days or more Everyday
- Do you bring your lunch from home?
  Yes No Sometimes

Question 2

This question tested the candidates' ability to:

- Construct and use frequency, pie charts, histograms, stem-and-leaf diagrams, box-and-whisker plots and cumulative frequency curves (ogives).
- Outline the relative advantages and disadvantages of frequency polygons, pie charts, bar charts, histograms, stem-and-leaf diagrams and box-and-whisker plots in data analysis.
- Determine or calculate the mean, trimmed mean, median and mode for ungrouped and grouped data.
- Calculate the range, interquartile range, semi-interquartile range, variance and standard deviation of ungrouped and grouped data.
- Work with data displayed as a pie chart and use the correspondence between the angle and frequency of sectors to determine total sample size, angle and frequency of sectors.

This question was done by all candidates 69 per cent of whom scored 16 marks or more.

Part (a) (i) was very well done by most candidates. However, some candidates could not convey the fact that the individual profits were lost. This set of candidates mainly focused on the loss of the frequencies. In Part (a) (ii), many candidates used the class limits instead of the class boundaries to calculate the class width and therefore obtained an incorrect answer.

Part (a) (iii) was very well done by most candidates. Some candidates ignored the values given on the *x-axis* and drew bar charts, instead of a histogram, using their own boundaries. For Part (a) (iv), many candidates were unable to draw the diagonals on the histogram to find the mode. Some of those who drew the diagonals were unable to read the value of the mode from the histogram accurately.

Eighty per cent of the candidates performed well on Part (a) (v). The remaining 20 per cent of the candidates had difficulty calculating the midpoints or did not know how to find fx. There were a few candidates who used 5 as the total frequency instead of the total from the frequency column.

In Part (a) (vi), candidates performed well, but lacked knowledge regarding how to use the correct formula to calculate the standard deviation. In most cases, the wrong formulae was used by those who were unable to score a high mark.

Part (b) was also well done. Those who did not do well had difficulty manipulating the ratios to find the total in the sample.

Solutions

(a) (i) All individual data values are lost. (ii) 74.5 – 59.5 = 15 (iii) Histogram (iv) Mode 53.5 (v) Mean = 62.5 (vi) S.D. = 19.03

(b) (i) 80 (ii) $36°$ (iii) 16 walkers

Module 2: Managing Uncertainty

Question 3

This question tested candidates' ability to:

- Calculate the probability of events and the intersection of two events.
- Draw a Venn diagram to display given information.
- Calculate probabilities and conditional probabilities from contingency tables.
- Enumerate possible sample selections in sampling without replacement and calculate probability and conditional probability of specified events.

Part (a) (i) a) was generally well answered given the options available. Many candidates erroneously considered $P(F \cap M) = 0.5 \times 0.45$. This was the most common mistake, while other candidates found the union of the two sets instead of the intersection. The majority of candidates did Part (a) (i) b) correctly, recognizing that they were to subtract their previous answer from 0.5.

Part (a) (ii) was not well done with more than half of the candidates only achieving one of the three available marks for this question. Most candidates were able to recognize that the Venn diagram required two intersecting circles, and were able to correctly insert the correct intersection into the diagram. Many, however, were unable to insert the correct values for F only and M only (0.5 and 0.45 respectively were the most common mistakes). Many candidates often neglected to include $P(F \cup M)'$ in their Venn diagram. Even when this value was excluded, approximately 70 per cent of the candidates were able to correctly answer Part (a) (iii) based on their Venn diagram.

Parts (b) (i) and (ii) were very well done with over 87 per cent of the candidates giving the correct responses of $\frac{4}{9}$ and $\frac{2}{5}$ respectively. The majority of candidates was able to identify the correct numerator of 12 for Part (b) (iii). However, several candidates gave the final answer as $\frac{12}{50}$ as opposed to the correct response of $\frac{12}{90}$.

Part (b) (iv) was not well done with only approximately 15 per cent of the candidates achieving full marks on this question. The majority of candidates correctly stated the formula for the conditional probability and most were also able to identify the correct denominator of $\frac{4}{9}$. However, there were several answers given in the numerator, with $\left(\frac{18}{90} \times \frac{4}{9}\right)$, $\left(\frac{1}{9} \times \frac{4}{9}\right)$ or $\left(\frac{28}{90} \times \frac{4}{9}\right)$ being the common errors. The correct numerator should have been $\frac{1}{9}$, giving a final answer of $\frac{1}{4}$. This answer could have been obtained straight from the table as well.

Part (c) (i) was attempted by the majority of candidates, with approximately 50 per cent of them achieving full three marks. Many candidates used a tree diagram to obtain the possibility space, but some of them did not list the final results. Several of the candidates who did not use a tree diagram only listed four or six of the possible eight combinations.

Part (c) (ii) was not well done by candidates, with less than 25 per cent of all responses gaining full marks. The common mistake was to take the correct response of $\frac{7}{13}$ and multiply it by $\frac{8}{14}$.

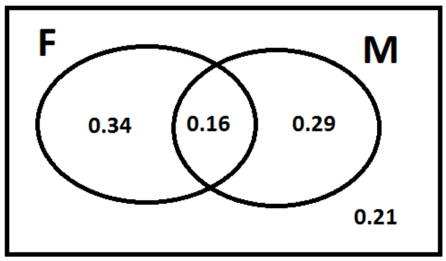
Part (c) (iii) was done satisfactorily. Many candidates were able to achieve the correct response of $\frac{30}{91}$. Candidates who calculated individual probabilities often miscalculated one or more of these leading to an incorrect answer. Some candidates only used two possible combinations instead of 3.2

Solutions

3. (a)

(i) (a) 0.16 (b) 0.34

(ii)



(iii) 0.21

(b) (i) $\frac{4}{9}$ (ii) $\frac{2}{5}$ (iii) $\frac{2}{15}$ (iv) $\frac{1}{4}$

(c) (i) CCC, CCV, CVC, CVV, VVV, VVC, VCV, VCC

(ii) $\frac{7}{13}$ (iii) $\frac{30}{91}$

Question 4

This question tested candidates' ability to:

- Construct a probability distribution table and compute probability and expected values.
- Identify a binomial distribution and state its parameters.
- Calculate probabilities using binomial distribution.
- Use a normal approximation to evaluate probabilities.

For Part (a) (i), most candidates were able to prepare the probability distribution table; however, a minority confused decimal places and significant figures. A small percentage of candidates used fractions thus losing the mark awarded for the correct number of decimal places.

In Part (a) (ii), most candidates were able to calculate P $(2 \leq X \leq 4)$, however, a small percentage of candidates interpreted this to be P $(X = 3)$. In Part (a) (iii), the majority of candidates had a general idea regarding how to work out the expected number of errors per page.

For Part (b), candidates knew the name of the distribution of Y and were able to state the correct parameters. Most candidates were able to calculate $P(Y = 4)$, however, a large number of them experienced difficulty finding P $(Y \geq 1)$. Some candidates summed P $(Y = 0)$ and $P(Y = 1)$ while others did not seem to understand the concept of *at least*.

For Part (c), many candidates failed to find the percentage of candidates who would get a Grade A, although having gone through all the correct steps. Some candidates added or subtracted 0.5 to or from 82 (Grade A). Most candidates demonstrated a keen grasp of using the distribution table.

Solutions
(a) (i)

Number of errors, X	0	1	2	3	4	5
P(X = x)	0.166	0.211	0.280	0.186	0.100	0.057

(ii) $P(2 \leq X \leq 4) = 0.566$ (iii) $E(X) = 2.01$

(b) (i) $Y \sim Bin(10, 0.3)$ (ii) $P(Y = 4) = 0.20$ $P(Y \geq 1) = 0.97$

(c) Grade A = 4.01%

Module 3: Analysing and Interpreting Data

Question 5

The question tested candidates' ability to:

- Calculate from a 95 per cent confidence interval for a population mean.
- Compute the sample mean, standard deviation and interval width.
- Identify the approximate distribution of the sample mean.
- Calculate the probability that the sample mean is less than a given value.
- State the null hypothesis H0 and an alternative hypothesis H1.
- Apply a two-tailed z-test procedure.
- Determine the critical value from tables for a given test and level of significance.
- Identify the critical region for a given test.
- State a valid conclusion of the given test.

This question was attempted by most candidates. In Part (a) (i), candidates correctly found the width of the interval, Part (a) (ii) was fairly well done, most candidates recognized that the midpoint of the interval was the mean. However, Part (a) (iii) proved to be a challenge to many who attempted it. Candidates did not use the fact that $1.96 \times \frac{\sigma}{\sqrt{60}} = 2.74$.

In Part (b), candidates overlooked that the question asked for the distribution of $\bar{X}$ and so few of them were able to write that $\bar{X}$ has a normal distribution. This then led to answering Part (b) (ii) incorrectly. Some candidates used a continuity correction, + 0.5 or – 0.5 in the calculation of $P(X < 28)$. Converting to Z was well done and candidates were able to use the probability tables correctly.

In Part (c) (i), candidates used the alternate hypothesis H_1: $\mu < 48$ even though the question asked for a change. The correct statement should have been H_1: $\mu \neq 48$. In Part (c) (ii), many of the candidates who used H_1 : $\mu < 48$ also used just one rejection region. Some candidates who had the correct statement for H_1 proceeded to read the wrong value from the table, a few candidates went to the t-distribution and used 49/2 as the sample size. Candidates who correctly set up their hypotheses were able to complete the question correctly.

Solutions

(a) (i) 5.48 (ii) mean = 16.8 (iii) S.D. = 10.8

(b) (i) $\bar{X} \sim N(25, \frac{144}{81})$ (ii) $P(\bar{X} < 28) = 0.9898$

(c) (i) $H_o: \mu = 48$; $H_1: \mu \neq 48$ (ii) $z < -2.054$ or $z > 2.054$ (iii) $Z_{test} = -2.43$ (iv) Reject H_0

Question 6

This question tested candidates' ability to:

- Distinguish between a dependent and independent variable in a bivariate pair of variables.
- Draw scatter diagrams, on graph paper provided, to represent bivariate data.
- Compute the sample mean point $(\bar{x}, \bar{y})$ and plot it on the scatter diagram.
- Draw the regression line of $y \text{ } on \text{ } x$ passing through $(\bar{x}, \bar{y})$ on a scatter diagram.
- Use the regression line to estimate a y-value from a given x-value.
- Apply a χ^2test for independence in a contingency table.
- Formulate the null hypothesis H_0 and the alternative hypothesis H_1.
- Compute expected contingency values.
- Determine the degree of freedom and critical values of the test from χ^2 tables for a given level of significance.
- State clearly the conclusion drawn from the test.

In Part (a), most candidates were able to identify the independent and dependent variables. In Part (b), some candidates had difficulty understanding the scale given for the graph, and therefore plotted some points incorrectly. Drawing the regression line also proved to be difficult for some. Some candidates did not observe that the regression line was to pass through the point $(\bar{x}, \bar{y})$.

Some candidates who plotted the point $(\bar{x}, \bar{y})$ did not find a second point to draw the line. In Part (b) (v), most candidates correctly used the line to make the estimate for the person who is 22 years old. Some candidates had difficulty rounding values correctly (for example, 7.98 rounded to the nearest whole number was given as 9).

In Part (c) (i), candidates confused the null and alternative hypotheses. In Part (c) (ii), the common error was incorrect reading of the table — candidates used the reading from the 5 per cent instead of the 95 per cent on the chi-squared test values table; calculating the degrees of freedom also gave some candidates a problem.

Part (c) (iii) was well done. In Part (c) (iv), whereas candidates could correctly determine whether to accept or reject the null hypothesis, this was affected by incorrectly stating the null hypothesis. Many candidates did not write a valid conclusion although they may have chosen to accept or reject the null hypothesis.

Solutions

(a) (i) Income – independent, monthly rent – dependent
(ii) Advertising – independent, sales – dependent
(iii) Damage done – independent, repairs – dependent

(b) (ii) $\bar{x} = 30$, $\bar{y} = 8$ (v) $x = 22$, $y = 6$

(c) (i) H_0: No association between school and grade
H_1: There is an association between school and grade.
(ii) c.v : $\chi^2(0.05)(6) = 12.59$ (iii) E(number of students) = 12.72 (iv) Accept H_0

Paper 032 – Alternative to the School-Based Assessment (SBA)

Module 1: Collecting and Describing Data

Question 1

This question tested candidates' ability to:

- Distinguish between qualitative and quantitative data as well as between discrete and continuous.
- Identify sampling methods.
- Describe and apply the systematic random sampling method.
- Construct a stem-and-leaf diagram.
- Determine the mode, median and interquartile range.
- Calculate the mean to a specified accuracy.
- Describe the shape of a distribution.

In Part (a), candidates correctly identified the types of variables. For Part (b), candidates were also able to identify the sampling methods. In Part (c), candidates were able to adequately explain the process of systematic random sampling. In Part (d) (i), a very common error was the failure to include a key for the stem-and-leaf diagram. Part (d) (ii) c), computing the interquartile range from the data was a challenge for some candidates.

Candidates recognized that the data was skewed in Part (d) (iv), but many of them identified it as positively skewed rather than negatively skewed.

Solutions

1. (a) (i) qualitative (ii) quantitative (iii) discrete

(b) (i) stratified (ii) random

(c) Arrange the names in some order (for example, alphabetical).

Randomly select a name.

Thereafter select every fifth name until 30 names are selected.

(d) (i)

Stem	Leaf							
2	5	8						
3	2	7	8					
4	2	3	5	8				
5	0	4	4	4	6	7	7	9
6	0	1	3					

(ii) (a) mode = 54 (b) median = 52 (c) IQR = 17

(iii) Mean = 48.15

(iv) nNgatively skewed

Module 2: Managing Uncertainty

Question 2

This question tested candidates' ability to:

- Construct a probability distribution.
- Identify elements of an event.
- Calculate the probability of an event.
- Calculate the expected value.
- Use the normal distribution to calculate a probability.
- Recognize the conditions for using the binomial distribution.
- Use the binomial distribution to calculate a probability.
- Apply the normal distribution as an approximation for the binomial distribution.

In Part (a) (i), candidates were able to state the values of the random variable X, but the calculations of the probabilities proved to be a challenge. Some candidates had probabilities that did not add to 1as expected.

Since Part (a) (ii) depended on the results of the table in Part (a) (i), most of the candidates who did not score well on Part (a) (i) also did not get this part correct. Some candidates also interpreted "*the probability that more than two rings will ...*) to be P(X ≥ 2) rather than P(X > 2).

The E[X] asked for in Part (a) (iii) depended on the result from Part (a) (ii), but many candidates were able to use the formula that E[X] = ∑ x P(X = x).

In Part (b), many candidates correctly stated the correct probability statement P(X > 75) but then in converting X to Z, many candidates divided by √20 instead of just dividing by 20; 20 was given as the standard deviation, candidates treated this as the variance. Another challenge in this part was recognizing that P (Z > z) = 1 – Ø (z). Some candidates did not subtract. The question asked candidates for a percentage of values, candidates did not convert their probability to a percentage.

In Part (c) (i), candidates correctly calculated the expected number of cars as E[X] = n.p. In P©art (ii) candidates were asked to calculate P(X is more than 90). Many candidates interpreted this as P(X ≥ 90).

The next major error in this part was the application of the continuity correction; either candidates did not use the correction, or they subtracted the 0.5, rather than adding the 0.5 since 90 should not have been in the range.

Module 3: Analysing and Interpreting Data

Question 3

This question tested candidates ' ability to:

- State fully the approximate distribution of the sample mean.
- Calculate unbiased estimates for the mean and the variance.
- Formulate null and alternative hypotheses using statistical symbols.
- Evaluate the z-test statistic.
- Apply the z-test and state clearly the conclusion drawn.
- Interpret parameters in the regression line equation.
- Use the regression line to estimate values of y.

Part (a) was well done. Candidates were able to correctly able to state the distribution for $\bar{X}$.

Candidates were able to correctly calculate the mean, but for the variance, many candidates did not multiply be the correction factor $\frac{60}{59}$.

Many candidates were able to set up the null and alternative hypotheses in Part (c) (i), but in Part (c) (ii) candidates had problems defining the critical regions. Although candidates recognized a two-tailed alternative in Part (c) (i), they gave only one side of the rejection, or they used the one-tailed region. Answers given were $z > 1.96$, $z < 1.96$ or $z > 1.645$.

For Part (c) (iii), in calculating Z_{test}, candidates used $\mu - \bar{x}$ rather than $\bar{x} - \mu$.

All candidates who attempted Part (d) got it correct.

Solutions

(a) $\bar{X} \sim N\left(20, \frac{169}{45}\right)$

(b) (i) The mean is 12.63 (ii) The variance is 168.67

(c) (i) H_0: $\mu = 55$

H_1: $\mu \neq 55$

(ii) Critical region: $z > 1.96$ or $z < -1.96$ (iii) $Z_{test} = -1.385$

(iv) Accept H_0. The mean mass of the packages is 55 grams.

Paper 031 – School-Based Assessment (SBA)

Generally, the standard and presentation of the samples submitted were satisfactory. In most cases, appropriate topics were selected based on the guidelines of the syllabus.

Project Title

Approximately 15 per cent of the titles in the samples were too vague. A typical example was "chubby soda kids". In addition, some titles were too wordy. It is recommended that students be encouraged to relate their title to real-life situations. Teachers are required to pay keen attention to the selected title being used.

Purpose of Project

Fifty per cent of students failed to identify the variables used and as a result of this, most of them scored only one out of two marks.

Method of Data Collection

This section was well done. However, some students listed the techniques but failed to clearly integrate them as part of their method of data collection. Also, several students gave definitions instead of sufficiently explaining how the project was executed.

Presentation of Data

Students were able to score well in this section. Many of them demonstrated mastery in the construction of histograms, pie charts, stem and leaf, box plots, the chi-square and regression correlation. In general, the presentation of data was well done; however, too many students presented long tables of data. Students should be reminded to append these at the end of the project in the appendix.

Statistical Knowledge/Analysis of Data

The majority of students demonstrated a good grasp of statistical concepts in the syllabus and scored well in this area. Some students who used chi-square tests were able to:-

- Apply the Yates correction on 2 X 2 contingency tables.
- Merge cells with frequencies less than 5.

However, some students erroneously utilized inappropriate testing methods for the data they were working with. For example, remembering to abandon the chi-square test if more than 20 per cent of expected frequency is less than 5.

Discussion of Findings/Conclusion

In several cases, this section was too verbose and much of it irrelevant to the purpose of the project. Approximately 90 per cent of students included and interpreted their findings and were able to earn a score of 3–5 marks out of a total of 5 marks. Students should be reminded to draw suitable conclusions based on their findings.

Communication of Information

The majority of candidates was able to obtain full marks in this area; however, students are encouraged to proof-read their work prior to final submission.

List of References

Very few candidates failed to list relevant references. Students should seek to include multiple references preferably using the APA style.

UNIT 2

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of approximately 63.6 out of 90, standard deviation of 18.7 and scores ranging from 16 to 90.

Paper 02 – Essay

Module 1: Discrete Mathematics

Question 1

This question tested candidates' ability to:

- Formulate a linear programming model in two variables from real-life-situations.
- Derive and graph linear inequalities in two variables.
- Identify the objective function and constraints of a linear programming problem.
- Determine the solution set that satisfies a set of linear inequalities in two variables.
- Determine the feasible region of linear programming problem.
- Determine, from a graph, the optimal solution (where it exists) of a linear programming problem.
- Determine the shortest and longest path in a graph.

For Part (a) (i), most candidates neglected to state *maximize* P when formulating the objective — profit function. Part (a) (ii) was poorly done. Many candidates seldom converted hours to minutes or vice versa when writing the inequalities. Very few candidates identified the inequalities, $x \geq 0$; $y \geq 0$, and rarely substituted the coordinates of all vertices into the profit function. For Part (a) (iii), most candidates selected where the lines intersected as the combination of bottles for the maximum profit instead of checking all boundary points.

In Part (b), many candidates had difficulty calculating the shortest path.

Solution

(a) (i) a) Let x be the number of cases of glass bottles.
Let y be the number of cases of plastic bottles.
Maximize $P = 20x + 15y$

b) $4x + 4y \leq 3000$, $3x + 6y \leq 3000$, $x \geq 0$, $y \geq 0$

(ii) a) Plot above inequalities of provided graph paper.
b) Identify feasible region on graph.

(iii) a) By substituting coordinates of vertices of feasible region in P, 750 cases of glass bottles and 0 plastic bottles will maximize profit.
b) Maximum profit is $15 000.

(b) Shortest path: SCDFG
Longest path: SDFEG or SCEDFG or SDCEFG

Question 2

This question tested candidates' ability to:

- Use the Hungarian algorithm to make optimal assignments.
- Calculate the optimal value.
- Use truth tables to determine whether a given proposition is a tautology or a contradiction.
- Use the laws of Boolean algebra to simplify Boolean expressions.
- Represent a Boolean expression for circuits.
- Draw circuits corresponding to Boolean expressions.

The overall handling of the question was satisfactory, with most candidates demonstrating a reasonably good grasp of the essential concepts.

Part (a) was the most difficult subpart which required the use of the Hungarian algorithm. Some of the candidates drew incorrect matrices and others also used alternative methods to derive a solution. An estimated 60 per cent of candidates demonstrated an acceptable grasp of the Hungarian algorithm.

The strongest area for the majority of candidates was Part (b) which required the construction of a truth table to determine whether the given expression was a tautology or contradiction. An estimated 90 per cent of candidates handled this adequately.

Part (c) was generally well answered. Candidates also demonstrated strong competency in writing the Boolean expression for the circuit in Part (c) (i), with over 85 per cent of them performing well. A notable proportion of candidates, however, struggled to simplify the Boolean expression.

Solutions

(a) (i) From Hungarian algorithm: Mrs Jones gets Class 1 or 2, Mr James is assigned to Class 4; Mrs Wright gets Class 2 or 1 (reverse from Mrs Jones); Ms Small is assigned to Class 3.

(ii) Total time: hours (30+30+31+29) =120 hours

(b) From truth table: $(p\wedge q)\rightarrow p$ is a tautology

(c) (i) $a\wedge[b\vee(a\wedge b)]\wedge[a\vee(\neg a\wedge b)]$

(ii) $(a\wedge b)$

Module 2: Probability and Distributions

<u>Question 3</u>

This question tested candidate' ability to:

- Identify and use the binomial distribution to calculate a probability.
- Solve an optimization problem based on the mean and standard deviation of a binomial distribution.
- Approximate a binomial distribution with a Poisson distribution to find a probability.
- Use the normal distribution with a continuity correction to approximate a Poisson distribution.
- Calculate a probability using a geometric distribution.

Candidates responded well to this question.

In Part (a) (ii), approximately 50 per cent of candidates, changed the inequality to an equation and solved. This resulted in an incorrect final solution. Many candidates simplified the equation obtained from the quotient of the formulas for standard deviation and mean, substituting the given value until the final stage of the solution, rather than substituting and then simplifying which involved less algebraic manipulation. Candidates should be encouraged to solve the inequality from the start to the end of the problem and not to change to an equation.

For Parts (b) and (c), more than 50 per cent of candidates did not use the appropriate approximations in the solution, and then in Part (c), many candidates did not use the continuity correction. Many candidates had difficulty reading the normal distribution tables.

In Part (d), candidates did not apply the correct formula for the geometric distribution. Many candidates interpreted the question as ‘less than or equal to’ rather than just ‘less than’.

Solutions

3. (a) (i) 0.833 (ii) $n = 401$

(b) 0.544

(c) 0.8907

(d) 0.657

Question 4

This question tested candidates' ability to:

- Adequately use the properties of probability density and cumulative functions.
- Carry out all required steps in a χ^2 goodness-of-fit test to determine whether a given data set may be modelled by a normal distribution.

For Part (a) (i), most candidates had difficulty performing the integration correctly. Many of them just substituted the value a = 2.2 in the expression $a+bx$ and equated it to 0 and therefore arrived at an incorrect value for *b*. Hence they were unable to prove that *a* = 2.2. Part (a) (ii) was well done.

For Part (b) (i), most candidates were able to obtain a value for 'm' and 'n'. However, a few candidates did not multiply by 400 as required to obtain the two values. In Part (b) (ii), approximately 25 per cent of candidates came to a conclusion without setting up a null or alternative hypothesis. Candidates also used n-1 to calculate the degrees of freedom.

Solutions

(a) (i) Integrate to obtain simultaneous equations to solve to obtain
a = 2.2, b = -2.4.

(ii) P(X > 0.5) = 0.2

(b) (i) m = 136.5, n = 63.5

(ii) H_0: data may be modelled by normal with a mean of 9 and variance of 1
H_1: data may <u>not</u> be modelled by normal with a mean of 9 and variance of 1
At 5% significance level and degrees of freedom of 3, the critical chi square value is 7.815.
The calculated chi square value is 84.024.
Conclusion: Reject the H_0 at 5% significance level and conclude that data may not be modelled by a normal distribution with a mean of 9 and variance of 1.

Module 3: Particle Mechanics

Question 5

This question tested candidates' ability to:

- Resolve forces.
- Apply Newton's laws of motion.
- Solve problems involving displacement, velocity and acceleration.
- Create first order differential equations for the motion of a particle in a straight line.

In Part (a) (i), the majority of candidates scored at least two out of the three marks. While the diagram was drawn accurately most candidate were unable to insert the normal reaction force and friction accurately on their diagram. In Part (a) (ii), most candidates did not identify all forces acting on the 5 kg mass. As a result, the second equation required was not obtained and consequently the question was poorly done.

Part (a) (iii) was not attempted by many candidates. However, of those who attempted the question about 50 per cent obtained the maximum four marks for the question.

Part (b) (i) was generally done well. The common errors for this question were

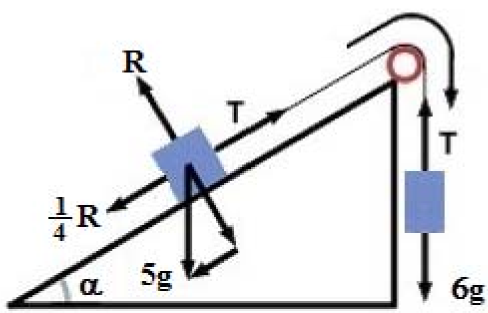
- direct substitution of $t = 3$ before differentiation
- incorrect differentiation.

Part (b) (ii) was attempted by most candidates with the majority of them obtaining at least two marks. The main area of difficulty for candidates was the process of integration. This was either done incorrectly or without the consideration of the components, *i* and *j*.

In Part (c), many candidates assumed constant acceleration and thus used the incorrect formula to determine the differential equation. The use of $F = ma$ and $a = \frac{dv}{dt}$ was widely applied, resulting in many candidates receiving half of the available mark. Most of the candidates who were able to obtain full marks for this question did so at the first step and then proceeded to solve the differential equation.

Solution

(a) (i)



(ii) $T = 36.8\ N$

$a = 3.87\ ms^{-2}$

(i) The force exerted by the string on the pulley = 54.6 N

(b) (i) *Acceleration* = $30i - j$

(ii) *Position of the particle* = $45 + \frac{15}{2}j$

(c) Differential equation: $m\frac{dv}{dt} = mg - kv$

Question 6

This question tested candidates' ability to:

- Apply the principle of conservation of linear momentum to the direct impact of two inelastic particles moving in the same straight line. (Knowledge of impulse required).

- Apply Newton's law of motion to a system of two connected particles.
- Apply the principle of the conservation of energy.

Part (a) was generally well done. Generally, candidates had knowledge of Newton's Second law of motion; however, many did not distinguish between acceleration and velocity. Newton's law F = ma was preferred over Ft = mv-mu (Impulse). Alternatively, a number of candidates used the power equation P = F/v to solve the problem.

Part (b) was generally well done by those candidates who attempted it. However, some candidates did not acknowledge the direction of the velocities. Most responses only contained the magnitude of the resultant velocity. Some candidates failed to acknowledge the fact that after the collision, the coupled trucks had the sum of the masses of the two trucks.

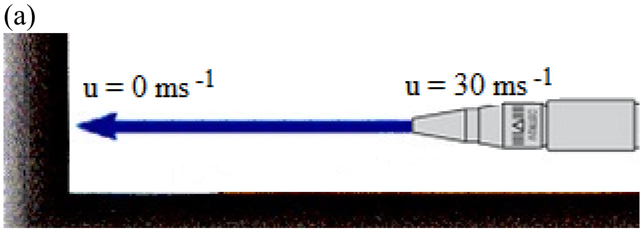
Part (c) was attempted by most candidates; however, many did not use or state the principle of conservation of mechanical energy. Additionally, many candidates failed to acknowledge that the 3M mass had no change in potential energy. Furthermore, candidates generally ignored the change in kinetic energy of the 1.5 M mass. Some candidates interpreted M to mean mega, that is, 10^6 while others ignored it completely.

Many candidates used a force balance along with Newton's equations of motion to solve the problem rather than use the principle of conservation of mechanical energy as specified in the question.

Generally most candidates did Part (d) well; however, some left off 'g' when forming their simultaneous equations to calculate the acceleration.

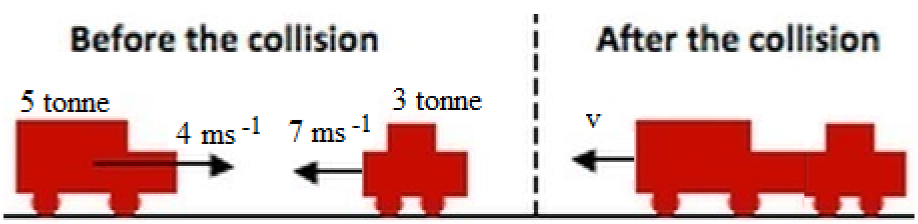
Solution

6. (a)



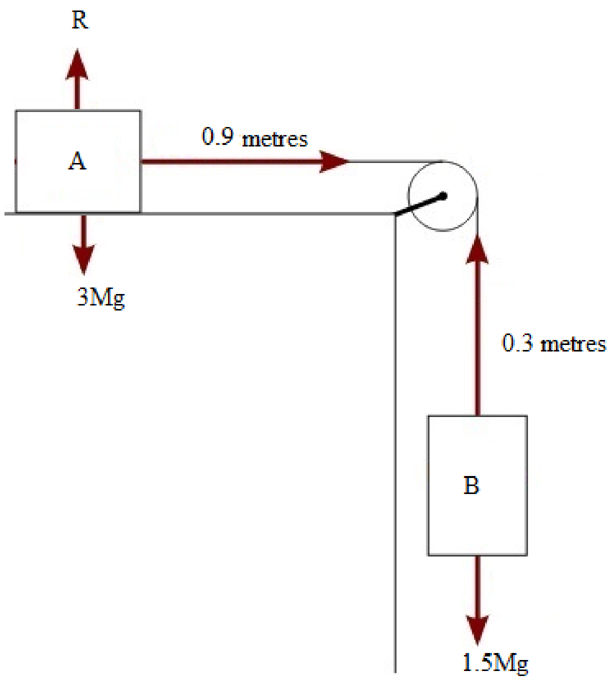
Force exerted on the wall = 300 N

(b)



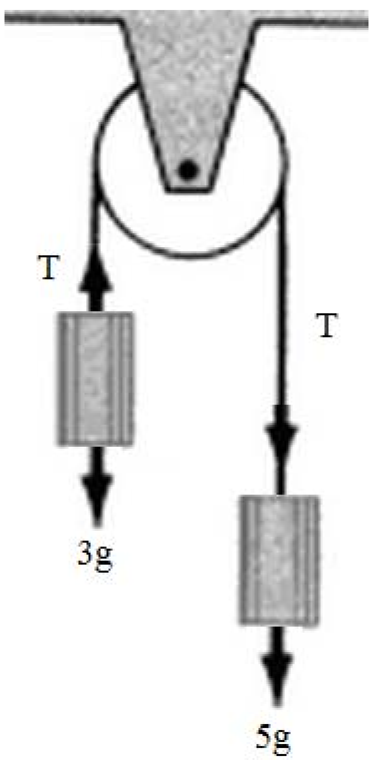
Velocity of coupled trucks = *0.125 ms*$^{-1}$
Direction is the same as the three-tonne truck.

(c)



Speed of the block A, when it reaches the edge of the table = *2.45* ms^{-1}

(d)



$a = 2.5\ ms^{-2}$

Paper 032 – Alternative to School-Based Assessment (SBA)

Module 1: Discrete Mathematics

Question 1

Candidates performed poorly on this paper. Understanding of the basic concepts was weak.

This question tested candidates' ability to:

- State the converse, inverse and contra-positive of implications of propositions.
- Derive a Boolean expression from a given switching or logic circuit.
- Establish the truth value of propositions that involve conjunctions, disjunctions and negations.

In Part (a), most candidates were able to correctly state the converse and the inverse of p => ~q but could not write the contra-positive. In Parts (b) (i) and (ii) only partial answers were given. In Part (c), candidates could not set up the proposition for the alarm to trigger, $s \wedge (w \vee d)$ and, therefore, had problems with the construction of the truth table.

Solutions

(a) ~p => p; ~p ∧ q; q ∧ ~p

(b) (i) [a ∨ (~b ∧ c)] ∧ (ii) [(A ∨ B) ∧ (A∨ B ∨ C)] ∨ (~B ∧ C)

(c) s ∧ (w ∨ d) =

s	w	d	(w ∨ d)	s ∧ (w ∨ d)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Module 2: Probability and Distributions

Question 2

This question tested candidates' ability to:

- Solve problems involving the probability density function and the cumulative distribution function.
- Use the Poisson distribution to calculate probabilities.
- Use the binomial distribution to calculate probabilities.
- Calculate the number of ordered arrangements of *n* object taken *r* at a time, with or without restrictions.

Candidates demonstrated a lack of understanding of basic probability concepts. Many of them could not derive the simultaneous equations required to solve the problem.

Candidates interpreted at least four incorrectly.

In Part (c), although candidates recognized that it was a binomial distribution, they calculated P(X = 3) rather than P(X≤ 3).

Candidates confused the concepts of permutations and combinations. Many candidates did not know how to calculate arrangements with repetitions.

Solutions

(a) a = $\frac{-1}{15}$, b = $\frac{1}{15}$
(b) P(Y ≥ 4) = 0.979 (3 s.f)
(c) P(X ≤ 3) = 0.984 3 s.f.)
(d) (i) 210 (ii) $\frac{8}{21}$
(e) 60 480

Module 3: Particle Mechanics

<u>Question 3</u>

This question tested candidates' ability to:

- Distinguish between distance and displacement, and speed and velocity.
- Calculate and use displacement, velocity, acceleration and time in simple equations representing the motion of a particle in a straight line.
- Formulate and solve first order differential equations as models in linear motion of a particle when the applied force is proportional to its displacement or its velocity.
- Resolve forces, on particles, in mutually perpendicular directions.
- Use the principle that for a particle in equilibrium, the vector sum of the forces is zero.
- Solve problems involving concurrent forces in equilibrium.

In Part (a), candidates mostly failed to identify the appropriate relationship between a and t^2, and so could not derive the appropriate differential equation to solve for the velocity.

In Part (b), candidates failed to demonstrate a grasp of basic concepts in force, work and displacement. Most candidates used the incorrect function (sine) instead of the correct trigonometric function (cosine) when resolving the resulting force horizontally. Candidates then had problems recognizing the use of simultaneous equations to find the forces X and Y.

Solutions

(a) $V = \frac{t^3}{14} + \frac{24}{7}$
(b) X = 55.8 N at 240° to the x-axis , Y = 63.9 N at 120° to the *x*-axis

Paper 031 – School-Based Assessment (SBA)

The topics chosen for this internal assessment were suitable for students at this level. Students mainly used aspects from the Discrete Mathematics portion of the syllabus to showcase their mathematical abilities with only a few of them venturing into the applications of Module 3 (Mechanics). Efforts made by students who chose to apply this module were highly commendable.

Statement of Task

Most students obtained full marks in this section. Approximately 30 per cent of students failed to clearly identify and define variables, thus obtaining two out of three marks. In some instances, students provided irrelevant information and seemed unsure about the requirements to fulfil their purpose.

Data Collection

This section was well done. The majority of students presented suitable data and were able to obtain full marks.

Mathematical Knowledge/Analysis

A substantial number of students gained maximum marks in this section and showcased a grasp of the mathematical concepts required.

Evaluation

Conclusions drawn by several students were directly related to their initial statement of task; however, marks were lost due to the fact that they failed to list any limitations encountered during their investigation. Furthermore, no recommendations were given; hence marks were lost in this area.

Communication of Information

This section was generally well done by students.

Strengths and Weaknesses

Strengths

- Good application of chi-square test
- Use of the z-table to find critical values
- Proper application of the Hungarian algorithm

Weaknesses

- Incorrect interpretation of r the product moment coefficient.
- Confusion regarding appropriate method of analysis relative to tasks.
- Failure to state the accurate alternative hypothesis when applying hypothesis testing. For example, if the test statistic is negative, then the alternative hypothesis must be less than or not equal to the value being tested.
- Using the chi-square contingency table to test for relationships instead of independence.

Recommendations

Students should practise more problems involving the use of the chi-squared test, normal distribution and mechanics.

Students need to have a comprehensive understanding of the various modules, paying special attention to the mechanics module of Unit 2.

Teachers should instruct students to cite references to inculcate good research skills from early.

Students should be encouraged to work independently. Teachers should guide students to choose distinct topics and carry out their own investigations.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2018

APPLIED MATHEMATICS

GENERAL COMMENTS

The CAPE Applied Mathematics examination is based on three papers for each unit covered in the syllabus: Paper 01, a multiple-choice paper consisting of 45 compulsory items, 15 from each of the three modules; Paper 02 consisting of six compulsory questions, two from each of the three modules; and Paper 032, an alternative practical paper for candidates who do not register for the School-Based Assessment (SBA) consisting of three compulsory questions, one from each module. Paper 02 is divided into three sections: Section A with two structured questions, from module 1, Section B with two structured questions, from Module 2 and Section C with two structured questions from Module 3. Each question on Paper 02 is worth a total of 25 marks.

Applied Maths is a two-unit course with each unit further divided into modules.

Unit 1, Statistical Analysis**,** is divided into three modules as follows:

- Collecting and Describing Data
- Managing Uncertainty
- Analysing and Interpreting Data

Unit 2, Mathematical Applications, is divided into three modules as follows:

- Discrete Mathematics
- Probability and Probability Distributions
- Particle Mechanics

The examination for each unit comprises three papers: Paper 01, consisting of 45 multiple-choice items, Paper 02, consisting of six essay questions, two questions from each module and Paper 031, the School-Based Assessment, which is a project examined internally by class teachers and moderated by CXC. Paper 032 is done by those candidates not registered in a school and who do not do the School-Based Assessment. This paper consists of three questions, one from each module.

This year marked the third time that the marking process was done as an online activity using RM Assessor. Markers were trained in the use of this medium for marking. The SBAs were fully marked using the online process this year.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

For Unit 1, the percentage of candidates earning acceptable grades (Grades I–V) was approximately 93 per cent compared with 95 per cent in 2017. There was a decline in the number of candidates attaining Grade I — 50 per cent in 2018 compared with 56 per cent in 2017. The paper had a mean score of approximately 67 out of a maximum of 90.

For Unit 2, the percentage of candidates earning acceptable grades (Grades I–V) was approximately 91 per cent, which was consistent with performance in 2017. There was a decline in the number of candidates attaining Grade I — 46 per cent in 2018 compared with 55 per cent in 2017. The paper had a mean score of approximately 70 out of a maximum of 90.

Paper 02 – Structured Responses

Module 1: Collecting and Describing Data

Question 1

This question tested candidates' ability to do the following:

- Distinguish between qualitative and quantitative data.
- Distinguish among the following methods: simple random, stratified random, systematic random, cluster and quota sampling.
- Use simple random, stratified random, systematic random, cluster and quota sampling to obtain a sample.
- Explain why sampling is necessary.
- Determine or calculate the mean, trimmed mean, median and mode for grouped or ungrouped data.
- Calculate the range, interquartile range, semi-interquartile range, variance and standard deviation of grouped and ungrouped data.

Solutions

(a) Classifying variables

• Number of performers at last month's Jazz	Discrete
• Times recorded for the athletes in the 100 m	Continuous
• The heights of the players in the basketball team	Continuous

(b) Classifying data as quantitative OR qualitative

- The texture of the material used — Qualitative
- The number of phone calls — Quantitative
- The heights of the parsley plants — Quantitative

(c) Classifying as a population or a sample

- The 100 cars sold — Population
- The 100 patients selected — Sample
- The 100 children — Population

(d) (i) Number of Year 1 males in the sample

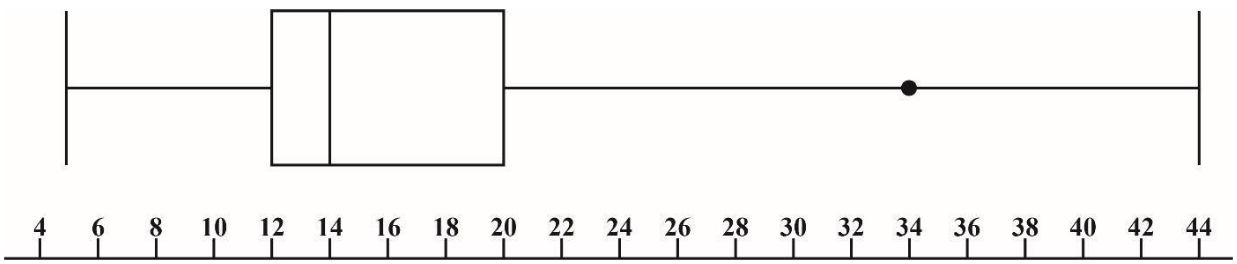
$\frac{175}{750} \times 30 = 7$

(ii) Advantages and disadvantages of stratified random sampling

Advantage	**Disadvantage**
All groups will be sampled. Proportional sample from each group. Good representation of the entire population Each stratum reproduces the characteristics of the population. Reduces bias (less bias).	Population must first be classified. Putting the population into unique groups may be difficult. Time consuming and tedious. Sampling frame known. Costly.

(e) (i) Mode = 14
Median = 14
Upper and lower quartiles = 20, 12
Mean = 17.1

(ii)



Parts (a), (b), (c) and (d) were generally very well done with many candidates getting full marks for each of these sections.

In Part (e), some candidates had problems calculating the values for each group using stratified random sampling.

In Part (f), candidates confused the definitions for cluster and quota sampling.

Question 2

This question tested the candidates' ability to do the following:

- Construct frequency distribution from raw data.
- Construct and use frequency polygons, pie charts, bar charts, histograms, stem-and-leaf diagrams.
- Outline the relative advantages and disadvantages of the mean, trimmed mean, median and mode as measures of central tendency for raw or summarized data.
- Determine or calculate the mean, trimmed mean, median and mode for grouped or ungrouped data.
- Determine quartiles and other percentiles from raw data, grouped data, stem-and-leaf diagrams, box-and-whisker plots and cumulative frequency curves.

Solutions

(a) (i)

2	3 9	
3	0 4 7	
4	5 7 9	
5	0 2 6 7	
6	0 1 5 5 8 9	
7	3 5 8	
8	0 1 8	key: 5|6 = 56

(ii) Mode = 65
Median = 60
Interquartile range = 71 – 46 = 25
60 marks or more = 13

(b) (i) Disadvantage – individual data values are lost.
(ii) Boundaries of the second class = 15.5, 20.5
(iii) Frequency density of the fifth class = 2.6

(c) (i) 40.9
(ii) 8.30
(iii) 40.6, 8.41
(iv) Mean = 41.5
Standard deviation = 15.2
Median class = 41 – 50
Median = 41.3

In Part (a), most candidates were able to construct the stem-and-leaf diagram, but forgot to include the key. Candidates were able to read off the mode and the median from the diagram, but still had difficulty with the quartiles, especially reading the lower quartile.

In Part (b), most candidates correctly stated the disadvantage of using grouped data. The boundaries proved difficult for candidates as they did not use the midpoints between the classes. Some candidates wrote the definition for calculating frequency density in the wrong order, using the class width as the numerator (using $\frac{class\ width}{frequency}$ instead of $\frac{frequency}{class\ width}$).

In Part (c) (iv), most candidates lost marks as a result of inaccurate calculations. Candidates need to recheck their calculations. Some candidates were able to identify the median class, but could not manipulate the formula for calculating the median.

Module 2: Managing Uncertainty

<u>Question 3</u>

This question tested candidates' ability to do the following:

- Calculate ($A \cup B$) and ($A \cap B$).
- Use the property of ($A \cap B$) = 0 or P ($A \cup B$) = P (A) +P (B), where A and B are mutually exclusive.
- Calculate the conditional probability (A/B), where the probability is that event A will occur given that event B has already occurred.
- Use the property that ($A \cap B$) = $P(A)P(B)$ or $P(A/B)$ = $P(A)$ where A and B are independent variables.

- Construct tree diagrams.
- Solve problems involving probability.

Solutions

(a) (i) (T) = 0.2 (ii) (L) = 0.8 (iii) $P(M \cap N)$ = 0.8; $P(M/N)$ = 0.5
(b) (i) p = 4, q = 3 (ii) 16; 12
(c) (i) 2332; 316; 117256

For (a), most candidates performed well. However, in Part (a) (iii), some candidates divided incorrectly by 0.48 instead of 0.56.

For Part (b), most candidates were able to attain the values of p and q but some interchanged the values. In Part (b) (ii), candidates were unable to find (6) but were able to determine $(H/6)$.

For Part (c), many candidates encountered problems inputting the data correctly on the tree diagram; however, most were able to follow through to calculate the probabilities in the subsequent part.

Question 4

This question tested candidates' ability to do the following:

- State the assumptions made in modelling data by a binomial distribution.
- Identify and use the binomial distribution as a model of data, where appropriate.
- Use the notation X~Bin $(n,)$, where n is the number of independent trials and p is the probability of a successful outcome in each trial.
- Calculate and use the mean of a binomial distribution.
- Calculate the probability $(X=a)$.
- Use the normal distribution as an approximation to the binomial distribution, where appropriate and apply a continuity correction.

Solutions

(a) (i) Assumptions:

- Independent trials
- Finite number of trials
- Each trial has two outcomes — success and failure
- Same probability of success for each trial

(ii) Not binomial — number of trials is not infinite; not binomial — dependent, probability is not the same for each trial; Not binomial — probability changes, dependent

(b) (i) $X \sim Bin\,(12{,}0.6)$; $P(X = 8) = 0.213$
(ii) $(X) = 48$
(iii) $(X > 55) = 0.0434$

For Part (a) (i), most candidates were able to identify all four assumptions. There were some who failed to state that p was constant. For Part (a) (ii), quite a number of candidates were able to identify that the given situations were not binomial; however, most were unable to state the correct reason to justify this.

For Part (b) (i), most candidates were able to state the correct distribution and its parameters. Many were able to identify and use the correct probability density function while there were some who used the wrong formula or values. In Part (b) (ii), most candidates were successful in computing the expectation.

Some candidates were able to successfully complete Part (b) (iii) while others made a variety of mistakes including the use of $\mu - \bar{x}$, the wrong standard deviation, and also the wrong continuity correction.

Module 3: Analysing and Interpreting Data

Question 5

This question tested candidates' ability to do the following:

- Calculate the unbiased estimates for the population mean and standard deviation.
- Construct a 90 per cent confidence interval for the population mean.
- Apply the central limit theorem in situations where $n \geq 30$.
- Apply a z-test for a population proportion when a large sample ($n \geq 30$) is drawn from a binomial distribution, using a normal approximation to the binomial distribution with an appropriate continuity correction.
- Formulate a null hypothesis, H0, and an alternative hypothesis, H1; identify the critical or rejection region for a given test and the level of significance.
- Evaluate, from sample data, the test statistic for testing a population mean or proportion.
- Apply a z-test for a population mean when a large sample ($n \geq 30$) is drawn from any other distribution of known or unknown variance, using the central limit theorem.

Solutions

(a) Unbiased estimate of $\mu = 1.791$, and unbiased estimate of $\sigma^2 = 0.662$

A symmetric 90 per cent confidence interval for $\mu = (1.696, 1.886)$

(b) Probability that in a random sample of 120 customers 77 or more will buy a hot drink
= $(Y \geq 77) = 0.027$

(c) (i) $H_0: \mu = 3.30$
$H_1: \mu \neq 3.30$
(ii) Critical region(s) for the test: z < -2.05 and z > 2.05 or $|z| > 2.05$
(iii) The value of the test statistic = $Z\text{-}_{test}$ = -1.789
(iv) Conclusions, giving a reason for your answer:

Since $Z\text{-}_{test}$ = -1.789 > -2.05

Do not reject H_0, so the mean crop is 3.30 kg.

Common errors for Part (a) included the following: confusing the two formulae for σ^2, the incorrect z-value and not dividing σ by √200.

In Part (b), common errors included not recognising that a normal approximation to binomial was needed and not applying continuity correction.

For Part (c) (i), a common error was not including μ in their hypotheses whereas for Part (c) (ii), the incorrect z-value and incorrect critical region were given.

For Part (c) (iii), the incorrect z-value was given. Additionally, candidates did not divide σ by √80. In Part (c) (iv), candidates did not state the reason for the decision made nor did they include the conclusion derived from their hypothesis. In addition, candidates used the term 'accept' instead of *do not reject*.

Question 6

This question tested candidates' ability to do the following:

- Formulate a null hypothesis, H_0, and an alternative hypothesis, H_1.
- Apply a one-tailed test or a two-tailed test, appropriately.
- Determine the critical values from tables for a given test and the level of significance.
- Identify the critical or rejection region for a given test and the level of significance.
- Evaluate the Chi-square test statistic, where O_i is the observed frequency, E_i is the expected frequency and N is the total frequency.
- Determine the appropriate number of degrees of freedom for a contingency table.
- Determine probabilities from χ^2 tables.

Solutions

(a) (i) H_0: no difference in proportion of advertisements on different channels
H_1: difference in proportion of advertisements on different channels OR
H_0: the proportion of advertisements for vehicles is independent of channels
H_1: the proportion of advertisements for vehicles is dependent on channels OR
H_0: there is no relationship between the proportion of advertisements and channels
H_1: there is a relationship between the proportion of advertisements and channels

(ii) Table which shows the observed (O) and the expected (E) frequencies

	Channel 1		**Channel 2**		**Channel 3**		**Total**
	O	E	O	E	O	E	
Honda	69	55.55	35	46.75	28	29.7	132
Kia	20	27.78	28	23.38	18	14.84	66
Toyota	12	17.67	22	14,87	8	9.46	42
Total	101		85		54		240

(iii) χ^2 test value = 15.535

(iv) Critical region of the test at the 5 per cent level of significance $\chi^2 > 9.488$

(v) Conclusion that may be drawn from this test

Since 15.535 > 9.488, reject H_0.

That is, there is evidence of difference in proportion of advertisements on different channels.

(b) (i) The equation of the regression line, c on v, in the form c = *a* + *b*v
c = 70.2 + 6.95v

(ii) An interpretation of the constants *a* and *b* in this context
a = number of sign-ups without an advert
b = number of extra sign-ups per million viewers of the advertisement

(iii) The number of customers that will sign up with the company the day after an advertisement is shown during a programme watched by 3.7 million viewers = 96

(iv) Two factors, other than number of viewers, that will affect the success of an advertisement in gaining new customers for the company

- Type of programme
- Length of advertisement

In Part (a) (i), candidates did not identify the proportion of advertisements and channels as variables. For Part (a), candidates made errors in their calculations to find the expected frequencies.

In Part (a) (iii), candidates made errors in calculating the χ^2 test value.

For Part (a) (iv), most candidates gave the incorrect degrees of freedom, incorrect tables value and incorrect critical region. Some had χ^2 test < 9.488 instead of χ^2 test > 9.488.

In Part (a) (v), candidates did not state the reason for the decision made nor did they include the conclusion derived from their hypothesis. Candidates also used the term ‘accept’ instead of *do not reject.*

For Part (b) (i), candidates incorrectly calculated *b* as $S_{vv}\ S_{vc}$ and made calculation errors finding *a.* In Part (b) (ii), candidates were not able to explain *a* and *b*. For Part (b) (iii), many candidates did not substitute correctly into their equation.

In Part (b) (iv), candidates were unable to identify two factors that would affect the success of an advertisement in gaining new customers for the company.

Paper 032 – Alternative to the School-Based Assessment

Module 1: Collecting and Describing Data

Question 1

This question tested candidates' ability to do the following:

- Distinguish among the following sampling methods — simple random, stratified random, systematic random.
- Use simple random, stratified random, systematic random to obtain a sample.
- Write a report of the findings obtained from collected data.
- Outline relative advantages of using a stem-and-leaf diagram.
- Determine the mode, median and inter-quartile range from a stem-and-leaf diagram.
- Calculate the mean for grouped data.

Solution

(a) (i) A: Simple random
B: Systematic
C: Stratified

(ii) Number of claims from 600 = 40

(iii) Number of claims to give a sample size of 80 = 1200

(iv) Cost of the investigation
Time for investigation
Distance of affected properties
Available manpower

(b) (i) All data values are shown
Easy to determine the mode, median and quartiles
Easy to determine the shape of the distribution

(ii) The range of the marks = 46
The modal mark = 27
The median mark = 17
The third quartile of the distribution = 27

(iii) Mean mark = 18.5

Candidates demonstrated sound capabilities in the recognition of simple random sampling, but stratified random and systematic sampling were not quite as clear.

Most candidates were able to calculate the sample size using stratified random sampling, but many could not calculate the number of claims needed to give a sample of 80.

Nearly all candidates were able to competently respond to the stem-and-leaf diagram. Computing the quartiles provided the biggest challenge for candidates, with most of them being unable to successfully complete this section.

Module 2: Managing Uncertainty

Question 2

This question tested candidates' ability to do the following:

- Use a contingency table to find probabilities.
- Use the notation X ~ Bin(n, p) where n is the number of independent trials and p is the probability of a successful outcome in each trial.
- Calculate probabilities using the binomial distribution.
- Calculate the mean and variance of a binomial distribution.
- Use the notation $X \sim N(\mu, \sigma^2)$ where μ is the population mean and σ^2 is the population variance.
- Solve problems involving probabilities of the normal distribution using z-scores.

Solution

(a) (i) 18/25 = 0.72

(ii) 5/13

(b) (i) n = 14
E(X) = 11.2
P(X = 12) = 0.25

(c) (i) P (X > 58) = 0.8944

(ii) n = 152 apples

Generally**,** the question was well done.

In Part (a) (i), many candidates were able to calculate the probability but the common error was that candidates did not consider the intersection of chocolate and large cake. For Part (a) (ii), whereas many candidates recognized the given probability, some of them tried to apply the formula for conditional probabilities, instead of reading the values from the table.

In Part (b), candidates had problems creating the equation npq = var. Hence, they could not calculate n. For those who calculated n, calculating E(X) and the P(X = 12) was very well done.

For Part (c), the common error was subtracting μ – x rather than x – μ when converting from x to z. In Part (d), candidates were able to read the z-tables.

Module 3: Analysing and Interpreting Data

<u>Question 3</u>

This question tested candidates' ability to do the following:

- Write an approximate distribution for $\overline{X}$.
- Apply the central limit theorem in situations where $n \geq 30$.
- Calculate and interpret the value of r, the product moment correlation coefficient.
- Calculate regression coefficients for the line y on x.

Solution

(a) (i) Distribution for $\bar{X}$: $\bar{X} \sim N(9, \frac{3.6}{80})$

(ii) $P(\bar{X} < 9.4) = 0.9704$

(b) (i) Pearson's product moment correlation coefficient

$r = 0.898$ strong, positive correlation

(ii) $y = 2.28 + 0.2x$

Candidates had difficulty applying the formula for calculating the correlation coefficient. Candidates who completed the calculation were able to interpret the value of r.

In Part (b), candidates had challenges using the formula for calculating the coefficient b. Many candidates, after calculating the values of a and b, omitted the regression equation.

Paper 031 – School-Based Assessment

The overall quality of the SBAs received was very good, and represented an improvement, when compared to 2017.

Project Titles

The reports had project titles that were relevant, concise and acceptable. There were only a few projects with unclear titles and the necessary comments attached.

Purpose of Project

The purpose of the projects was clearly stated, and this is commendable. However, the variables stated were not relevant for a number of the projects seen.

Method of Data Collection

Data collection methods were clearly described and were appropriate, in most projects. This is commendable and a clear improvement from the previous years' projects.

Presentation of Data

This section was well organized, showing clear bar charts, histograms and even scatter diagrams. Many projects included other diagrams/charts such as the box-and-whisker, which is commendable. Students should be encouraged to include other diagrams and charts in their projects.

Statistical Knowledge

The calculations were generally good. There were quite a number of projects with only one mathematical calculation, and this requires some attention. If only one calculation is used, candidates will not have enough evidence to formulate a conclusion.

Discussion of Findings

This section of the project was well done, and this is commendable. Students spoke about the evidence they found in their statistical calculations, so the projects with few calculations had little to discuss.

Communication

The communication of ideas and analysis were well done. Some candidates failed to carefully read over and check for grammatical errors.

References

More students included references in their projects and this is commendable.

UNIT 2

Paper 01 – Multiple Choice

This paper consisted of 45 items, 15 items from each module.

Paper 02 – Structured Responses

Module 1: Discrete Mathematics

Question 1

This question tested the candidates' ability to do the following:

- Use the activity network algorithm in drawing a network diagram to model a real-world problem.
- Calculate the earliest start time, latest start time and float time.
- Identify the critical path in an activity network.
- Use the critical path in decision-making.
- Use truth tables to determine if propositions are equivalent.
- Use the laws of Boolean algebra to simplify Boolean expressions.

Solution

(a) Earliest and latest start times

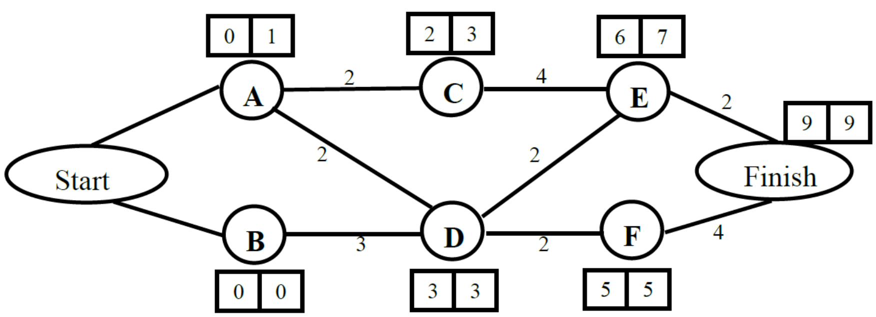
(i)

Activity	Earliest Start Time	Latest Start Time
A	0	1
B	0	0
C	2	3
D	3	3
E	6	7
F	5	5

(ii) Critical path: Start – B – D – F – Finish

(iii) Minimum completion time = 9 days

(iv)



(v) New critical activities: Start – A – C – E – Finish

New minimum completion time = 10 days

(b) (i)

p	q	~p	~pvq	p ⇒ q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

p	q	~p	~pvq	p ⇒ q
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

Part (a) was attempted by almost all candidates, with most being able to complete it. In Part (a) (i), candidates' start times were incorrect based on their diagram. For Part (a) (iii), candidates' calculation of the minimum completion time was incorrect. In Part (a) (iv), candidates did not have a start and finish and provided incorrect durations.

For Part (a) (v), candidates incorrectly calculated the new minimum completion time.

Part (b) was attempted by most candidates, who were successful in Part (b) (i) but not in Part (b) (ii). One common mistake made in Part (b) (i) was that candidates did not substitute values correctly into the statements. In Part (b) (ii), there were several common mistakes including the incorrect use of the associative, distributive and De Morgan's laws and the use of truth tables to prove the Boolean algebra.

Question 2

This question tested the candidates' ability to do the following:

- Derive and graph linear inequalities in two variables.
- Determine the solution set that satisfies a set of linear inequalities in two variables.
- Determine the feasible region of a linear programming problem.
- Identify the objective function and constraints of a linear programming problem.
- Determine a unique optimal solution for a linear programming problem.

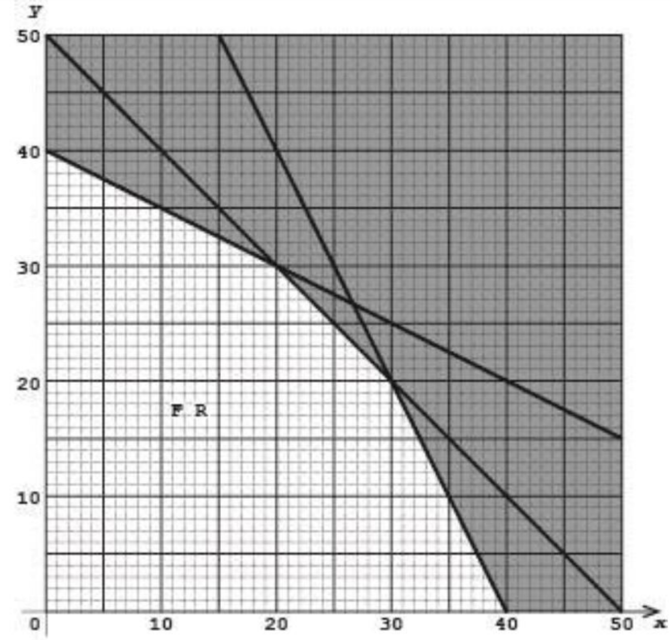
Solution

(a) x: the number of bags sold of Type X
y: the number of bags sold of Type Y

(b) $4x + 4y \leq 200$; $6x + 3y \leq 240$; $3x + 6y \leq 240$; $x, y \geq 0$

(c) Objective function: $P = 20x + 25y$

(d) and (e)



(f) Coordinates of the feasible region: V1 (0, 40), V2 (20, 30), V3 (30, 20), V4 (40, 0), V5 (0, 0)

(g) Maximum profit = $\$1150$

(h) $x = 20, y = 30$

For (a), most candidates did not attain full marks because the variables were stated incorrectly. For (b), most candidates were able to obtain the correct inequalities, but some forgot to state the trivial inequality ($x, y \geq 0$) and others used the wrong inequality. For Part (c), most candidates were able to state the correct objective function.

For Part (d), while most candidates were able to complete this question, some drew the wrong line. For Part (e), most candidates were able to identify the feasible region. For Part (f), while most candidates were able to identify the correct vertices, some omitted $(0, 0)$.

Part (g) was well done except that some candidates did not substitute all their vertices into the profit equation.

Most candidates attempted Part (h) correctly.

Module 2: Probability and Distributions

Question 3

This question tested candidates' ability to do the following:

- Calculate the number of selections of n distinct objects taken r at a time, with or without restrictions.
- Calculate the number of ordered arrangements of n objects taken r at a time, with or without restrictions.
- Calculate probabilities of events which may be combined by unions or intersections using appropriate counting techniques.
- Calculate and use probabilities associated with conditional, independent or mutually exclusive events.
- Apply the properties of the probability density function f of a continuous random variable X.
- Calculate expected value, variance, median and other quartiles.

Solutions

(a) (i) $^{14}C_5 = 2002$

(ii) $^{8}C_3 \times {}^{6}C_2 = 840$

(iii) $\frac{840}{2002} = 0.420$

(b) (i) $^{5}P_3 = 60$

(ii) $^{4}P_2 = 12$

(iii) Probability = $\frac{12}{60} = 0.2$

(c) (i) k = 1/9

(ii) $E(X) = \frac{9}{4}$

(iii) $Var(X) = \frac{27}{80}$

(iv) $P(X > 2) = \frac{19}{27}$

In Part (a) (i), many candidates used permutation instead of combination. In Part (a) (ii), candidates added instead of multiplied.

In Part (a) (ii), many candidates divided their two previous answers, but either divided in the wrong order, or since their previous answers were wrong, many of them got probabilities which were larger than 1. Similarly, in Part (b), candidates confused the permutations and combinations.

In Part (c) (i), some candidates tried to calculate k without using the equation that the total probability = 1, thus $k\int_0^3 x^2\,dx = 1$. Few candidates realized that if they were not able to calculate the value of k, then they could use the value given in the next part of the question. Whereas many candidates were able to calculate the mean in Part (c) (ii), many were not able to calculate the variance. Candidates did not subtract the mean squared or did not subtract at all.

In Part (c) (iii), candidates tried to use the normal distribution to calculate the p $(X > 2)$. Candidates also calculated the probability as if it was discrete.

<u>Question 4</u>

This question tested candidates' ability to do the following:

- Solve problems involving probabilities of the normal distributions using z scores
- Calculate a goodness of fit test using the chi-squared test.

Solutions

(a) (i) $V \sim N(500, 160)$

(ii) $T \sim N(540, 169)$

(iii) $P(T < 510) = 0.0105$

(b) (i) Expected frequencies = 20

(ii) H_0: the number of haircuts is the same each day.
H_1: the number of haircuts is not the same each day.
OR
H_0: the distribution is uniform.
H_1: the distribution is not uniform

$\chi^2 calc = 5.20$
$\chi^2(0.1)(5) = 9.236$
Do not reject H_0.

For Part (a) (i), candidates did not treat the biscuits as independent so many candidates multiplied the variance by 10^2 rather than by 10. In Part (a) (ii), candidates did not add the variances. Some candidates added the standard deviations, some multiplied the variances rather than adding.

In Part (a) (iii), most candidates were able to use the normal distribution to calculate the required probability. However, the common errors of (i) incorrectly subtracting – $(\mu - \bar{x})$ rather than $\overline{(x} - \mu)$ appeared too often and (ii) dividing by the variance rather than the standard deviation to find z.

Some candidates did not correctly use the ϕ conversion for reading the z-tables.

Most candidates did Part (b) very well. The most common error was seen in writing the hypotheses, where candidates wrote the hypotheses in the wrong order. Most candidates wrote the correct number of degrees of freedom and were able to get the correct reading from the tables.

Module 3: Particle Mechanics

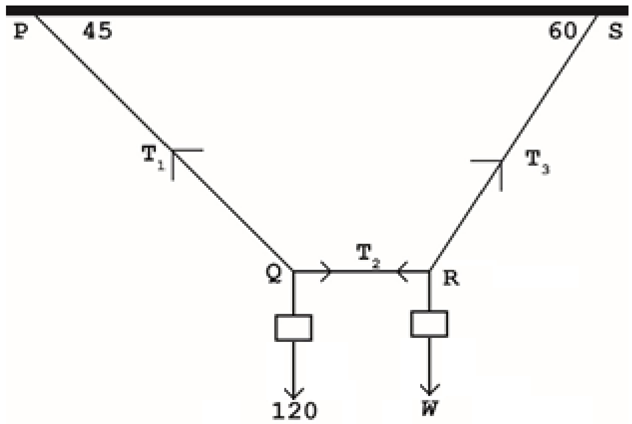
Question 5

This question tested candidates' ability to do the following:

- Identify forces (including gravitational forces) acting on a body in a given situation.
- Represent the contact force between two surfaces in terms of its normal and frictional component.
- Calculate the resultant of two or more coplanar forces.
- Use the principle that for a particle in equilibrium, the vector sum of its forces is zero, (or equivalently the sum of its components in any direction is zero).
- Use the appropriate relationship $F = \mu R$ or $F \leq \mu R$ for two bodies in limiting equilibrium.
- Distinguish between distance and displacement, and speed and velocity.
- Resolve forces, on particles, in mutually perpendicular directions (including those on inclined planes).

Solutions

(a) (i) Clearly labelled diagram showing all the forces (tensions and weights), and the points P, Q, R and S

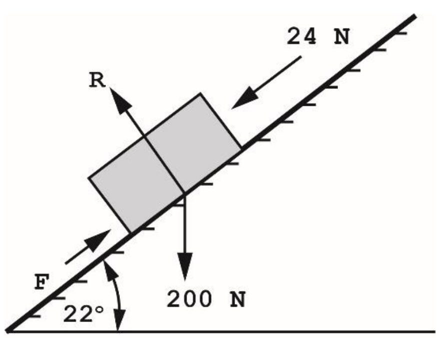


(ii) Tension in string PQ, $T_1 = 120\sqrt{2} = 169.7$ N

Tension in string QR, $T_2 = 120\sqrt{2} \times \frac{1}{\sqrt{2}} = 120\ N$

Weight, $\boldsymbol{w} = 120\sqrt{3}\ N = 207.85$ N

(b) (i) Diagram which illustrates the forces acting on the particle



(ii) The coefficient of friction, μ = 0.533

(iii) The least force which, acting along the slope of the plane, will just cause the particle to move upwards = 173.8 N.

In Part (a) (i), common errors were that candidates did not include all forces on the system, label all the tensions as 'T', label string QR. They also did not draw QR as a horizontal line.

For Part (a) (ii), common errors were that candidates did not resolve nor use Lami's theorem correctly.

For Part (b) (i), common errors were that candidates did not include all forces on the system and the normal reaction. They also incorrectly drew the 24 N acting up the slope instead of down the slope.

In Part (b) (ii), a common error was that candidates equated the frictional force to 24 N and did not consider the component of the weight acting down the slope. For Part (b) (iii), candidates incorrectly resolved parallel to the slope.

Question 6

This question tested candidates' ability to do the following:

- Calculate and use displacement, velocity, acceleration and time in simple equations representing the motion of a particle in a straight line.
- Apply the principle of conservation of linear momentum to the direct impact of two inelastic particles moving in the same straight line. (Knowledge of impulse is required. Problems may involve two-dimensional vectors).
- Calculate the work done by a constant force.
- Solve problems involving kinetic energy and gravitational potential energy.
- Apply the principle of conservation of energy.
- Solve problems involving power.
- Apply Newton's laws of motion to a constant mass moving in a straight line under the action of a constant force.

Solutions

(a) (i) Speed, $v = 1 \text{ ms}^{-1}$
Loss of kinetic energy in the collision = 0.8 *J*

(ii) The speed, *vc*, of the combined particles after the collision = 2 ms^{-1}

(b) (i) Time taken to reach a speed of 30 ms^{-1} = 9 seconds

(ii) Magnitude of F = 9 kN
Work done against the braking force = 810 *kJ*

In Part (a) (i), candidates incorrectly substituted into the formula for impulse to find the speed, *v*, and used the incorrect formula for loss in kinetic energy.

For Part (a) (ii), common errors were that candidates did not use the combined masses of the particles for the momentum after collision, nor did they use '0' as the initial velocity of the stationary particle.

In Part (b) (i), candidates did not use the correct formula, $\frac{1}{2}mv^2 = pt$, to find the time and used '90' to substitute for power in the formula instead of 90×10^3 or *90,000*. For Part (b) (ii), candidates were not able to find the correct acceleration of the truck, and so were unable to find the correct magnitude of the force, F. They also did not find the correct distance travelled by the truck after the engine was disengaged.

Paper 032 – Alternative to the School-Based Assessment

Module 1: Discrete Mathematics

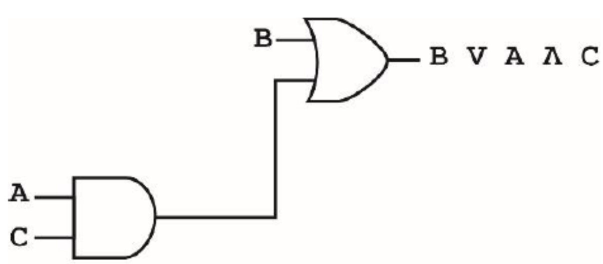
<u>Question 1</u>

This question tested candidates' ability to do the following:

- Use the laws of Boolean algebra to simplify Boolean expressions.
- Represent a Boolean expression by logic circuit.
- Solve a minimum assignment problem by the Hungarian algorithm.

Solutions

(a) (i)



(b) (i)

	A	B	C	D	E
T1	5	0	8	3	3
T2	2	8	19	0	2
T3	10	0	3	7	3
T4	0	14	5	3	8
T5	11	0	21	6	6

	A	B	C	D	E
T1	5	0	5	3	1
T2	2	8	16	0	0
T3	10	0	0	7	1
T4	0	14	2	3	6
T5	11	0	18	6	4

	A	B	C	D	E
T1	5	0	4	2	0
T2	3	9	16	0	0
T3	11	1	0	7	1
T4	0	14	1	2	5
T5	11	0	17	5	3

(ii) Allocation: A – T4, B – T5, C – T3, D – T2, E – T1

Minimum completion time: 27 hours

Most candidates performed poorly in Part (a) (i) as they were unable to use the laws of Boolean algebra correctly to simplify the expression. Part (a) (ii), however, was well done as candidates correctly drew the logic circuit.

Part (b) was well done by most candidates; however, there were some who forgot to reduce the matrix by the column minimum.

Module 2: Probability and Distributions

<u>Question 2</u>

This question tested candidates' ability to do the following:

- Apply the properties $0 \leq P(X = x_i) \leq 1$, and $\sum_{i=1}^{n} P(X = x_i) = 1$ for all x_i
- Calculate and use the expected values and variance of linear combination of independent random variables.
- Calculate and use probabilities associated with conditional events.

Solutions

(a) (i) $p = 0.25$

(ii) $E(X) = 2.35, Var\ (X) = 1.53$

(iii) $P(Y = 3) = 0.135, E(Y + 3) = 7.70, Var(Y + 3) = 6.12$

(b) (i) 0.2

(ii) 0.1

For Parts (a) (i) and (ii), most candidates performed well. However, in Part (a) (iii), most were unable to use the linear combination of independent variables to compute the required mean and variance.

Most candidates performed well in Part (b) (i) as compared to Part (b) (ii). Candidates were unable to equate the conditional probability to that of finding P(BBR).

Module 3: Particle Mechanics

Question 3

This question tested candidates' ability to do the following:

- Resolve forces, on particles, in mutually perpendicular directions (including those on inclined planes).

- Use the equations of motion for a projectile to determine
  (i) the magnitude and direction of the velocity of the particle at any time, t
  (ii) the position of the projectile at any time, t
  (iii) the time of flight and the horizontal range of the projectile
  (iv) the maximum range
  (v) the greatest height.

Solutions

(a) (i) Greatest height the golf ball reaches above the horizontal ground = 50.62 m

(ii) Using $x = ut + \frac{1}{2}at^2$

Vertical motion: $-20 = (35 \sin 45^o)\,t - \frac{1}{2}(10)t^2$

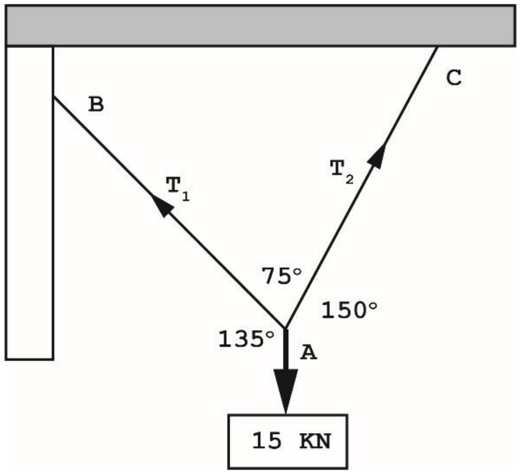
Time taken for the golf ball to travel from P to Q is approximately 5.657 seconds

(iii) Immediately before its impact with the ground at Q

Speed of the golf ball = 40.34 ms^{-1}

Direction of the golf ball = -52.15^o or 127.85^o with the positive *x*-direction

(b) (i)



(ii) Applying Lami's theorem

$$\frac{15\ kN}{\sin 75^{\circ}} = \frac{T_1}{\sin 150^{\circ}} = \frac{T_2}{\sin 135^{\circ}}$$

T_1 = 7.76 kN

T_2 = 10.98 kN

This question was challenging for candidates. Many of them seemed ill-prepared. However, most of those who attempted the question got high marks.

In Part (a) (i), candidates used the correct equation to find the greatest height reached above P, but did not add the height of the cliff (20 m) to get the correct answer. In Part (a) (ii), a common error was that candidates did not use '–20' in the equation for the vertical motion of the projectile.

In Part (a) (iii), candidates used either the vertical or the horizontal component of the velocity for the answer instead of finding the resultant of these two velocities.

For (b) (i), candidates did not insert the correct angles into the diagram.

Paper 031 – School-Based Assessment

This report serves as a summary of observations made during the marking of the SBA. It is hoped that the comments will motivate students so that they can take their research abilities even further than this level. The overall quality of the SBAs received was very good and represented an improvement, when compared with 2017.

Statement of Tasks

The projects highlighted the statement of tasks. Most projects clearly described the plan for carrying out the task. Some projects did not have a definition of variables, in this section or any other part of the project.

Data Collection

This section was very well done, and this is commendable. Students ensured that the type of modelling, experimenting or investigating used in the project was included and mentioned in detail.

Mathematical Analysis

The calculations and analysis employed were accurate, and this is commendable. However, quite a few projects did not carry out enough mathematical analysis, which was not sufficient to make sound conclusions.

Evaluation

Evaluations related to the analysis and this section was done well overall. Students neglected to mention limitations and recommendations and this requires some attention.

Communication

Students communicated information exceptionally well in most of the projects. There was considerable improvement in grammar and spelling. Calculations were followed by analysis and clear explanations.

Further Comments

Strengths

- Good application of chi square
- Use of the z- table to find critical values
- Proper application of the critical path analysis and the Hungarian algorithm
- Improvement seen in writing hypotheses.

Weaknesses

- Confusion regarding appropriate method of analysis relative to tasks
- Using the chi square contingency table to test for relationships instead of independence
- Very few projects used mechanics for investigation.

In some cases, it was evidenced that students were repeating projects that were done in previous years. This is a habit which should be discouraged. It should be noted that whereas CXC accepts group projects, each student in the group would be expected to get the same mark.

Recommendations

- Candidates should practise more problems involving the use of the chi squared tests, normal distribution and mechanics.
- Candidates need to have a comprehensive understanding of the various modules, paying special attention to the mechanics module of Unit 2.
- Teachers should instruct students to cite references to inculcate good research skills from early.
- Candidates should still be encouraged to work independently. Teachers should guide students to choose distinct topics and carry out their own investigations.